中心의 誘電體 막대가 있는 圓筒形空胴의 電磁波吸收

論 文
28～2～3

EM WAVE PENETRATION INTO A CYLINDRICAL CAVITY WITH A CENTER DIELECTRIC-ROD

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Abstract

The penetration of an electro-magnetic wave through an aperture in a cylindrical structure with a center dielectric-rod is investgated. By using a standard mode matching procedure, the electrical and magnetic fields in a cavity are determined as a function of position inside the cavity and frequency of the incident field. For the given parameters, computed data are obtained and the results exhibited in form of amplitude curves of the nor malized field and energy densities of functions of position and frequency. Depending on the increase of the relative dielectric constant of center dielectric-rod, the resonance frequecies of the cavity vary as the cavity size decrease. The stored electromagnetic energy varies very rapidly as a function of position inside the cavity and of the source frequency. Its peak value can be two orders of magnitude greater than the incident energy density. The frequencies where the peaks occur can be identified approximately as the resonance frequencies of the cavity.

I. INTRODUCTION

A problem of considerable interest in EMP(electromagnetic pulse) and biological studies is the penetration of an electromagnetic wave through an aperture into a cylindrical cavity. Unfortunately, it is also a problem of great difficulty with little progress has been made even in the most elementary situation. One of the earliest works on this subject was done by A. Sommerfeld in 1949 [1]. He considered a two dimensional problem where a circular cylinder with a longitu dinal slot is illaminated by a normally ncident plane wave. Using a Fourier analysis, the problem was reduced to that of a system of infinitely many linear equations and it was thought as an unpractical problem. Of course, mordern computer facilities have changed this situation. Related problems of radiation from a slot or an aperture on a cylinder have been even more widely studied and extensive numerical results obtained [2]～[7]. Very recently, the problems of the penetration through an aperture in a cylindrical cavity has been studied [8]～[12].

The purpose of the present paper is to study a problem similar to A. Sommerfeld's but with a center dielectric-rod. As shown in Fig. 1, a cylindrical cavity with a center dielectric-rod is fitted into an infinitely long cylinder. By using a standard mode matching procedure, the problem is formulated and a system of infinitely many

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linear equations derived. These are solved by computer for various truncations in order to determine the number required for consistent results. By varying the relative dielectric constant of the center dielectric-rod, the position inside the cavity and the source frequency, the resonance frequ encies and the stored electromagnetic energy are obtained.

Ⅱ. FIELD REPRESENTATION

Consider a infinitely long conducting cylinder of radius a as shown in Fig. 1. The cylinderhas a center dielectric-rod of radius b and two conducting plates at $z=h$ in its interior. On the surface of the cylinder, a rectangular aperture of dimension $2c\times 2d$ is centered at $x=a,\ y=0,\ z=0$, where the width $2c$ is assumed to be small in terms of wavelength, i.e.,

$$2kc \ll 1 \tag{2.1}$$

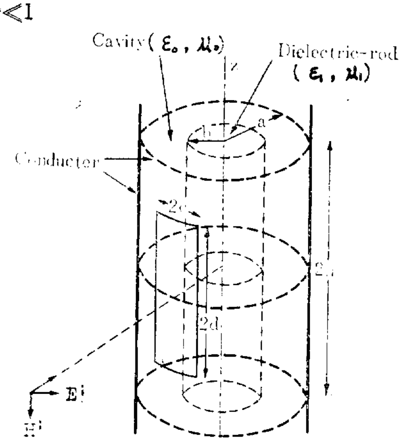

Fig. 1. Geometry of infinitely long conducting cylinder with a coaxial neter dielectricrod which has cavity in its interier and aperture in its surface.

The cylinder is illuminated by a normally incident plane wave described by

$$E=\hat{y}e^{jkx}$$

$$H=-\hat{z}\sqrt{\frac{\epsilon_0}{\mu_0}}e^{jkx} \tag{2.2}$$

where the time factor $\exp(+j\omega t)$ has been dropped.

As usual in problems of this type, we express the fields in terms of TE and TM potential vectors that have only z components

$$TE: F=\hat{z}\Psi \tag{2.3a}$$

$$TM: A=\hat{z}\Psi \tag{2.3b}$$

by the relations

$$E=-\nabla\times(\hat{z}\Psi)+\frac{1}{j\omega\epsilon}\nabla\times\nabla\times(\hat{z}\Psi) \tag{2.4a}$$

$$H=\nabla\times(\hat{z}\Psi)+\frac{1}{j\omega\mu}\nabla\times\nabla\times(\hat{z}\Psi) \tag{2.4b}$$

Because of the assumption in (2.1), (2.2), we may ignore the field variation along in the aperture and only the TE field can exist in cavity. For TE field, expanding Eqs (2.4a), (2.4b) in cylindrical coordinates, the result is

$$E_\rho=-\frac{1}{\rho}\frac{\partial\Psi}{\partial\phi} \qquad H_\rho=\frac{1}{j\omega\mu}\frac{\partial^2\Psi}{\partial\rho\partial z}$$

$$E_\phi=\frac{\partial\Psi}{\partial\rho} \qquad H_\phi=\frac{1}{j\omega\mu\rho}\frac{\partial^2\Psi}{\partial\phi\partial z} \tag{2.5}$$

$$E_z=0 \qquad H_z=\frac{1}{j\omega\mu}\left(k^2+\frac{\partial^2}{\partial z^2}\right)\Psi$$

The scalar Helmholtz equation is

$$(\nabla^2+w^2\mu\epsilon)\Psi=0 \tag{2.6}$$

We can form solution to the Helmholtz equation in cylindrical coordinates as

$$\Psi=\begin{Bmatrix}J_m(\rho k_\rho)\\ Y_m(\rho k_\rho)\end{Bmatrix}\begin{Bmatrix}e^{jm\phi}\\ e^{-jm\phi}\end{Bmatrix}\begin{Bmatrix}e^{jk_z z}\\ e^{-jk_z z}\end{Bmatrix} \tag{2.7}$$

where $k^2=k_\rho{}^2+k_z{}^2$, and $\{J_m, Y_m\}$ are Bessel functions.

We will express the fields in the aperture, inside and outside the cylinder, in terms of appropriate modes with unknown coefficients. By matching the field at the interface $\rho=a$, matrix equations are derived for the unknown coefficients.

First, consider the total field inside the cylinder.Because of the symmetry in incident field and the geometry and the boundary conditions in cavity, the total field E,H everywhere must have following properties;

The symmetry conditions are

i) for $-\pi\leq\phi\leq\pi$, E_ρ : odd E_ϕ : even
H_ρ : even H_ϕ : odd in ϕ (2.8)

ii) for $-h\leq z\leq h$, E_ρ : even E_ϕ : even
H_ρ : odd H_ϕ : odd in z

The boundary conditions in cavity are

i) at $z=\pm h$, $E_\rho=E_{\rho 1}=0$, $E^\phi=E_{\phi 1}=0$
$H_z=H_{z1}=0$

ii) at $\rho=b$, $E_\phi=E_{\phi 1}$, $H_z=H_{z1}$ (2.9)

where subscript 1 denotes the field in dielectric-rod.

From (2.8) and (2.9), we derive

in dielectric-rod($\rho<b$, $|z|<h$)

$$\Psi^1=\sum\sum_{m,n=0}^{\infty} A_{mn}J_m(\rho U_{n1})\cos(m\phi)\cos\frac{(2n+1)\pi}{2h} \tag{2.10a}$$

in cavity ($b<\rho<a$, $|z|<h$)

$$\Psi=\sum\sum_{m,n=0}^{\infty}[B_{1mn}J_m(\rho U_n)+B_{2mn}Y_m(\rho U_n)]\cos(m\phi)\cos\frac{(2n+1)\pi}{2h}z \tag{2.10b}$$

From (2.9), (2.10) and (2.5), we derive

$$\Psi=\sum\sum_{m,n=0}^{\infty}B_{mn}\left[\frac{J_m(\rho U_n)}{X_{mn}^{(1)}}-\frac{Y_m(\rho U_n)}{X_{mn}^{(2)}}\right]\cos(m\phi)\cos\frac{(2n+1)\pi}{2h}z \tag{2.11}$$

where

$$X_{mn}^{(1)}=U_{n1}J_m(bU_{n1})J_m'(bU_n)-U_nJ_m'(bU_{n1})J_m(bU_n)$$
$$X_{mn}^{(2)}=U_{n1}J_m(bU_n)Y_m'(bU_n)-U_nJ_m'(bU_{n1})Y_m(bU_n) \tag{2.12a}$$

$$k^2=w^2\mu_0\epsilon_0$$
$$k_1^2=w^2\mu_1\epsilon_1 \tag{2.12b}$$

$$U_n=\begin{cases}+\sqrt{k^2-\left(\frac{(2n+1)\pi}{2h}\right)^2}, & \text{if } k\geq\frac{(2n+1)\pi}{2h}\\ -j\sqrt{\left(\frac{(2n+1)\pi}{2h}\right)^2-k^2}, & \text{if } k<\frac{(2n+1)\pi}{2h}\end{cases}$$

$$U_{n1}=\begin{cases}+\sqrt{k_1^2-\left(\frac{(2n+1)\pi}{2h}\right)^2} & \text{if } k_1\geq\frac{(2n+1)\pi}{2h}\\ -j\sqrt{\left(\frac{(2n+1)\pi}{2h}\right)_2-k_1^2} & \text{if } k_1<\frac{(2n+1)\pi}{2h}\end{cases} \tag{2.12c}$$

the constant $\{B_{mn}\}$ are to be determined.

From (2.11) and (2.5), we have

$$E_\phi=\sum\sum_{m,n=0}^{\infty}B_{mn}\left[\frac{J_m'(\rho U_n)}{X_{mn}}-\frac{Y_m'(\rho U_n)}{X_{mn}^{(2)}}\right]\cos(m\phi)\cos\frac{(2n+1)\pi}{2h}z \tag{2.13a}$$

$$H_z=\frac{1}{jw\mu_0}\sum\sum_{m,n=0}^{\infty}B_{mn}U_n^2\left[\frac{J_m(\rho U_n)}{X_{mn}^{(1)}}-\frac{Y_m(\rho U_n)}{X_{mn}^{(2)}}\right]\cos(m\phi)\cdot\cos\frac{(2n+1)\pi}{2h}z \tag{2.13b}$$

To represent the field in aperture, it is most convenient to regard, that tactically, the cylinder wall has a small thickness such that the aperture is a rectangular waveguide with transverse dimension $2c\times2d$, The potential for the TE field may be represented by

$$\Psi=\lim_{\rho\to a}\sum_{n=0}^{\infty}\cos\frac{(2tn+1)\pi}{2d}z[\bar{C}_n\cos\gamma_n(\rho-a)+\bar{D}_n\sin\gamma_n(\rho-a)] \tag{2.14}$$

If (2.14) is substituted into (2.5), the tangential components of the aperture field can be readily calculated with the results, valid for

$\rho=a$, $|\phi|\leq\frac{c}{a}$, $|z|\leq d$,

$$E_\phi=\sum_{n=0}^{\infty}D_n\cos\frac{(2n+1)\pi}{2d}z \tag{2.15a}$$

$$H_z=\sqrt{\frac{\epsilon_0}{\mu_0}}\sum_{n=0}^{\infty}C_n\cos\frac{(2n+1)\pi}{2d}z \tag{2.15b}$$

where $\{C_n\}$ and $\{D_n\}$ are unknown constants to be determined.

Finally, consider the field representation outside the cylinder. We define the total field there as

$$\Psi=\Psi^i+\Psi^r+\Psi^s \tag{2.16}$$

The incident field potential Ψ^i is derived from (2.2) by the Jacobi-Anger expansion

$$\Psi^i=\frac{1}{jk}\exp(jk\rho\cos\phi)=-\frac{1}{jk}\sum_{m=-\infty}^{\infty}j^m J_m(k\rho)\exp(jm\phi) \tag{2.17}$$

Ψ^r is the reflected field from the cylinder at $\rho=a$ when the aperture is closed ($c=d=0$), and may be derived from boundary condition at $\rho=a$

$$\Psi^v=-\frac{2}{jk}\sum_{m=0}^{\infty}\frac{1}{\epsilon_m}j^m\frac{J_m'(ak)}{H_m^{(2)\prime}(ak)}H_m^{(2)}(\rho k)\cos(m\phi) \tag{2.18}$$

where $\epsilon_m=\begin{cases}2 & \text{if } m=0\\ 1 & \text{if } m\neq0\end{cases}$

The third term Ψ^s in (2.16) is the scattered field. By applying a Fourier integral instead of discrete modes, namely,

$$\Psi^s=\sum_{m=0}^{\infty}\cos(m\phi)\int_{-\infty}^{\infty}e^{j\alpha z}F_m(\alpha)H_m^{(2)}(\rho\beta)d\alpha \tag{2.19}$$

where

$$\beta=\begin{cases}+\sqrt{k^2-\alpha^2}, & \text{if } k\geq\alpha\\ -j\sqrt{\alpha^2-k,} & \text{if } k<\alpha\end{cases}$$

The function $\{F_m(\alpha)\}$ are to be determined.

From (2.16), (2.19) and (2.5), we derive

$$E_\phi=\sum_{m=0}^{\infty}\beta\cos(m\phi)\int_{-\infty}^{\infty}e^{j\alpha z}F_m(\alpha)H_m^{(2)\prime}(\rho\beta)d\alpha$$

(2.20a)

$$H_z=\frac{1}{jw\mu_0}\Big[\frac{k}{j}e^{jk\rho\cos\phi}-\frac{2k}{j}\sum_{m=0}^{\infty}\frac{1}{\epsilon_m}j^m\cos(m\phi)$$

$$\frac{J_m'(ak)}{H_m^{(2)\prime}(ak)}H_m^{(2)}(\rho k)+\sum_{m=0}^{\infty}\beta^2\cos(m\phi)\int_{-\infty}^{\infty}e^{j\alpha z}$$

$$F_m(\alpha)H_m^{(2)}(\rho\beta)d\alpha\Big] \qquad (2.20b)$$

Ⅲ. FIELD MATCHING

We expressed the fields in the aperture, inside and outside the cylinder, in terms of appropriate modes with unknown coefficients. By assumption (2.1), (2.2), H_ϕ at $\rho=a^+$ and that at $\rho=a^-$ are nearly zero, thus, we can match the field E_ϕ, H_z at $\rho=a$, and the unknown coefficients can be solved.

The matching of E_ϕ and H_z at $\rho=a$ is carried out by Galerkin's method [5]. Consider first the matching of E_ϕ at $\rho=a$. From (2.13a) and (2.15a) we find

$$\sum_{m,n=0}^{\infty}\sum B_{mn}U_n\Big[\frac{J_m'(\rho U_n)}{X_{mn}^{(1)}}-\frac{Y_m'(\rho U_n)}{X_{mn}^{(2)}}\Big]\cos(m\phi)$$

$$\cos\frac{(2n+1)\pi}{2h}z=\Omega(r)\sum_{n=0}^{\infty}D_n\cos\frac{(2n+1)\pi}{2d}z \qquad (3.1)$$

$-\pi\leq\phi\leq\pi,\ -h\leq z\leq h$

where $\Omega(r)$ is a characteristic function of the aperture.

$$\Omega(r)=\begin{cases}1 & \text{if } r \text{ is in the aperture}\\ 0 & \text{otherwise}\end{cases} \qquad (3.2)$$

Let us multiply the following operater to both side of (3.1)

$$\Big(\frac{1}{\pi\epsilon_p}\int_{-\pi}^{\pi}d\phi\cos(p\phi)\Big)\Big(\frac{1}{h}\int_{-h}^{h}dz\cos\frac{(2q+1)\pi}{2h}z\Big)$$

This manipulation leads to

$$B_{pq}U_q\Big[\frac{J_p'(aU_q)}{X_{pq}^{(1)}}-\frac{Y_p'(aU_q)}{X_{pq}^{(2)}}\Big]=\sum_{n=0}^{\infty}D_n$$

$$\frac{2\sin(pc/a)}{p\pi\epsilon_p}\ \frac{\Delta_{nq}}{h} \qquad p,q=0,1,2\cdots \qquad (3.3a)$$

where

$$\Delta_{nq}=\frac{(2n+1)\pi(-1)^n\Big[\cos\Big(\frac{2q+1}{2}\Big)\frac{\pi d}{h}\Big]}{d\Big\{\Big(\frac{(2n+1)}{2d}\pi\Big)^2-\Big(\frac{(2q+1)}{2h}\pi\Big)^2\Big\}} \qquad (3.3b)$$

Similarly, matching of H_z at $\rho=a^-$ and E_ϕ, H_z at $\rho=a^+$, we find

$$\frac{1}{jk}\sum_{m,n=0}^{\infty}\sum B_{mn}U_n^2\Big[\frac{J_m(aU_n)}{X_{mn}^{(1)}}-\frac{Y_m(aU_n)}{X_{mn}^{(2)}}\Big]$$

$$\frac{\sin(mc/a)}{mc/a}\ \frac{\Delta_{qn}}{d}=C_q$$

$$q=0,1,2\cdots\cdots \qquad (3.4)$$

$$\beta\, F_p(\alpha)H_p^{(2)\prime}(\alpha\beta)=\sum_{n=0}^{\infty}D_n\frac{\sin(pc/a)}{p\pi^2\epsilon_p}J_n(\alpha)$$

$$p=0,1,2\cdots\cdots \qquad (3.5)$$

$$\Big[-\frac{1}{2c/a}\int_{-c/a}^{c/a}\exp(jka\cos\phi)d\phi+\sum_{m=0}^{\infty}\frac{2}{\epsilon_m}j^m$$

$$\frac{J_m'(ak)}{H_m^{(2)\prime}(ak)}H_m^{(2)}(ak)\frac{\sin(mc/a)}{mc/a}\Big]\frac{4(-1)^q}{(2q+1)\pi}$$

$$+\frac{1}{jk}\sum_{m=0}^{\infty}\frac{\sin(mc/a)}{mc/a}\ \frac{1}{d}\int_{-\infty}^{\infty}d\alpha\ J_q(\alpha)$$

$$\beta^2F_m(\alpha)H_m^{(2)}(\alpha\beta)=C_q \quad q=0,1,2,\cdots\cdots \qquad (3.6)$$

where

$$J_n(\alpha)=\frac{-\pi(2n+1)(-1)^n\cos(\alpha d)}{\alpha^2-[\pi(2n+1)/2d]^2}\ \frac{1}{d} \qquad (3.7)$$

From the four sets of equations (3.3), (3.4), (3.5) and (3.6), we can derive a set of equations whose only unknowns are $\{D_n\}$. The result is

$$\sum_{n=0}^{\infty}\lambda_{mn}D_n=I_m \qquad m=0,1,2,\cdots\cdots \qquad (3.8)$$

$$\lambda_{mn}=\lambda_{mn}^{(1)}+\lambda_{mn}^{(2)} \qquad (3.9a)$$

$$\lambda_{mn}^{(1)}=\frac{2c/a}{d}\int_0^{\infty}d\alpha\beta J_m(\alpha)J_n(\alpha)$$

$$\Big\{\sum_{l=0}^{\infty}\Big(\frac{\sin(lc/a)}{lc/a}\Big)\cdot\frac{H_l^{(2)}(a\beta)}{\epsilon_lH_l^{(2)\prime}(a\beta)}\Big\} \qquad (3.9b)$$

$$\lambda_{mn}^{(2)}=\frac{-2c/a}{d\pi h}\sum_{r=0}^{\infty}U_r\Delta_{mr}\Delta_{nr}\Big\{\sum_{s=0}^{\infty}\Big(\frac{\sin(sc/a)}{sc/a}\Big)^2\frac{1}{\epsilon_s}$$

$$\Big(\frac{J_s(aU_r)X_{sr}^{(2)}-Y_s(aU_r)X_{sr}^{(1)}}{J_s'(aU_r)X_{sr}^{(2)}-Y_s'(aU_r)X_{sr}^{(1)}}\Big)\Big\} \qquad (3.9c)$$

$$I_m=\frac{4(-1)jk}{(2m+1)\pi}\Big\{\frac{1}{2c/a}\int_{-c/a}^{c/a}\exp(jka\cos\phi)\,d\phi$$

$$-\sum_{n=0}^{\infty}\frac{2}{\epsilon_m}j^n\frac{J_n'(ak)}{H_n^{(2)\prime}(ak)}H_n^{(2)}(ak)\frac{\sin(nc/a)}{nc/a}\Big\}$$

(3.10)

The infinite set of linear equations in (3.8) will be truncated at an appropriate number by moment method [4]. Once $\{D_n\}$ are solved from (3.8), they may be used to calculate $\{B_{mn}\}$ from (3.3) and then the field E_ϕ, H_z inside the cylinder from (2.13)

$$E_\phi=\sum_{m,n=0}^{\infty}\sum\left[\frac{J_m'(\rho U_n)X_{mn}^{(2)}-Y_m'(\rho U_n)X_{mn}^{(1)}}{J_m'(aU_n)X_{mn}^{(2)}-Y_m'(aU_n)X_{mn}^{(1)}}\right]$$
$$\cos(m\phi)\cos\frac{(2n+1)\pi}{2h}z\cdot\frac{\sin(mc/a)}{mc/a}$$
$$\frac{2c}{a\pi h\epsilon_m}\sum_{l=0}^{\infty}D_l\Delta_{ln} \qquad (3\cdot 11a)$$

$$H_z=\frac{1}{jw\mu_0}\sum_{m,n=0}^{\infty}\sum U_n$$
$$\left[\frac{J_m(\rho U_n)X_{mn}^{(2)}-Y_m(\rho U_n)X_{mn}^{(1)}}{J_m'(aU_n)X_{mn}^{(2)}-Y_m'(aU_n)X_{mn}^{(1)}}\right]$$
$$\cos(m\phi)\cdot\cos\frac{(2n+1)\pi}{2h}z\cdot\frac{\sin(mc/a)}{mc/a}\ \frac{2c}{a\pi h\epsilon_m}$$
$$\sum_{l=0}^{\infty}D_l\Delta_{ln} \qquad (3.11b)$$

As shown by (3.11), the E_ϕ and H_z become in finite at a set discrete frequencies $k=\{k_{pq}\}$, where k_{pq} is the root of the equation.

$$J_m'(aU_n)X_{mn}^{(2)}-Y_m'(aU_n)X_{mn}^{(1)}=0 \qquad (3.12)$$

We identify $\{k_{pq}\}$ as the resonance frequencies of TE modes in the cavity when the aperture is absent. To circumvent this difficulty of infintie (3.8) and (3.11), (3.8) must be modified in the manner descrived below. At an exact resonance frequency $k=k_{pq}$, (3.4) is rewritten as

$$\frac{1}{jk}\sum_{\substack{m=0\\m\neq p}}^{\infty}\sum_{\substack{n=0\\n\neq q}}^{\infty}B_{mn}U_n^2\left[\frac{J_m(aU_n)}{X_{mn}^{(1)}}-\frac{Y_m(aU_n)}{X_{mn}^{(2)}}\right]$$
$$\frac{\sin(mc/a)}{mc/a}\ \frac{\Delta_{ln}}{d}+\frac{1}{jk}B_{pq}U_q^2$$
$$\left[\frac{J_p(aU_q)}{X_{pq}^{(1)}}-\frac{Y_p(aU_q)}{X_{pq}^{(2)}}\right]\frac{\sin(pc/a)}{pc/a}\ \frac{\Delta_{lq}}{d}=C_l$$
$$l=0,1,2,\cdots\cdots \qquad (3.13)$$

From (3.13) and (3.3), we can derive

$$\sum_{n=0}^{\infty}\lambda_{mn}D_n+\eta_mB_{pq}=I_m \quad m=0,1,2,\cdots\cdots \qquad (3.14)$$

where

$$\lambda_{mn}=\lambda_{mn}^{(1)}+\lambda_{mn}^{(2)} \qquad (3.15a)$$
$$\lambda_{mn}^{(2)}=\frac{-2c/a}{d\pi h}\sum_{\substack{r=0\\r\neq q}}^{\infty}\sum_{\substack{s=0\\s\neq p}}^{\infty}U_r\Delta_{mr}\Delta_n\left[\frac{\sin(sc/a)}{sc/a}\right]^2\frac{1}{\epsilon_s}$$
$$\cdot\left[\frac{J_s(aU_r)X_{sr}^{(2)}-Y_s(aU_r)X_{sr}^{(1)}}{J_s'(aU_r)X_{sr}^{(2)}-Y_s'(aU_r)X_{sr}^{(1)}}\right] \qquad (3.15b)$$
$$\eta_m=-U_q^2\left[\frac{J_p(aU_q)}{X_{pq}^{(1)}}-\frac{Y_p(aU_q)}{X_{pq}^{(2)}}\right]\frac{\sin(pc/a)}{pc/a}\ \frac{\Delta_{nq}}{d}$$
$$(3.16)$$

Since the denominator of (3.15b) no longer contains a factor given on the left hand side of (3.12). The set of linear equations in (3.14) now replace (3.8). From the (3.14), $\{D_n\}$ and B_{pq} are sloved, while the remaining $\{B_{mn}\}$ are next calculated from (3.3).

Ⅳ. COMPUTED DATA AND DISCUSSION

Now, we will present a set of data of the energy density distribution inside the cavity calculated with the following parameters.

$a=3.97$cm　$b=1.57$cm　$h=15.03$cm
$c=0.183$cm　$d=2.93$cm　　(4.1)

For numerical computations, we truncate (3.10) at $n=5$ ($n=4$, or $n=5$ is sufficient [11]).

With a specific set of values for parameters (4.1), two sets of resonance frequencies are found. One set of the resonance frequencies of the (TE_p modes in the cavity, the other set includes all of the resonance frequencies in the aperture.

i) Resonance frequency of aperture

The resonance frequencies of the aperture is defined by

$$f_m=\frac{(2m-1)c_0}{4d} \quad m=1,2,\cdots\cdots \qquad (4.2)$$

where c_0 is the velocity of light in free space. By given parameters of aperture the resonance frequencies are calculated as follows;

$f_1=2.56$Ghz
$f_2=7.68$Ghz
:

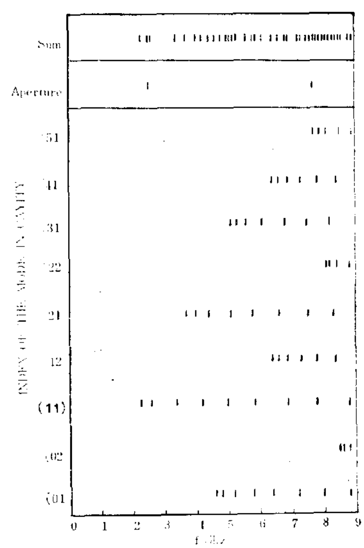

Fig. 2. Resonance frequency spectrum of aperture-cavity configuration in Fig. 1 with its parameter specified in (4.1). $\varepsilon_r=1$, $\mu_r=1$ (dielectric-rod).

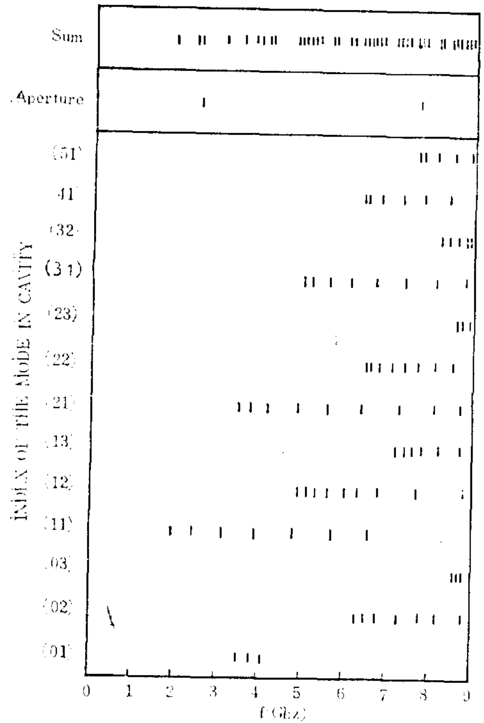

Fig. 3. Resonance frequency spectrum of aperture-cavity configuration in Fig. 1 with its parameter specified in (4.1). $\epsilon_r=4$, $\mu_r=1$ (dielectric-rod).

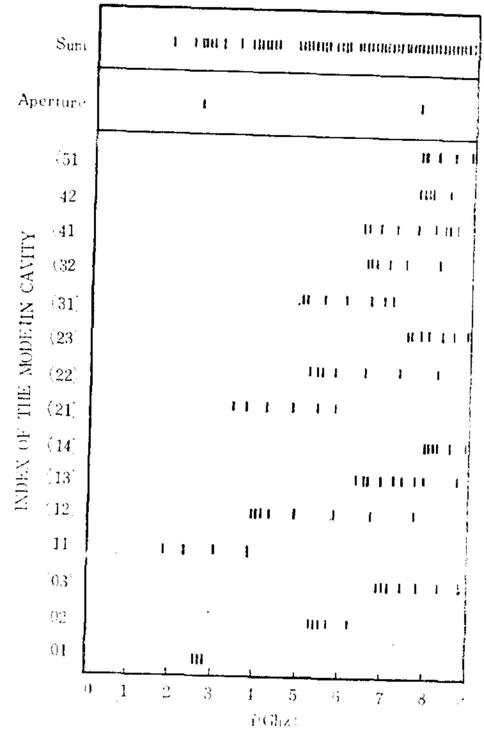

Fig. 4. Resonance frequency spectrum of aperture-cavity configuration in Fig. 1 with its parameter specifed in (4.1). $\epsilon_r=8$, $\mu_r=1$ (dielectric-rod).

ii) Resonance frequency of cavity

From (3.12) the resonance frequencies of cavity are caculated

$$J_m'(aU_n)X_{mn}^{(2)}-Y_m'(aU_n)X_{mn}^{(1)}=0 \qquad (3.12)$$

In Fig. 2, the resonance frequencies of the first nine TE modes are desplayed. The frequencies next to [21], for example, are those which satisfy (3.12) with $p=2$ and $q=1$. As shownin Fig. 2~Fig. 4, depending on the increase of the relative dielectric constant ϵ_r of center dielectric-rod, the resonance frequencies of cavity vary as the cavity size decrease.

iii) When input parameters are $f=2.9$ Ghz, $\epsilon_r=4$ (dielectricrod) and (4.1), normalized electric field E_ϕ inside the cavity is plotted as a function of ρ in Fig. 5. The electric field decrease rapidly away from the aperture. In Fig. 6 and Fig. 7 normalized electric field E_ϕ is plotted as a function of ϕ and z, the peak values of E_ϕ are occured in aperture region.

iv) In Fig. 8~Fig. 12, the normalized electric and magnetic stored energy densities are presented as functions of source frequency f (2~4Ghz) for five different locations inside the cavity. Normalized energy densities defined by

$$W_e(r,f)=\frac{|E|^2}{|E^i|^2}$$

$$W_m(r,f)=\frac{|H|^2}{|H_i|^2} \qquad (4.3)$$

As a function of the source frequency, W_e and W_m assume local peaks approximately at the resonance frequencies listed in Fig. 3 (resonance freq. of cavity : 1.92, 2.35, 3.06, 3.40, 3.51, 3.77, 3.89 Ghz, resonance freq. of aperture : 2.56Ghz). This phenomenon is clearly demonstrated in Fig 8. Their values of resonance frequencies are close. Thus, the peak values of W_e and W_m as function of the frequency, apear approximately at the resonance frequencies of the cavity.

For f between 2 and 4Ghz, the absolute peak of W_e is about 312 (at $f=3.60$Ghz in Fig. 8).

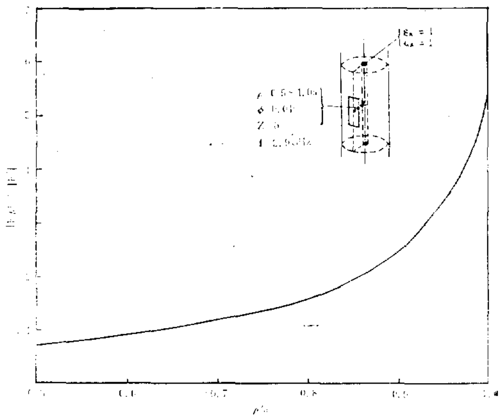

Fig. 5. Electric field E_ϕ inside cavity as function of ρ

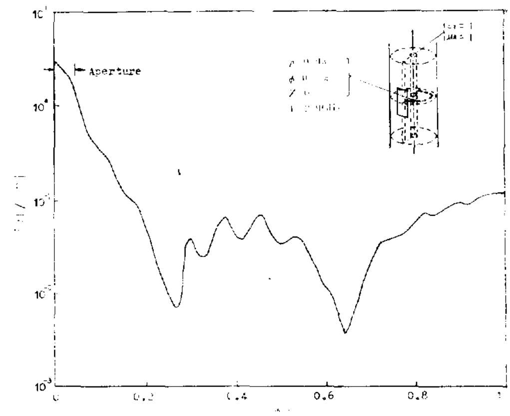

Fig. 6. Electric field E_ϕ inside cavity as function of ϕ

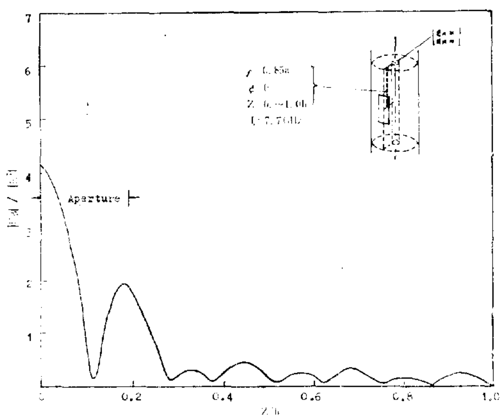

Fig. Electric field E_ϕ inside cavíty as fuction of z

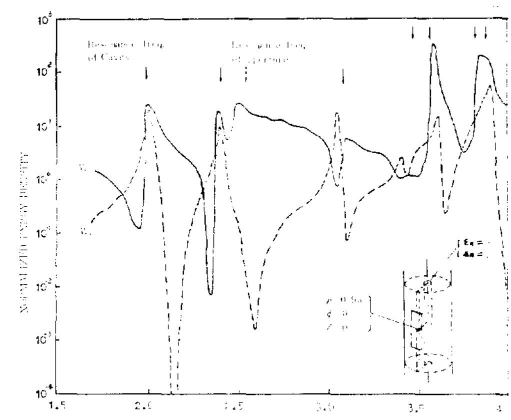

Fig. 8. Normalized energy densities inside cavity due to incident field in (2.2) for parameters given in(4.).

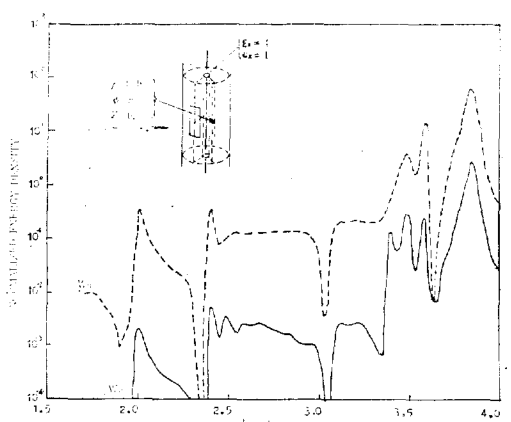

Fig. 9. Normalized energy densities inside cavity due to incident field in (2.2) for parameters given in(4.1).

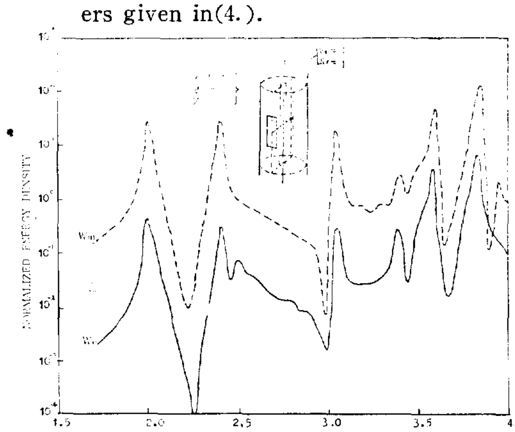

Fig. 10. Normalized energy densities inside cavity due to incident field in (2.2) for parameters given in(4.1).

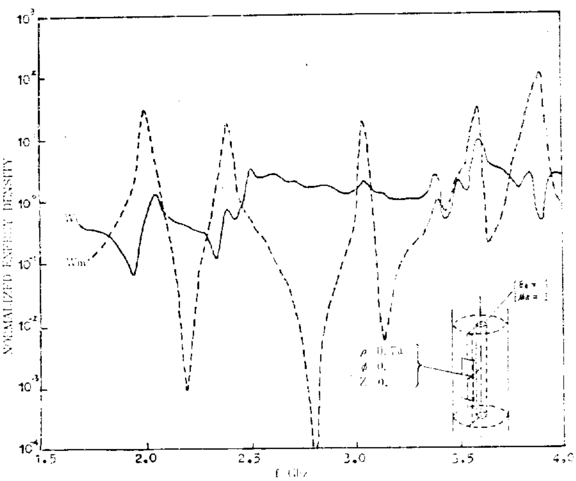

Fig. 11. Normalized energy densities inside cavity due to incident field in(2.2) for parameters given in(4.1).

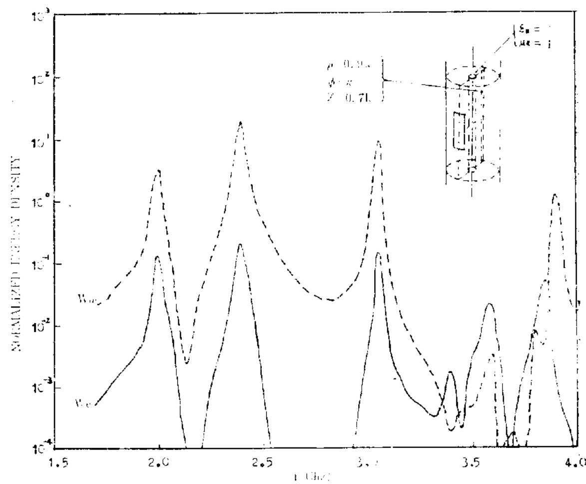

Fig. 12. Normalized energy densities inside cavity due to incident field in(2.2) for parameters given in(4.1).

Thus the energy density inside the cavity can be two orders of magnitude larger than the incident one and can be larger than the magnitude without dielectric-rod[6](about 97 [12]).

A study of Fig. 8 through Fig. 12 shows that the stored energy is a rapidly varying function of position inside the cavity and of the source frequency. It should be remarked that, in construction of those figures, W_e and W_m are calculated only at discrete frequencies(with an increment of 0.5 Ghz). Therefore, those figures show only general variations of energy densities versus frequency, but not the fine details. Fortunately, this calculation can be done with a reasonable amount of effort and computer time, as demonstrated in this paper.

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