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A Mathematical Model of a Central District Heating System for an Urban Residential Community

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Abstract

A mathematical model is developed in order to describe the network configuration and heating distribution to a Central District Heating System for an Urban Residential Community.

The purpose of using this model is to optimize operating costs and to distribute heat to the Residential Community efficiently. In particular, because of the inherent nonlinearity and dual optimization of the problem a dyamic programming approach is taken.

It is turned out that the optimal cost of the system is a strong non-linear function of the network. In particular, it is found that increasing N, the number of houses, may not necessarily imply increased costs. It is felt that past failure of producing economical systems may be due to the improper attention given to the network.

Introduction

In this paper the concept of lower-cost housing is extended to include lower operating costs as well as construction costs. Specifically, the operating costs related to heating the housing unit are investigated. These costs comprise the major operating costs and hence their minimization contributes to the efforts of making lower-cost housing a reality in the long-run, especially in the light of increasing fuel costs.

In particular, it is felt that this objective could be approached by utilizing "Central District Heating Systems" (CDHS) more extensively. The term 'District Heating' refers to supplying heat from a central power station to groupbuildings. The advantages of district heating are better combustion efficiency with less expensive fuel, reduction of maintenance costs per unit of output, and a decrease in the overall investment reguired if seperate heating units were to be installed in each housing unit. In the past, some case studies of residential communities utilizing CDHS have shown that such an endeavor can be sucussful in reducing costs and meeting the financial satisfaction of its consumers. But some case studies have contradicted these results. It is the purpose of this paper, because of the large investment and benefits involued in implementing

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CDHS, to investigate why some systems have succeeded and other failed in past, and also in what circumstances it is beneficial to implement such a system?

The model is basically one which results in a central power plant distributing the required heat to the housing units through the network of pipes.

A Computer Simulation Model for the CDHS has developed and concentrated on minimizing the cost of transporting the heat. This particular cost related to a reasonable fraction of the operating costs of a CDHS, and it is felt that this cost can be controlled by optimally planning the heat transportation network.

The optimization of the transportation network is determined from the spatial design of the community by inputting the relative distance between the central plant and the housing units. The heat distribution and related costs are assumed to be a function of these distances.

This simple model of a CDHS can then be analyzed for minimal operating costs and sensitivity to the network configuration. The opmitization may be approached by the methods of dynamic programming once the above assumptions are made. This modelling technique has also applied to residential communities with non-residential buildings in order to further decrease the operating costs per residential buildings.

The Basic Model of a Central District Heating Systems

The system is comprised of these components: (a) the housing units which will receive the heat, (b) a pipeline network which will distribute the heat, and (c) a central heating plant which generates the heat (See Figure 1).

Figure 2 shows the relative interactions of these components.

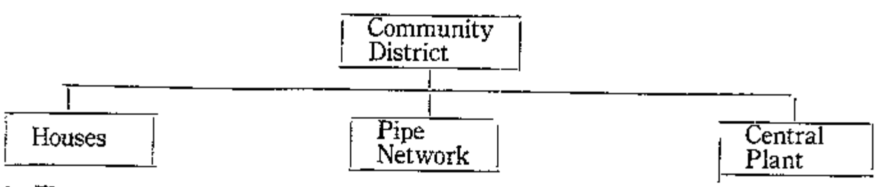

Figure 1 : The components of a community district in terms of a central district heating system.

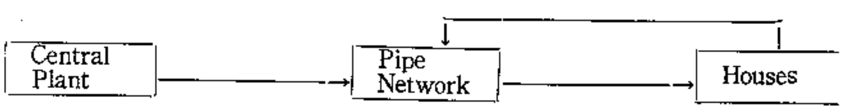

Figure 2 : The relative interactions of the components of a central district heating system.

The cost related to this system is C_1, the cost of supplying power from the central plant, C_2, the cost of heating each housing unit to a required amount, and C_3, the cost of distributing the heat from the central plant to the houses and among the houses. The cost of producing the heat, C_1 is fixed once the total amount of power required is known.

Likewise the costs related to heating each house to a required level is also fixed. It is the cost, C_3, related to the particular distribution network configuration which will vary depending on the network configuration chosen to connected the central plant to the housing units. It is then hoped that the total cost of the CDHS may be minimized by finding the optimal network configuration.

Suppose that the cost of the systems is related to a sum of the costs of producing a certain amount of power, heating N houses, and the heat losses between nodes, repectively:

$$C_1=f(S_p)$$

$$C_2=g\left(\sum_{i=1}^{N}\sum_{j=1}^{N} H_{ij}^{R}\right), \text{ and}$$

$$C_3=h\left(\sum_{i=1}^{N}\sum_{j=1}^{N} H_{ij}^{L}\right)$$

Where S_p is the power output of the central plant, H_{ij}^{R} is the heat required by housing unit located at (i, j) and H_{ij}^{L} is the heat loss when going from a node (i) to a node (j)

Then the objective function can be stated as follows:

$$\text{Minimize } (C_1+C_2+C_3) \tag{1}$$

Subject to

$$\sum_{i=1}^{N}\sum_{j=1}^{N} H_{ij}^{R}+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} H_{ij}^{L} \leq S_p \tag{2}$$

The constraint (Eq. 2) merely states that the power station must supply at least enough heat for all the houses including the heat loss that is incurred as each house is serviced. As previously mentioned the C_1 and C_2 of the objective function (Eq.1) can be considered fixed and determinable by other methods (cf. (1), (3)), but C_3 depends on the particular network configuration and distribution of heat. Since C_3 may be a nonlinear function of heat loss, a dynamic programming approach is most applicable.* Once the problem is formulated as a dynamic programming problem the simulation is relatively straight forward and its generality makes it applicable to all functional reationships of cost and heat loss as a function of distance. The following sections will be devoted to the dynamic programming formulation and on heuristic procedure which can be used to carry out the simulation.

Furthermore, the following conditions must be met in order to develop the model.

i) There will be one central heating station whose location is fixed at node (1, 1). **

ii) A grid of housing units with coordinates (i, j) must be supplied external to the model with the total amount of heat which each house requires (H_{ij}^{R}).

iii) A functional representation of the total heat loss related to transporting the heat from node (i) to node (j) expressed as a function of distance.

iv) Each housing unit may act as a source or sink for every other housing unit, but the central station will be considered only as a source.

The Dynamic Programming Approach

In order to apply the dynamic programming method to the model it must be defined the necessary stages of the simulation. Each stage can be taken from the view point of each node measuring its respective relationship to all other (N−1) nodes. This is implicitly done when solving for the minimal spanning network (see the Appendix).

* For a discussion of dynamic programming supply-demand models see reference 4.

** Economic studies of CDHS, have shown that the economical location of the plant will be near the distribution center in order to simplify and shorten the distribution piping. Actually, after an optimal distribution is determined, the location of the central station may also be varied and optimized for least relative to the entire distribution of houses. But this is beyond the scope of this present paper.

The distribution of heat over a particular link is more explicitly decided, because it depends only on the allocation of heat on the previous link. The cost of the making of these decisions is then simply determined, once knowing where to send the heat and how much heat to send.

Let

X_{ij}=the heat transferred from node i to node j.

d_{ij}=the distance between node i and j.

C_{ij}=the cost of transporting heat between node i and node j.

ϕ_{ij}=the probability the link between node i and node j is included in the network.*

Then the objective function is given as follows:

$$\text{Min}\left(\sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij}\ (d_{ij})\ C_{ij}\ (d_{ij})\ \phi_{ij}\right) \tag{3}$$

Subject to

$$X_{ij} \geq H_{ij}^{R} + H_{ij}^{L} \tag{4a}$$

$$C_{ij} \geq 0 \tag{4b}$$

$$\phi_{ij} = \begin{cases} 0, & if\ X_{ij}=0 \\ 1, & if\ X_{ij} \neq 0 \end{cases} \tag{4c}$$

where $i=1, 2, \cdots\cdots\cdots, N$, and $j=1, 2, \cdots\cdots\cdots, N$

Hence, the dynamic recursive equations for each state are given as follows:

$$f_{rj}(X_{ij}) = \underset{\text{network}}{\text{Min}} \{C_{ij}X_{ij}d_{ij} + f_{(r-1)j}\ [V_{ij}(X_{ij}, d_{ij})]\} \tag{5a}$$

$$r=1, 2, \cdots\cdots\cdots, (N-1)$$

and

$$f_{oj}=0 \text{ for all } j \tag{5b}$$

The cost at each stage is determined by,

$$f_r = \sum_{j=1}^{N} f_{rj} \tag{5c}$$

The state equation for the distribution is,

$$(X_{ij})_{t_l} = V_{ij}(X_{ij},\ \phi_{ij})$$

$$= \left\{ (X_{ij})_{ti} + \left(\sum_{m=1}^{N} X_{jm}\phi_{jm}\right)_{ti} \right\}(\phi_{ij})_{ti} \tag{6}$$

where t_l is the period immediately after t_i.

The state eqation for the minimization of the network can not be explicitly written but is implicitly followed when solving the minimal spanning network. In particular the set of all links included in the minimal spanning network at stage i depends on the links which were in set at $(i-1)$ or

$$S_i = S_{i-1} \cup e_i \tag{7}$$

The reader to referred to the Appendix for more discussion.

The Heuristic Approach for the Computer Simulation

Since the amount of heat transferred, X_{ij}, and cost C_{ij}, are a function of the link connecting

* For the following discussion ϕ_{ij} is taken to be either 0 or 1 depending on if the link is not included in the network or is included in the network respectively.

node(i) and node (j). the minimization over the network is solved by the use of a shortest spanning tree algorithm described in the Appendix. The spanning tree is then described uniquely by a decision matrix, ϕ, with elements ϕ_{ij}. The elements ϕ_{ij} take on the value 1 or 0 depending whether the link is included in minimal spanning netework or not, respectively. The procedure used in solving the dynamic program is given in Fig. (3). The step begins by solving the minimal spanning tree as given in the appendix and determining the decision matrix, ϕ. And then allocate the amount of heat for each of the nodes included in the last row of the decision matrix. This corresponds to picking out the links which are at most the last link which is their branch. The procedure then proceed to the (N−1) row of the decision matrix.

Here, for $\phi_{ij}=1$, the program picks up the links that have at most one link attached to it. This continues until row 1 of the decision matrix which has links connected to at most (N−1)

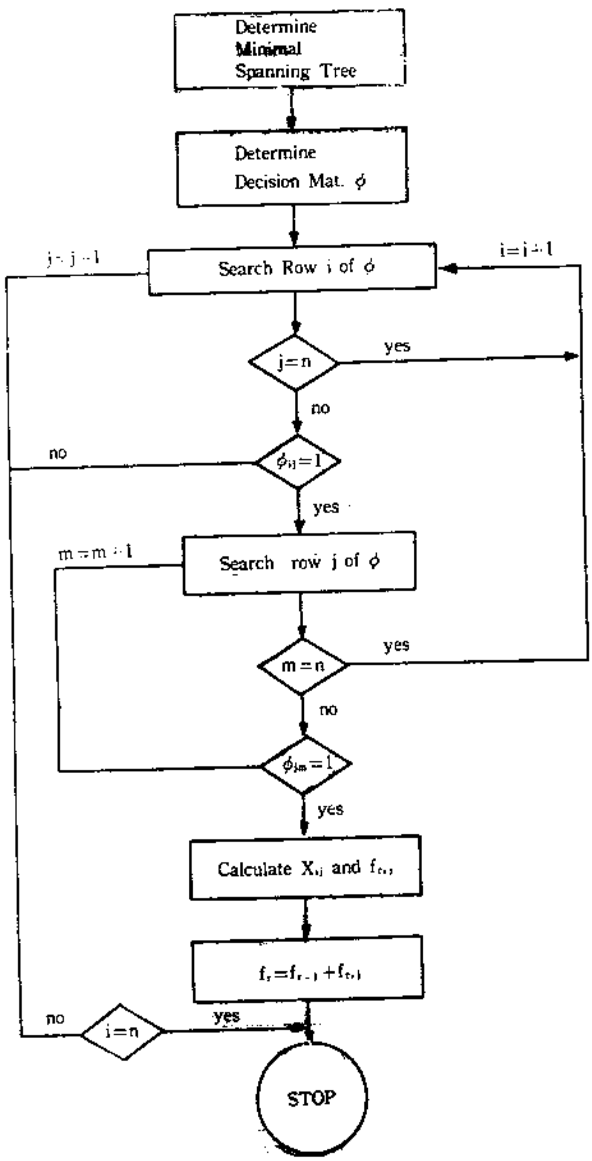

Figure 3. The Main Flowchart for the Central District Heating Systems.

other links.

Therefore, the allocation of X amount of heat between *i* and *j* nodes depends only on the allocation from m to i where link (m,i) received an allocation, X in the previous stage. Hence, the Markevian property of dynamic programming is explictly evident. The cumulative cost and transferred heat is determined by summing over all the stages.

Results

A computer program was written and model was tested for simulated situations. Applicable data was not available for a proper simulation to be carried out. The results presented in this section serve to illustrate the applicability of the model and are not necessarily representative of an actual CDHS. To simplify the presentation of our results we considered a CDHS with a network containing N=16 housing units with hypothetical cost and heat loss functions based on distance.* The input to the computer program is an N×N matrix containing all possible distances between nodes. The entire distribution of houses and central station is then uniquely defined relative to every other node.

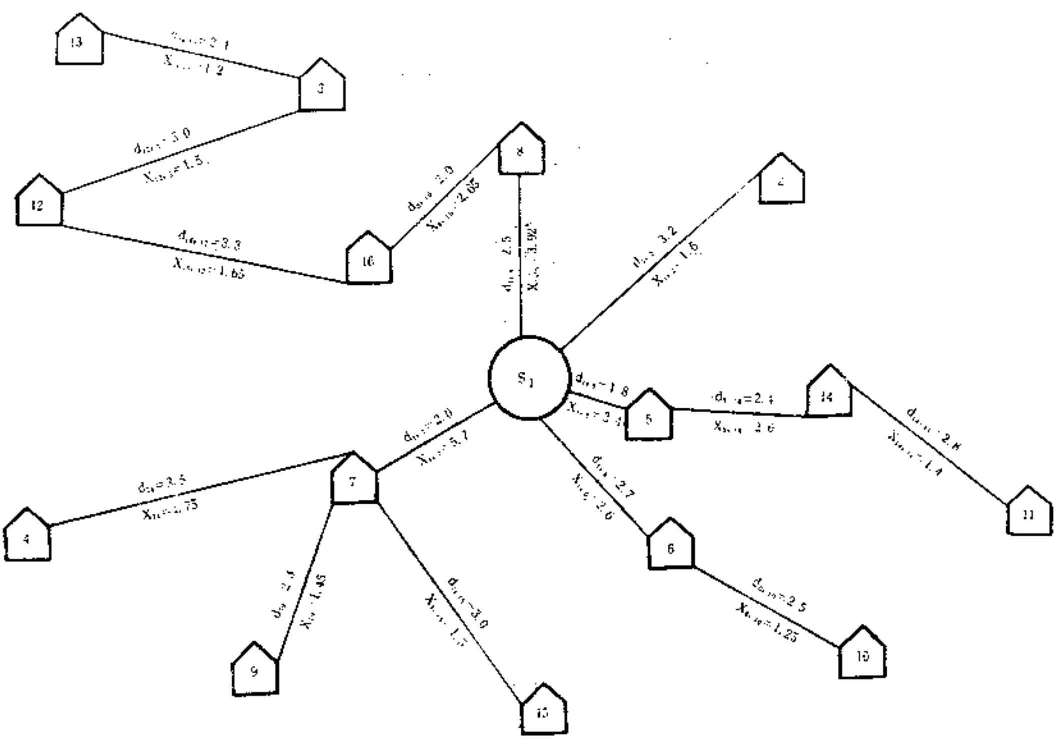

Figure 4 : The optimal network configuration and distribution of heat for a CDHS with 16 housing units and one central station. The heat loss function and related costs are taken as a linear function of distance.

* The particular cost and heat loss function we have assumed in this paper are not crucial to our analysis, as long as they are based on the variation with distance. Past case studies of CDHS indicate that these are reasonable assumptions. (cf.5)

By applying the algorithm presented in the previous section the optimal network and distribution of heat is obtained for a cost and heat loss function which is proportional to distance (See fig. (4)). Thus we have obtained from all possible networks which may exist, the particular network and its corresponding heat distribution which minimizes the total over all costs. The particular breakdown of the costs can be seen in Table (1). Here F_{TOT} represents the comulative cost of the system after each stage and X_{TOT} the comulative heat transferred at each stage. The column represented as link corresponds to the decision link, (*i.e.*, $d_{ij}=1$) which minimizes the cost in that stage. The columns represented as F and X give, the cost, F, of transferring amount of heat, X, across that link. Furthermore, at the last stage F_{TOT} represent the total operating cost of the entire CDHS and the total power requirement of the CDHS. These values can now be used to estimate the propriate size of a central power plant needed to supply X_{TOT} amount of heat to the CDHS.

In order to test the hypothesis that the CDHS will respond differently to variations in the cost heat loss function and network configurations, it was simulated a situation where the cost and heat loss function could be proportional to the distance squared. Fig. (5) shows the different responses obtained. The vertical axis is the cumulative heat after each stage of the simulation and the horizontal axis is the stage of the simulation. The higher costs are obtained when the heat loss function varies with a high order of magnitude than the cost function. Note that these responses are not representative of an actual CDHS, but the fact that variations in the responses may occur with different cost and heat loss functions are indicated by this graph.

Concluding Remarks

It has been taken a simple model of the Central District Heating Systems and analyzed it for minimum operating costs and sensitivity to network configuation can be solved for minimum operating cost by taking a dynamic programming approach once the relative distance between the central station and the buildings are known. It is important to note, that even though our discussion has centered on residential homes, a total community setting including stores, schools, etc., can be considered with the present model. In particular, if the decision matrix is interepreted as a relative probability transition matrix, then the ϕ_{ij} can take on values from o to 1 depending what its probability of service, say over a day. These probability distributions can also be modified to take into account seasonal variations. This study shows that the operating costs are sensitive to the network configuration, and the particular cost and heat loss functions. (See Fig. (5)). In fact, past case studies of district heating systems show that some implemented district heating systemes were not economical (cf. (2)). It is felt that this may have been due to the particular network configuration chosen. In fact, a study of Danish District Heating Systems has shown that 49% of the total cost is attributed to the piping and 6.8% attributed to the distribution (cf. (6)). This comprises more than 50% of the entire costs attributed to CDHS. Hence, if district heating is to become economically feasible, then the optimization should give primary attention to the network configuration.

It is felt that district heatings should be utilized more extensively in residential community settings, but better planning and system engineering is needed in district heating designs, in order to make them economically feasible. It seems to tempting to suggest that district heating would be a practical partial solution to the increasing fuel shortages, energy crisis, higher house maintenance costs, and environmental pollution problems which present residential homes have created due to the global trends of urbanization. It would be a shame if such a solution would not be implemented or implementation delayed because of poor planning and engineering.

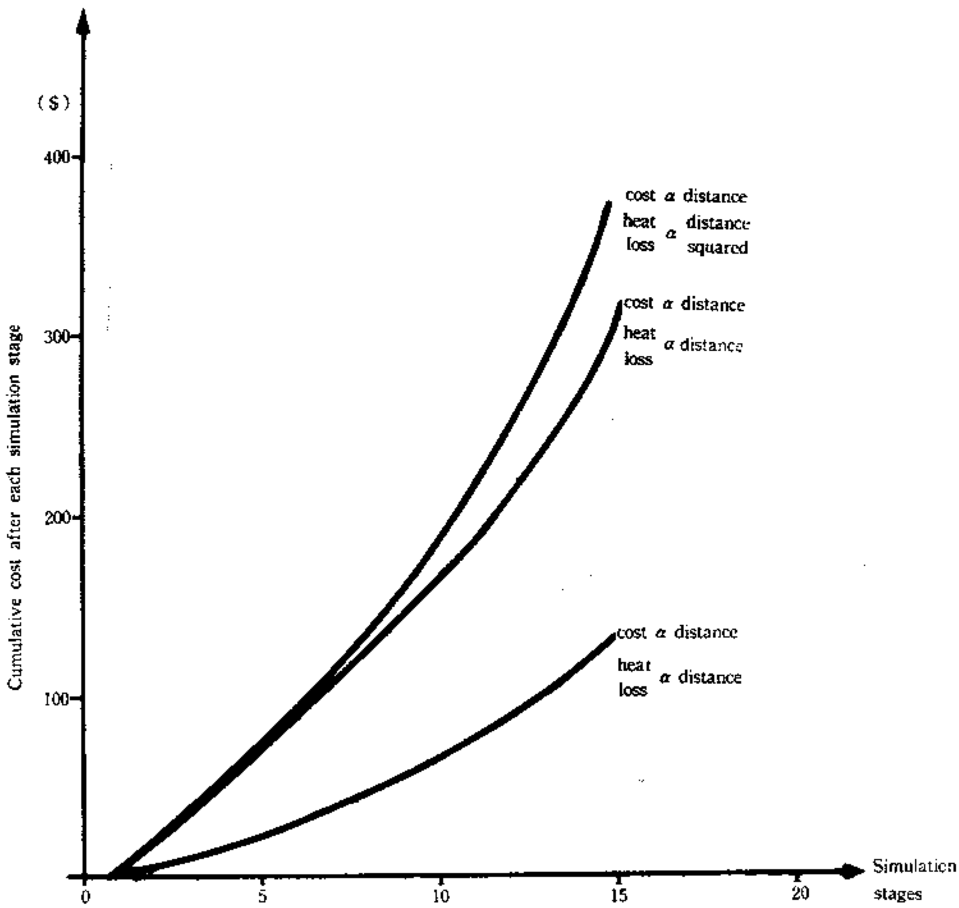

Figure 5 : The response of the CDHS simulation model for different cost and heat loss functions.

Table 1 : The resulting costs and heat distribution at each stage of the computer simulation.

Stage	F_{TOT}	X_{TOT}	Link	F	X
1	5.445	1.650	(16, 12)	5.445	1.650
2	9.365	3.050	(14, 11)	3.920	1.400
3	15.145	4.750	(14, 13)	5.780	1.700
4	18.025	5.950	(13, 3)	2.880	1.200
5	25.325	9.600	(8, 16)	5.300	2.650
6	31.450	11.350	(7, 4)	6.125	1.750
7	35.655	12.900	(7, 9)	4.205	1.450
8	40.155	14.300	(7, 15)	4.500	1.500
9	43.290	15.550	(6, 10)	3.125	1.250

10	62.720	23.650	(5, 14)	10.320	4.300
11	57.840	25.250	(1, 2)	5.120	1.600
12	77.280	31.150	(1, 5)	8.160	5.100
13	97.945	35.100	(1, 6)	7.020	2.600
14	115.245	49.750	(1, 7)	11.400	5.700
15	128.120	53.900	(1, 8)	9.750	3.900

Appendix

Technique for Obtaining a Minimal Spanning Network

A technique for finding the minimal spanning network which is based on the labeling method (cf. (7)) is used. The matrix manipulation method (M-method) enables sovling for the minimal spanning network within a matrix describing the relative distance of all the nodes.

The algorithm proceeds as follows:

Step 0 : Define S_i to be the set of all rows to be considered at stage i. Initially set $i=0$, $S_0=\{1\}$ and $e_1=\{1\}$ where e_i is the column index of the smallest element at stage i.

Step 1 : Set $i=i+1$ and search over all rows whose index is included in S_{i-1} and find the smallest element over all columns whose index is not included in set S_{i-1}.

Step 2 : Set e_i equal to the column index chosen in step 1. Set $S_i=S_{i-1}\cup e_i$.

Step 3 : If S_i contains all columns of the original distance matrix, stop; otherwise go to step 1.

The final set S_{N-1} for a N×N matrix will contain elements corresponding to the minimal spanning network.

References

1. Freeman, L.H., "Central Plants for Large Complexes," *Building Systems Design Journal.*
2. Ediamant, R.M., "Total Energy," Oxford (pub.) (1970).
3. Clifford, B.D., "Total Energy Plant," *Compressed Air Magazine,* (Sept. 1969).
4. Gluss, B., "An Elementary Introduction to Dynamic Programming," *Allyn and Bacon,* Inc. (pub.) (1972).
5. Genicq, P., "District Heating Serves Belgian Community," *Heating, Piping and Air Conditioning,* June 1967.
6. Field, A.A., "Danish Ideas in District Heating," *Heating, Piping and Air Conditioning,* June 1970.
7. Buscaker and Saaty, "Finite Graphs and Networks," *McGraw-Hill Book Company,* 1965.