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ON THE GAME OF GO

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1. Introduction

There are books ([3], [5]) for persons who want to know, practically, how to play the game of Go. There are books for persons who want to improve therir skill (see [7], [17], [24]). E. Thorp ([25], [26], [27]), W. Walden ([25], [27]), A. Zobrist [29] and D.B. Benson([1], [2]) have studied Go-game in connection with the computer science. We first give a definition of a Go-game using graphs. Then we give a definition of a three person Go-game and discuss problems of Go-games. The final part of this paper has a theorem (which is a kind of minimax theorems) on an upper bound of number of stones with which every Go-game can be played without exchanging black and white stones during the paly time.

2. The two person Go-game on a 2-dimensional board

We define a Go game by using mappings and graphs. Let Z_+ be the set of all positive integers. Let $n_i \in Z_+$ with $n^i \geqq 6$ $(i=1,2)$. Let $I^i=\{n\in Z_+ : n\leqq n_i\}$ and let $B=I_1\times I_2$. B will be called the *go-board*. $(i,j)\in B$ is called a *point* or an *intersection*. Let $S=\{b,w\}$ be a set of two distinct objects. (We call b a *black stone* and w a *white stone.*) If (y,a) is a member of $y\times B$, then (y,a) is called a *vertex with y-color* or a *y-vertex* $(y\in S)$.

Let $a_1=(i,j)$ and $a_2=(s,k)$ be two members of B. a_1 and a_2 are *adjacent* if one the following conditions holds:

(1) $s=i+1$ and $j=k$. (2) $i=s+1$ and $j=k$.
(3) $k=j+1$ and $i=s$. (4) $j=k+1$ and $i=s$.
(5) $s=i+1$ and $k=j+1$. (6) $i=s+1$ and $k=j+1$.
(7) $s=i+1$ and $j=k+1$. (8) $i=s+1$ and $k=j+1$.

a_1-a_2 means that a_1 and a_2 are adjacent and $a_1 \not- a^2$ means that a^1 and a_2 are not adjacent.

DEFINITION 1. *An edge.* Let $y\in S$. Let $v_i=(y, a_i)\in y\times B (i=1, 2)$. (v_1, v_2) is said to be an *edge* if a_1-a_2. For $v_1=(y,a_1)$, define $c(v_1)=a_1\in B$.

Any subset $G(y)$ of $y\times B$ becomes a graph by the definition of an edge, and

$G(y)$ is called a ***y-graph*** or a ***graph of y-stones***. $G(y\times B)$ denotes the set of all y-graphs. For $G(y)$ in $G(y\times B)$, define $c(G(y))=\{c(v) : v$ is a vertex of $G(y)\}$. Let $v_1=(y, a_1)$ and $v_m=(y, a_m)$ be two vertices of $G(y)$. v_1 and v_m are *connected* if there exists a sequence $(y, a_i)(i=2, 3, \cdots, m-1)$ of vertices (y, a_i) of $G(y)$ such that $a_i - a_{i+1}$ $(i=1, 2, \cdots, m-1)$. $G(y)$ is said to be *connected* if any two vertices of $G(y)$ are connected. Define $|G(y)|=m$ as the total number of vertices of $G(y)$ and will be called the ***order*** of $G(y)$.

Define $A\backslash B=\{x\in A : x\notin B\}$ for any two sets A and B. Let $B_1=\{(i,j)\in B : i=1$ *or* $i=n_1\}$, $B_2=\{(i,j)\in B : j=1$ *or* $j=n_2\}$ and $B(B)=B_1\cup B_2$, which may be called the ***border*** of B.

DEFINITION 2. ***A simple closed graph.*** A connected graph $G(y)=\{(y, a_i) : i=1, 2, \cdots, m\}$ of order $m=|G(y)|\geqq 3$ is said to be a ***simple connected graph with two terminal vertices*** (y, a_1) and (y, a_m) if a_1 is not adjacent with a_m and if $G(y)\backslash(y, a_i)$ $(1\neq i\neq m)$ is not a connected graph. $G(y)=\{(y, a_i) : i=1, 2\}$ with a_1-a_2 (an edge) is also called a ***simple connected graph.*** Let $G(y)=\{(y, a_i) : i=1, 2, \cdots, m\}$ $(m\geqq 2)$ be a simple connected graph with two terminal vertices (y, a_1) and (y, a_m). If $\{a_1, a_m\}\subset B(B)$, then $G(y)$ is called a ***weak simple closed graph.*** A graph $G(y)$ is said to be a ***strong simple closed graph*** if $|G(y)|\geqq 4$ and if, for all $i, G(y)\backslash(y, a_i)$ is a simple connected graph with two terminal vertices (y, a_{i-1}) (when $i=1$, we take $m-1$ as $i-1$) and (y, a_{i+1}) (when $i=m$ we take 1 as $m+1$). $CG(y\times B)$ denotes the set of all weak and strong simple closed y-graphs and any member of $CG(y\times B)$ will be called a ***simple closed graph*** (or a ***simple polygon***). Let $G(y)\in CG(y\times B)$. Then $G(y)$ divides the go-board B into two separated regions $R(G(y))$ and $\bar{R}(G(y))$ such that $R(G(y)\cup\bar{R}(G(y))\cup c(G(y))=B$ and $R(G(y))\cap\bar{R}(G(y))\cap c(G(y))=\phi$, the empty set. If $a=(i,j)\in R(G(y))$, then we say that the region $R(G(y))$ contains a point $a=(i,j)$. (If $|R(G(y))|<|\bar{R}(G(y))|$, then $R(G(y))$ will be called the ***inner*** ***(or interior) region*** of $G(y)$ and $\bar{R}(G(y))$ will be called the ***outer (exterior) region*** of $G(y)$.

EXAMPLE 1. There are exactly four weak simple closed graphs $G(y)=\{(y, c), (y, d)\}$ of order $2=|G(y)|$ with one point inner region $R(G(y))$, where $\{c, d\}=\{(1,2), (2,1)\}$, $\{c, d\}=\{(n_1, n_2-1), (n_1-1, n_2)\}$, $\{c, d\}=\{(n_1-1, 1), (n_1, 2)\}$ and $\{c, d\}=\{(1, n_2-1), (2, n_2)\}$. There is a strong simple closed graph $G(y)=\{(y, a_i) : i=1, 2, 3, 4\}$ of order with one point inner region $R(G(y))=(i+1, j+1)$, where

$a_1=(i+1,j)$, $a_2=(i+2, j+1)$, $a_3=(i+1, j+2)$ and $a_4=(i,j+1)$.

DEFINITION 3. *A graph $G(x)$ is completely surrounded by a simple closed graph $G(y)$*. Let $G(x)$ be a graph and let $G(y)\in CG(y\times B)$. If $c(G(x))\subset R(G(y))$, then we say that $G(x)$ is *surrounded* by $G(y)$ and we may denote this by $G(x)\subset G(y)$. If $c(G(x))=R(G(y))$, then we say that $G(x)$ is *completely surrounded* by $G(y)$ and we may denote this by $G(y)(G(x))$.

DEFINITION 4. Let $G_i(y)\in CG(y\times B)$ $(i=1,2,\cdots,m)$. Suppose that $G_i(y)\subset G_1(y)$ $(i\neq 1)$ and $c(G_j(y))\subset \bar{R}(G_i(y))$ $(j\neq i\neq 1)$. Then the region $R(G_1(y)\setminus\bigcup_{i\neq 1} R(G_i(y))\bigcup_{i\neq 1} c(G_i(y))$ is called the *region* of $G(y)=\bigcup_{i=1}^{m} G_i(y)$ and we denote that region by $R(G(y))=R(\bigcup_{i=1}^{m} G_i(y))$. Let $R(\bigcup_{i=1}^{m} G_i(y))$ be the region of $G(y)=\bigcup_{i=1}^{m} G_i(y)$ the union graph of graphs $G_i(y)$. (If $a\in R(G(y))$, then $R(G(y)\cup(y,a))=R(G(y))\setminus a$ is also called the *region* of $G'(y)=G(y)\cup(y,a)$, as a special case of $R(G(y))$. If $c(G(x))\subset R(G(y))$, then we say that $G(x)$ is *surrounded* by $G(y)$ and we may denote this by $G(x)\subset G(y)$. If $c(G(x))=R(G(y))$, then we say that $G(x)$ is *completely surrounded* by $G(y)=\bigcup_{i=1}^{m} G_i(y)$ and we denote this by $G(y)(G(x))$. Definition 4 is a generalization of Definition 3.

We now define a Ko.

DEFINITION 5. *Ko.* Let $G(y)$ be a simple closed graph with the inner region $R(G(y))=(i,j)$ of one point such that $2\leqq|G(y)|\leqq 4$. Let $x\in S$ with $x\neq y$. Let $G(x)$ be a graph such that $1\leqq|G(x)|\leqq 3$, $c(G(x))\cap c(G(y))=\phi$, $G(x)\cup(x,(i,j))$ forms a simple closed graph and $G(x)$ is not a simple closed graph. If there is a vertex (y,a) of $G(y)$ such that $R(G(x))\cup(x,(i,j))=a$, in the graph $G(x)\cup(x,(i,j))\cup G(y)\setminus(y,a)$. Then we say that $G(y)$ and $G(x)$ form a *Ko* and we shall denote this by Ko(G(y), $G(x)$, $(i,j),(y,a)$). We shall also say that P_y (the person with y-stones) *initiated* the Ko.

$G(y)\cup G(x)$ is called an (x,y) *graph* when $c(G(y))\cap c(G(x))=\phi$.

DEFINITION 6. *A move Function f and a capture function g*. Let G_i be a sequence of (b,w) graphs. Let $S^\circ=S\cup\phi$. We define a *move function* $f: Z_+\longrightarrow Z_+/S^\circ/B$ as a function satisfying the following three conditions. (1) $f(1)=(1,G_1)$, where $G_1=b\times V\subset b\times B$. $|f(1)|$ is defined as $|V|$.

(2) For all $n \in Z_+$,

$f(2n-1)=(2n-1, b, v)$, a black move by p_b,

$(2n-1, \phi, v)$, a pass move by p_b,

$f(2n)=(2n, w, v)$, a white move by p_w,

$(2n, \phi, v)$, a pass move by p_w,

where $v \in B$. We define $|f(i)|=1$ for a non-pass move and $|f(i)|=0$ for a pass move for $i \geqq 2$. We often write $f(i)=\phi$ for a pass move $f(i)$.

(3) If $f(i+1)=(i+1, x, v)$ and $x \neq \phi$, then $v \in B \backslash c(G_i)$. (We may identify $(1, G_1)$ $=f(1)$ with G_1 and $f(i+1)=(i+1, x, v)$ with (x, v).) We now define a *capture function* $g: Z_+ \longrightarrow G(b \times B) \cup G(w \times B) \cup \phi$ by the following: (4) Let $f(i)=(i, x, v)$ $(v \in B)$. If there exists a set $\{G_i(x): i=1, 2, \cdots, m\}$ of x-graphs $G_i(x)$ in G_{i-1} and if there exists a graph $G_0(y)$ in G_{i-1} such that either (a) $\bigcup_{i=1}^{m} G_i(x) \cup (x, y)$ makes the region $R(\bigcup_{i=1}^{m} G_i(x) \cup (x, v))$ as defined in Definition 4 and $\bigcup_{i=1}^{m+1} G_i(x)(G_0(y))$, where $G_{i+1}=(x, v)$, or (b) $G_i(x) \cup (x, v)$ $(i=1, 2, \cdots, m)$ forms a simple closed graph and $G_i(x) \cup (x, v)(G_i(y))$, where $G_i(y)$ is a part of $G_0(y)$ with $G_0(y)=\bigcup_{i=1}^{m} G_i(y)$. Then $g(i)=G_0(y)$. (We may say that p_x *captures* a group of y-stones of $G_0(y)$ when $g(i)=G_0(y)$.)

(5) (Suicide is illegal.) If there is a simple closed graph $G_0(y)$ in G_{i-1} such that $G_0(y)(G_0(x) \cup (x, y))$ for a graph $G_0(x)$ in G_{i-1} $(x \neq y)$, then $g(i)=G_0(x)$ $\cup (x, v)$.

(6) (Suicide is illegal.) If, in G_{i-1}, there exists a set $\{G_i(y): i=1, 2, \cdots, m\}$ of y-graphs $G_i(y)$ and $G_0(x)$ such that $\bigcup_{i=1}^{m} G_i(y)(G_0(x) \cup (x, v))$ (see Definition 4), then we define $g(i)=G_0(x) \cup (x, v)$. If $m=1$, then (6) returns to (5). (We may say that p_x *loses* a group of x-stones of $G_0(x) \cup (x, v)$ by his move $f(i)$.)

(7) If (4), (5) and (6) are not applicable, then $g(i)=\phi$. Now we can define a sequence G_i inductively: $f(1)=(1, G_1)$ with

$|G_1| \geqq 1$. For $f(i+1)=(i+1, x, v)$, $G_{i+1}=G_i$ if $x=\phi$,

$G_i \cup (x, v) \backslash g(i+1)$ if $x \neq \phi$.

We introduce one of important concepts on Go games. Ko-rule. For any graph $G(b, w)$, $k(G(b, w))$ denotes the set of all Kos in the (b, w)-graph $G(b, w)$.

(1) If a move $f(n)=(n, y, v)$ forms the first Ko=Ko$(G(y), G(x), (i, j), (y, a))$. Then the player px can take a move of the form $f(n+1)=(n+1, x, (i, j))$ and

consequently $g(n+1)=(y, a)$. If $f(n+1)=(n+1, x, (i, j))$ and $g(n+1)=(y, a)$, then we say that p_x *moves* to the Ko and captures a y-stone of (y, a). Now we have $G_{n+1}=G_n\cup (x, (i, j))\backslash(y, a)$. Ko-rule is that the moving $f(n+2)=(n+2, y, a)$ by P_y with $g(n+2)=(x, (i, j))$ is illegal. Alternative rule is that (Suicide is illegal), if $f(n+2)=(n+2, y, a)$, then $g(n+2)=(y, a)$. Notice that Ko-rule is to make the go-game finite.

(2) Suppose that $k(G_n)=\mathrm{Ko}(G(y), G(x), (i, j), (y, a))$, $f(n)=(n, y, v)$ and $g(n)\neq(x, (i, j))$. If $f(n+1)=(n+1, x, (i. j))$ and $g(n+1)=(y, a)$, then we say that P_x *moves* to the Ko. Suppose that P_x moved to the Ko. Then Ko-rule is that the move $f(n+2)=(n+2, y, a)$ with $g(n+2)=(x, (i, j))$ by $P_y(x\neq y)$ to any Ko of $k(G_{n+1})$ is illegal. If $f(n)=(n, y, v)$ with $g(n)=(x, (i, j))$, then a move $f(n+1)=(n+1, x, (i, j))$ with $g(n+1)=(y, a)$ is illegal. We generalize this Ko-rule. Let $m_1\geqq 1$.

(3) Suppose that $k(G_n)=\{\mathrm{Ko}(G_t(y), G_t(x), (i_t, j_t), (y, a_t)): t=1, 2, \cdots m_1\}\cup\{\mathrm{Ko}(G_t(y), G_t(x), (i_t, j_t), (x, a_t)): t=m_1+1, m_1+2, \cdots, m_1+m_2\}$. Suppose $f(n)=(n, y, v)$ with $g(n)\neq(x, (i_t, j_t))$ $(t\in\{1, 2, \cdots, m_1\})$. If P_x takes a move to a Ko of $k(G_n)$, then a move by P_y to any Ko in $k(G_{n+1})$ is illegal. (If $f(n+1)=(n+1, x, (i_t, j_t))$ with $g(n+1)=(y, a_t)$ $(t\in\{1, 2, \cdots, m_1\})$, then $f(n+2)=(n+2, y, (i_t, j_t))$ and $f(n+2)=(n+2, y, (i_s, j_s))$ $(s\in\{m_1+1, m_1+2, \cdots, m_1+m_2\})$ are both illegal).

DEFINITION 7. *The end of the game.* The game ends with the final graph G_n if $f(n+1)$ and $f(n+2)$ are first two consecutive pass moves. We shall say that the game *ends* at the move $t=n+2$. For a (b, w)-graph G_n, we can write $G_n=G(b)\cup G(w)$. Let $G(y)\in\{G(b), G(w)\}$.

SCORING. Let $G_n=G(b)\cup G(w)$ be the final graph of a game. Let $\Pi=\{G_i(y): i\in I\}$ be the set of all simple closed y-graphs in $G(y)$. Let $R=\{R_j: j\in J\}$ be the set of all regions determined by Π. We see that $R(G_i(y))$ and $\overline{R}(G_i(y))$ are members of R. (1) Consider $R(G_i(y))\in R$. Suppose $|R(G_i(y))\cap c(G(x)|=m_1$ and $|R(G_i(y))\cap c(G(y))|=m_2$. If $m_1=0$, then we define $(\|RG_i(y))\|=|R(G_i(y))|-m_2$. If $m_1\neq 0$, then there exists a graph $G_t(x)$ such that $c(G_t(x))\subset R(G_i(y))$. There are two cases. (2) There exists a positive integer k such that by k moves $f(n+2+i)$ $(i=1, 2, \cdots, k)$, it is possible to obtain $g(n+2+k)$ which contains $G_t(x)$. This is the case, then we set $\|R(G_i(y))\|=|R(G_i(y))|-m_2+m_1$. (3) If it is not possible to obtain such $g(n+2+k)$ containing $G_t(x)$ by a finite number (k)

of moves, then we set $\|R(G_i(y))\|=0$. (4) Let $R_j \in R$. Suppose that there is a subset K of $I(|K| \geqq 2)$ so that the union graph $\bigcup_{i \in K} G_i(y)$ makes the region $R(\bigcup_{i \in K} G_i(y))$ as defined in Definition 4. Then we take $|R_j \cap c(G(x))|=m_1$ and $|R_j \cap c(G(y))|=m_2$, where $R_j=R(\bigcup_{i \in K} G_i(y))$. If $m_1=0$, then we define $\|R_j\|$ as $\|R_j\|=|R_j|-m_2$. If $m_1 \neq 0$. Then there exists a graph $G_i(x)$ such that $c(G_i(x)) \subset R_j$. We now follow (2) and (3) for R_j. We define c_y by $c_y=\sum_{i \in J}\|R_i\|+\sum_{g(j) \in G(x \times B)} g(j)$, as the final score of $P_y (x \neq y)$. If $c_x > c_y$, then we say that P_x with x-stones win the game by (c_x-c_y).

DEFINITION 8. A *two person go-game* is a set $\{f, g, S, B\}$ of a move function which obey the Ko-rule, a capture function g, a set $S=\{b, w\}$ and a two dimensional board B.

3. Life and Seki

We introduce the terms of Safe and Seki. Let $\{f, g, S, B\}$ be a two-person go-game on B with the final graph $G_n=G(b) \cup G(w)$. Let $G(x) \in \{G(b), G(w)\}$. Suppose that there exist a simple closed x-graph $G_1(x)$ in $G(x)$ and a y-graph $G_1(y)$ in $G(y)$ $(x \neq y)$ such that $G_1(y) \subset G_1(x)$. Let $f(n)=(n, x, v)$.

(1) If a finite number $2k$ of moves $f(i)$ defined by

$f(n+2+j)=$ a pass move if $j=1, 3, \cdots, 2k-1$,

a non-pass (y) move if $j=2, 4, \cdots, 2k$,

with $g(n+2+j)=\phi$ $(j<2k)$, it is not possible to have $g(n+2+2k)=G_1(y)$. Then we say that $G_1(y)$ is *Safe*.

(2) If a non-pass move by any player P_x (or P_y) into a set $R(G_1(x) \backslash c(G_1(y))$ is unfavorable for that player, then we say that $G_1(x)$ and $G_1(y)$ form a *Seki* [27, p. 10]. (n can be replaced by $k<n$).

4. Examples

We have following examples of go-games.

EXAMPLE 2. Let $\bar{B}=B \backslash \{(1,1), (1,2), (2,1)\}$. Let $\{f, g, S, B\}$ be a two-person go-game on B defined by f and g:

$(i, b, a_i)(a_i \in \bar{B})$ if $i=2n+1$ $(n=0, 1, \cdots, (n_1 n_2-4))$,

$f(i)=$ a pass move if $i=2n$ $(n=1, 2, \cdots (n_1 n_2-4))$,

$(i=2(n_1 n_2-3), w, (1, 2))$,

a pass move if $i=2(n_1 n_2-3)+1$,

$(i=2(n_1n_2-3)+2, w, (2, 1)),$

a pass move if $i=2(n_1n_2-3)+3, 2(n_1n_2-3)+4,$ and

$g(i)=b\times\overline{B}$ if $i=2(n_1n_2-3)+2,$

ϕ if $i\neq 2(n_1n_2-3)+2.$

We can see that $|g(2(n_1n_2-3)+2)|=|\overline{B}|=n_1n_2-3$. Let G_n be the final graph of this game. Then $G_n=\{(w, (1, 2)), (w, (2, 1))\}$. It is not difficult to see that $n\leqq 2(n_1n_2-2)$. If this game takes place on $\overline{B}$, then the total number n' of moves of the second game is less than or equal to $2(n_1n_2-4)$. If the game repeatedly takes the place, then the (grand) total number $n+n'+\cdots$ of moves from the first game to the last game is less than or equal to

$$2(n_1n_2-2) + \sum_{i=4}^{n_1n_2} 2(n_1n_2-i).$$

EXAMPLE 3. Let $\{f, g, S, B\}$ be a go-game. Suppose that n moves $f(i)$ with $g(i)=\phi$ $(i=1, 2, \cdots, n)$ formed the graph $G_n=G(b)\cup G(w)$ consisting of Kos. If P_b and P_w take moves $f(n+i)$ $(i=1, 2, \cdots, k)$ with $|g(n+i)|\leqq 1$ and this game ends with the final graph G_{n+k} (after two consecutive pass moves $f(n+k+1)=\phi$ and $f(n+k+2)=\phi$), then it is not difficult to show that $n+k\leqq 3(n_1n_2)-4$.

EXAMPLE 4. Let $\{f, g, S, B\}$ be a 2-person go-game on B such that $g(i)=\phi$ for all i. Then the total number n of moves is less than or equal to $2n_1n_2$, where n is defined by the final graph G_n of the game.

PROBLEM. (1) Find the total number of stones with which every go-game can be played without exchanging black and white stones during the play time. (2) Find an upper bound of n, where n is defined by G_n, the final graph of the go-game.

5. Three person go-game on a 2-dimensional board

Let $S=\{b_1, b_2, b_3\}$ be a set of three distinct objects b_i. Let y be a member of S. Any subset of $y\times B$ is called a y-graph. Let $G(b_i)$ be a graph. $G=\bigcup_{i=1}^{3} G(b_i)$ is called a *graph* if $c(G(b_1))\cap c(G(b_2))\cap c(G(b_3))=\phi$. Let $G(y)$ be a simple closed graph and let $G(x)\cup G(y)\cup G(z)$ be a graph $(x, y, z\in S)$. If $c(G(x))\cup c(G)z))=R(G(y))$, then we say that $G(x)\cup G(z)$ is *completely surrounded* by $G(y)$ and we denote it by $G(y)(G(x) \cup G(z))$.

DEFINITION 9. *Ko.* Let $\mathrm{Ko}(G_1(y), G_1(x), (i_1, j_1), (y, a_1))$, $\mathrm{Ko}(G_2(x), G_2(z),$

$(i_2, j_2), (x, a_2))$ and $\mathrm{Ko}(G_3(z), G_3(y), (i_3, j_3), (z, a_3))$ be three Kos. Then we say that these three Kos form a *Ko* in a three person go-game on a 2-dimensional board B. We can define a move function f and a capture function g as defined before. We also can define Ko-rule and SCORING as defined in 2. We define a three-person go-game on a two-dimensional board B as a set $\{f, g, \{b_1, b_2, b_3\}, B\}$ of a move function f, a capture function g, $S=\{b_1, b_2, b_3\}$ and a two-dimensional board B. There are problems of Life and Seki. We give an example of a 3-person go-game on a 2-dimensional board B.

EXAMPLE 5. From this example, it shows that Seki is complicated in a 3-person go-game on B.

The following theorem is a partial answer to Problem 1.

THEOREM. *Let* $\{f, g, \{b_1, b_2, b_3\}, B\}$ *be a three person game of go on B with the final graph* $G_n = G(b_1) \cup G(b_2) \cup G(b_3)$. *Then* $|G(b_1)| + |G(b_2)| + |G(b_3)| + \sum_{i=1}^{n} |g(i)| < (n-2)n_1 n_2$.

PROOF. It is clear that $|G(b_1)| + |G(b_2)| + |G(b_3)| < n_1 n_2$. It is also clear that $|g(1)| + |g(2)| = 0$ and $|g(3)| = 0$. The theorem follows from $|g(i)| < n_1 n_2$ for all $i > 3$.

REMARK. The above theorem is true for an n-person go-game on a 2-dimensional go-board. The number $(n-2)n_1 n_2$ is not realistic because of n which is not known. We can define an n-person go-game on an m-dimensional board. In a 3-person go-game $\{f, g, \{b, w, y\}, B\}$, there will be two cases for $f(2)$: Let $f(1) = (1, G_1)$ and $g(1) = \phi$, where G_1 is a subset of $b \times B$. Case (1). $f(2) = (2, w, v) = (2, (w, y))$ with $g(2) = \phi$, where $v \in B \backslash c(G_1)$. Case (2). $f(2) = (2, w, V) = (2, G(w))$ with $g(2) = \phi$, where $G(w) = w \times V$ and $V \subset B \backslash c(G_1)$.

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