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The Mathematical Kinetics for Activated Sludge Process

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Abstract

A complete mixing aeration only activated sludge system was operated for the mathematical analysis. The unit, 4 liter capacity, was fed by an industrial waste containing a much amount of glutamic acid in a laboratory study. The observed results were compared with the generated data presented by the previous mathematical models.

Introduction

Many kinetics models (formulars) have been presented for the activated sludge process in the biological wastewater treament. Some of which have been described by Michalis-Menton(1931), Monod(1942), Eckenfelder and O'connor (1960—1971) and McKinney (1968—1970). It seemd that the confusion inherent in the existence of the above mentioned models were severely redarding the desiable and wide application of the activated sludge process analysis. The objective of this paper is to compare the parameters used in each mathematical model through a logical and stepwise developement.

The parameters used for comparison were 1) substrate removal rate, 2) sludge production rate, 3) effluent total BOD and 4) oxygen requirement. It is found that the models of Eckenfelder and McKinney are more complete and efficient and no real conflict between these two models actually existed for the activated sludge process. The model used by Michaelis-Menton is limited to be estimate only effluent soluble BOD and Monod model can be estimate effluent soluble BOD and volume of sludge to be produced.

Comparision of Parameters in Mathematical Models

1. Substrate removal rate

Michaelis-Menton reported that relationship between food and microbial growth rate coefficient can be expressed as

$$\frac{dF}{dt}=\frac{K_sFM_a}{K_m+F} \quad\cdots\cdots(1a)$$

Expressing average removal amount per unit time in eq. (1a)

$$\frac{S_r}{M_at}=\frac{K_s\,F}{K_m+F} \quad\cdots\cdots(1b)$$

where

F, F_i : substrate concentration in effluent, influent, mg/l

K_m : microbial growth rate coefficient

K_s : maximum removal rate at F_i concentration

x : biodegradable fraction of the volatile suspended solids

M_a : active microbial mass, xX_v

S_r : F_i-F

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If food (F) is very smaller than microbial growth rate coefficient in effluent

$$\frac{S_r}{M_a t}=\frac{K_s F}{K_m}=KF \tag{1c}$$

Rearranging eq. (1c)

$$\frac{F}{F_i}=\frac{1}{1+KM_a t} \tag{1d}$$

Monod (1942) published that relationship between food and microbial growth rate coefficient in a microbe limiting condition rather than a food limiting condition could be expresssed as

$$u=u_{max}\frac{F}{K_s+F} \tag{2a}$$

where

u = specific growth rate, hr^{-1}

u_{max} = maximum growth rate

K_s = substrate concentration at

$$u=\frac{1}{2}u_{max}$$

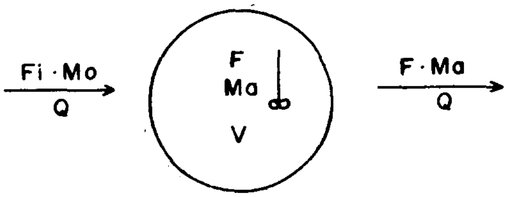

Fig. 1. Mass balance for the activeted aeration only system.

In Fig. 1 it can be seen that a mass balance for the mass of microbes in reactor system can be written as

$$\left\{\begin{matrix}\text{organism change}\\ \text{in reactor}\end{matrix}\right\}=\left\{\begin{matrix}\text{organisms}\\ \text{in reactor}\end{matrix}\right\}$$

$$+\left\{\begin{matrix}\text{growth of}\\ \text{organisms}\end{matrix}\right\}-\left\{\begin{matrix}\text{loss of organisms}\\ \text{in effluent}\end{matrix}\right\}$$

$$-\left\{\begin{matrix}\text{lose due to}\\ \text{decay}\end{matrix}\right\}$$

$$V(dM_v)\text{net}=M_oQdt+V(dM_v)g-M_vQdt$$

$$-K_eM_vdt \tag{2b}$$

Assuming that the rate of microbes growth is very small in influent, eq. (2b) can be rearranged for rate of microbes growth.

$$\left(\frac{dM_v}{dt}\right)\text{net}=uM_v-K_eM_v-M_v\frac{1}{t} \tag{2c}$$

where

$\left(\frac{dM_v}{dt}\right)$net : net change in organisms concentration in reactor

M_o : organisms concentration in influent

M_v : organisms concentration in effluent

K_e : endogenous respiration rate

$\left(\frac{dM_v}{dt}\right)g$: organisms growth rate : $u\ M_v$

Considering steady–state condition $\left(\frac{dM_v}{dt}=O\right)$, eq. (2c) can be simplified as

$$u=K_e+\frac{1}{t} \tag{2d}$$

A mass balance for the mass of food in reactor system can be expressed as

$$\left\{\begin{matrix}\text{substrate change}\\ \text{in reactor}\end{matrix}\right\}=\left\{\begin{matrix}\text{substrate in}\\ \text{influent}\end{matrix}\right\}$$

$$-\left\{\begin{matrix}\text{consumption}\\ \text{by organisms}\end{matrix}\right\}-\left\{\begin{matrix}\text{loss of substrate}\\ \text{in effluent}\end{matrix}\right\}$$

$$V(dF)\ \text{net}=F_iQdt-V(dF)g-FQdt \tag{2e}$$

Eq. (2e)can be rearranged for the rate of food concentration

$$\left(\frac{dF}{dt}\right)=\frac{F_i}{t}-\frac{uM_v}{Y}-\frac{F}{t} \tag{2e}$$

Considering steady–state condition $\left(\frac{dF}{dt}\right)=O$

And $$M_v=\frac{Y(F_i-F)}{1-K_e t} \tag{2g}$$

The values of Y, K_e can be determined from eq. (2h) which is transferred eq. (2g)as a linear form.

$$\frac{F_i-F}{M_v}=\frac{K_e t}{Y}+\frac{1}{Y} \tag{2h}$$

With reference to eq. (2a), (2d), and eq. (2i) can be determinded as

$$u=K_e+\frac{1}{t}=\frac{1+K_e t}{t}=\frac{u_{max}+F}{K_s+F} \tag{2i}$$

Rearranging eq. (2i)

$$F=\frac{K_s(1+tK_e)}{tu_{max}-(1+tK_e)} \tag{2j}$$

The values of K_s, and u_{max} can be determined from eq. (2k) which is transferred eq. (2i)as a linear form

$$\frac{t}{1+K_e t}=\frac{K_s}{u_{max}}\ \frac{1}{F}+\frac{1}{u_{max}} \tag{2k}$$

Michaelis–Menton and Monod model are desiable to the pure culture and the organic matter, but not reliable in the real wastewater which are composed of complex, mixed biological populations.

Eckenfelder and O'connor proposed a mathematical model for activated sludge wastewater treatment. This model was subsequently modified and expanded from 1960 to 1971.

A mass blance for the food with reference Fig. 1 in aeration only system by Eckenfelder expressed as

Input=Output−Accumulation

$$QF_i = QF - \frac{dF}{dt} V \cdots\cdots\cdots\cdots (3a)$$

Assuming the substrate removal rate is considered as first order reaction, eq. (3a) becomes as a differential

$$\frac{dF}{dt} = K^1 M_v F \cdots\cdots\cdots\cdots (3b)$$

By integrating eq. (3b)

$$F = \frac{F_i}{K^1 M_v t + 1} \cdots\cdots\cdots\cdots (3c)$$

McKinney has been developed a mathematical model for the complete mixing activated sludge (CMAS) treatment systems from 1962 to 1968.

A mass balance for the food in Fig. 1 can be expressed as

$$QF_i = QF - KFV \cdots\cdots\cdots\cdots (4a)$$

Rearranging eq. (4a)

$$F = \frac{F_i}{K_m t + 1} \cdots\cdots\cdots\cdots (4c)$$

The substrate removal rate coefficient used in Michalis-Menton, Eckenfelder and McKinney model(KM_a, K^1M_v, K_m) can be expressed to be identical, representing terms of influent and effluent BOD loading according to the hydraulic detention time.

$$KM_a = K^1 M_v = K_m = \frac{F_i - F}{Ft} \cdots\cdots\cdots\cdots (4d)$$

2. Sludge production rate

The total mixed liquor suspended solids(MLSS) concentration in aeration tank is computed as the sum of the active, endogenous, nonbiodegradable organic and inert inorganic matter.

2−1. Active microbial mass concentration (M_a)

Amass balance for the active micrabial mass in Fig, 1 is:

$$\left[\begin{array}{l}\text{active microbial}\\ \text{concentration in}\\ \text{effluent}\end{array}\right] = \left[\begin{array}{l}\text{active microbial}\\ \text{concentration}\\ \text{by synthesis}\end{array}\right] - \left[\begin{array}{l}\text{active microbial}\\ \text{concentration by}\\ \text{endogenous respiration}\end{array}\right]$$

and for the designations given Fig. 1 by Eckenfelder the above mass balance can be expressed

$$M_a Q = QaKmF - M_a K_e V \cdots\cdots\cdots\cdots (5a)$$

Dividing Q and rearranging for the active microbes

$$M_a = \frac{aKmF}{\frac{1}{t} + K_e} \cdots\cdots\cdots\cdots (5b)$$

Also eq. (5b) can be modified into eq. (5e) by substuting eq. (3c)

$$M_a = \frac{aS_r/t}{\frac{1}{t} + K_e} \cdots\cdots\cdots\cdots (5c)$$

where

a: mass yield rate

And the constants of a, K_e can be obtained from the empirical eq. (5d)

$$\frac{\Delta M_v}{M_v} = \frac{aSr}{M_v t} - xK_e \cdots\cdots\cdots\cdots (5d)$$

By McKinney a mass balance for the active-microbial mass Fig. 1 can be expressed as follows

$$QM_a = VK_s F - M_a K_e V \cdots\cdots\cdots\cdots (5e)$$

Dividing Q and rearranging M_a

$$M_a = \frac{K_s F}{\frac{1}{t} + K_e} \cdots\cdots\cdots\cdots (5f)$$

The synthesis rate(K_s) is the multiplication of a synthesis total energy ratio(K_1), a reciprocal of oxygen equivalent of cell yield(K_2), ultimate BOD/5−day BOD ratio (K_3) and substrate removal rate (Km) as indicated in eq. (5f), (5g)

$$K_s = K_1 \ X \ K_2 \ X \ K_3 \ X \ Km \cdots\cdots\cdots\cdots (5g)$$

The synthesis/total energy ratio(K_l) can be approximately as follows:

$$K_1 = S/T - E/T = 1 - (COD_{inf} - COD_{eff})/COD_{inf} \cdots\cdots\cdots\cdots (5h)$$

The endogenous respiration rate and a reciprocal of oxygen equivalent of cell yield(K_2) can be determined as the empirical equation(5i)[6] used by McKinney.

$$\frac{F_i-F}{M_v}=\frac{K_2}{Y}+\frac{K_2K_et}{Y}-\left(1-a'K_2\frac{(F_i-F)}{M_v}\right) \cdots\cdots(5i)$$

where

Y : reciproal of ICOD/VSS(K_2)

a' : nonbiodegradable organic fration in cell mass($a=0.2^{(2)}$)

E : energy

T : total energy

2—2. Volatile suspended solids (M_{tv})

The total volatile suspended solids(M_{tv}) concentration in aeration tank is computed as the sum of active (M_a), endogenous (M_e) and nonbiodegradable suspended solids (M_i)

Eq. ($2g$) used by Monod was considered the only pure microbial concentration except nonbiodegradable volatile suspended solids(NBDVSS) in influent. Therefore, the total volatile suspended solids (M_{tv}) can be expressed as

$$M_{tv}=Mv+Mi=\frac{Y(Fi-F)}{1+K_et}+Mi \cdots\cdots(6a)$$

By eq. (5d) the total volatile suspended solids(Mtv) used by Eckenfelder can be written as

$$M_{tv}=\frac{\frac{aSr}{t}}{\frac{1}{t}+xKe}+Mi \cdots\cdots(6b)$$

Also microbial mass concentration by endogenous respiration(Me) can be expressed as

$$Me=Mv-Ma=\frac{\frac{aSr}{t}}{\frac{1}{t}+xKe}-\frac{\frac{aSr}{t}}{\frac{1}{t}+Ke} \cdots\cdots(6c)$$

By McKinney, microbial mass concentration by endogenous respiration can be empirically expressed as

$$M_e=0.2K_eM_at \cdots\cdots(6d)$$

Nonbiodegradable volatile suspended solids (NBDVSS;Mi) can be determined as

$$\mathrm{Mi}=\mathrm{NBDVSS}=\mathrm{VSS}\times\frac{\mathrm{NBDICOD}}{\mathrm{ICOD}} \cdots(6e)$$

Thus, the total volatile suspended solids (M_{tv}) can be written as

$$M_{tv}=Ma+Me+Mi=\frac{KsF}{\frac{1}{t}+Ke}+0.2KeMat+VSS\times\frac{NBDICOD}{ICOD} \cdots\cdots(6f)$$

2.3. Inert inorganic mass concentration (Mii)

The concentration of inert inorganic mass retults from the entrapment of influent inorganic solids within the treatment system and buildup of inert inorganic end products of endogenous metabolism.

By Eckenfelder and McKinney inert inorganic mass concertration (Mii) can be equally expressed s

$$Mii=Miiinf+0.1\ (Ma+Me) \cdots\cdots(7a)$$

It should be noted that the total votalite suspended solids, the sum of active and endogenous microbes can not be caculated in Monod model, but active and endogenous microbial concentration can be completely caculated in Eckenfelder and McKinney models.

3. Total BOD in effluent.

It should be recognized that the effluent total BOD_5 results from the presence of unmetabolized influent BOD_5 and effluent BOD_5 producing by microbial mass.

The effluent total BOD_5 concentration as expressed by Eckenfelder at steady—state is as follows:

$$\text{Effluent } BOD_5=F+K_oMv \cdots\cdots(8a)$$

Also by McKinney

$$\text{Effluent } BOD_5=F+Ko\,Ma\,eff \cdots\cdots(8b)$$

where

Ko : BOD_5 equivalence of effluent VSS ($K_o=0.8^{(8)}$)

The difference in two models is that both active and nonactive microbe yield BOD_5 in Eckenfelder model, but the only active microbe yield BOD_5 in McKinney model.

4. Oxygen Requirement

It has been shown that the total oxygen reqirements in aerobic condition are related to the oxygen consumed to supply energy for synthesis and the oxygen consumed for endogenous respiration in which a' is the fraction of organics consumed to supply energy for synthesis b'is the autoxidation rate of sludge.

By Eckenfelder the oxygen requirement can be expressed as

$$\frac{dO}{dt}V = a'S_rQ - b'MvV - K^oQ \cdots\cdots\cdots\cdots\cdots\cdots (9a)$$

Neglecting K^o

$$\frac{dO}{dt} = \frac{a'Sr}{t} + K_2xKeMv \cdots\cdots\cdots\cdots\cdots\cdots (9b)$$

where

K_2 = reciprocal of $ICOD/VSS$

a' = $E/T \quad \frac{BODult}{BOD_5} = \frac{E/T}{1-10^{-kt}}$

= synthesis oxygen demand rate. BOD ult. or COD basis

b' = K_2xKe

= endogenous respiration oxygen demand rate

K^o = immediate oxygen demand rate

McKinney[7)] reported the empirical equation for oxygen requirement in the aerobic treatment system.

$$\frac{dO}{dt} = \frac{1.5(Fi-F)}{t} - \frac{1.42(Ma+Me)}{ts} \cdots (9c)$$

In aeration only system the sludge retention time(ts) can be substituted as the hydrualic detention time (t).

Experimentals

The experimental apparatus was constructed with thick plexiglas reactors (4 liters), a feed tank, constant head devices, diffusers for air supply and effluent storage tanks as shown Fig. 2.

The operating of these units started by growing an activated sludge culture in a jug on a batch basis. Continous operation was always initiated from long run to short run after achieving steady-state condition.

The detention time conducted with the 4 liter units consisted of the 12, 9.6 and 3 hour studies.

The flow control device was changed from a clamp connected to the internal diameter 10mm rubber tube. Clogging problems inside the tube was eliminated by providing constant head level.

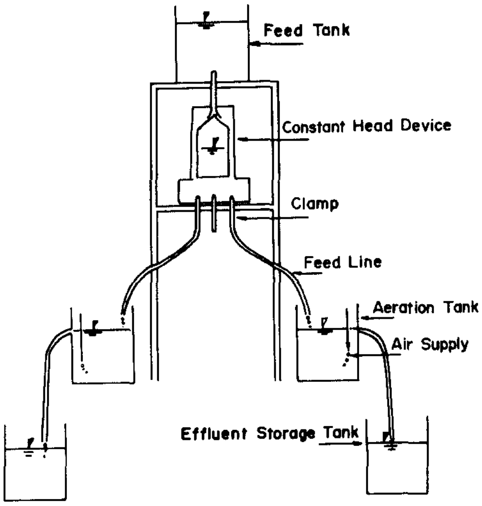

Fig. 2 Unit setup.

Aeration was made by using diffusers produc-

Table. 1. Glutamic acid characteristics and nutrient requirement

Analytical item	Values	Nutrient requirement [9)] for microbial growth
pH	3.4−3.8	
Ca++	0.48−0.55%	0.06%
Mg++	0.20−0.24%	0.04%
Fe++	0.014%	0.01%
Cl^-	5.5−6.6%	
SO_4=	0.5−1.0%	0.13%
P_2O_5	0.28−0.35%	0. 4%
K_2O	2.5−2.9%	0.05%
Total−N	3.95−4.46%	
NH_3−N	2.5−2.9%	
BOD	$150\times10^3-160\times10^3 mg/l$	
COD	$360\times10^3-380\times10^3 mg/l$	

ing a small buble from a porous medium.

Composite sampling collection for each unit was done during 24 hours for analysis. All analytical techniques were done according to the Standard Method (1971)[10] and/or Methods for Chemical Analysis of Water and Wastewater (1971).

It was assumed that the precision and accuracy given in the Standard Method was inherent to the analytical results unless otherwise denoted.

The feed solution was dilluted 250 times to the raw glutamic acid in order to maintain the proper *COD*(1500*mg*/*l*)and *BOD*(630*mg*/*l*) conceentraion. Table 1 shows the summary of the glutamic acid characteristics and nutrient requirement for the microbes growth.

Results

The results obtained from the laboratary units studies are presented in summary form in Table 2. The pH in the reactors ranged from 7.1 to 7.5 with dissolved oxygen range of 5 to 6*mg*/*l*

It was conducted the ultimate *BOD*(20day *BOD*) to determine the ability of biological treatment as shown Fig. 3 and BOD_{ult}/BOD_5 ratio appeared about 1.45.

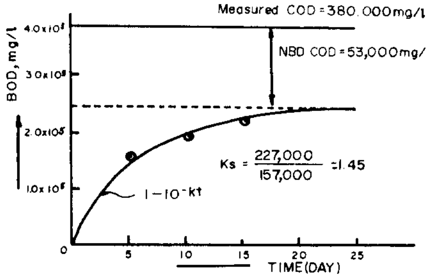

Fig. 3. Ultimate *BOD* versus 5—day *BOD*

The most important application of aeration in waste treatment is the transfer of oxygen to biological treatment proceses. Oxygen transfer was measured 56/*hr* and oxygenation capacity was 515*mg*/*l*/*hr* with range of 2—9 detention times.

Table 2. Summary of Opeating Results

Influent(*mg*/*l*)					Effluent(*mg*/*l*)						$\frac{ICOD}{VSS}$	K_1	$\frac{Fi-F}{t}$	$\frac{Fi-F}{Mv}$	$\frac{\triangle M_v}{Mv}$
					COD		*BOD*		*SS*						
Runs No.	t	Temp(°C)	*Fi*	*COD*	Total	Soluble	total	Soluble	M_T	*Mv*					
1	2.9	25	629	1445	937	421	367	59	466	402	1.28	0.65	196.6	1.41	0.34
2	5.7	25	626	1438	912	413	322	35	453	387	1.29	0.64	104	1.53	0.18
3	8.9	25	629	1445	903	412	265	22	430	378	1.30	0.63	68.2	1.61	0.11
4	11	25	626	1438	897	407	255	17	404	353	1.39	0.62	55.4	1.73	0.09

Microscopic observations indicated that the units operated from5.7 to 11 hour detention time contained protozoa, bacteria floc., but the units operated as 2.9 hour detention time contained filamentous microbes occuring sludge bulking problems in final sedimentation tank.

Discussion

The aeration only activated sludge process is a basic system when the evaluation of design constants for understanding greatly of other biological waste treatment processes. The key to the application of the each models equation to particular wastewater treatment problem lies in the evaluation of the constants and factors associated with the various mathematical models, therefore, an understanding of the effects of these for variables is necessary if an engineer is to make reasonable prediction of plant desings and performance.

In this study, it was shown that the percent removal of soluble *BOD* was about 92—97. Also, the efflucent *BOD* and *VSS* were shown in Fig. 4 according to aeration times.

Substrate removal rate

The values of $(Fi-F)/t$ and F from Table 2 were plotted to obtain substrate removal rate coefficient as decribed eq. (4d). The slope of the regression line gives the substrate removal rate coefficient and the value of the slope was approximately about 3.36 hr^{-1} at 25°C temperature by the least square computation with a correlation coefficient of 0.96 as shown Fig. 5.

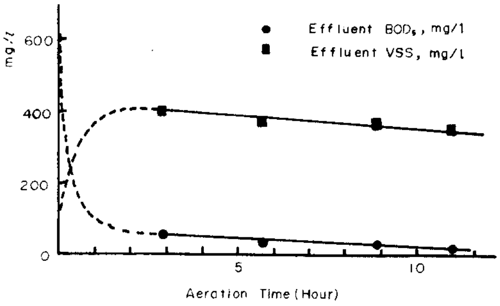

Fig. 4 Effluent soluble *BOD* and *VSS* with aeration time.

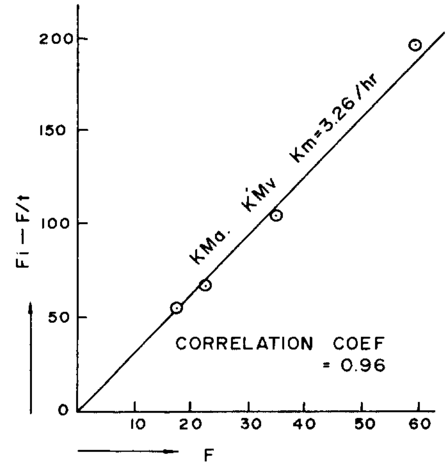

Fig. 5. Substrate removal rate

The substrate removal rate coefficients have been reported variously as 15/hr[12], 8.5/hr[13], 7.2/hr[6] etc. Rencent analysis of the pure okygen pilot plant data yieled the range of 2—67/hr(14, 15) in municipal wastewater treatment, a mean value of 15/hr [14] at 20°C seems reasonable but the value of the substrate removal rate coefficient in the glutamic acid wastewater is smaller than the value of the municipal wastewater as shown 3.26/hr.

The expected values of K and K'used by Michaelis-Menton and Eckenfelder models for various values of Ma, Mv are indicated in Table 4.

Table 4. Comparison of substrate removal rate coefficient in models.

t	Michaelis-Menton		Eckenfelder		McKinney
	$Ma(mg/l)$	K	$Mv(mg/l)$	K'	$Km(hr$-1)
2.9	322	0.0101	402	0.0081	3.26
5.7	310	0.0105	387	0.0084	3.26
8.9	302	0.0107	378	0.0086	3.26
11	283	0.0115	353	0.0092	3.26

From Table 4, it is possible to see that the values of Ma, Mv decrease with increasing detention time. Thus it is not surprising that K, and K' increase as Ma and Mv decrease. Also, the values of K and K' used by Michaelis-Menton and Eckenfelder models are depend on detention time, but Km used by McKinney model is independent. Therefore it seems that the use of Km is more convenient than those of KMa and $K'Mv$.

In Monod model, the kinetic constant values of Y, Ke, U_{max} and $K_s$from eq. (2h)(2i) were resp

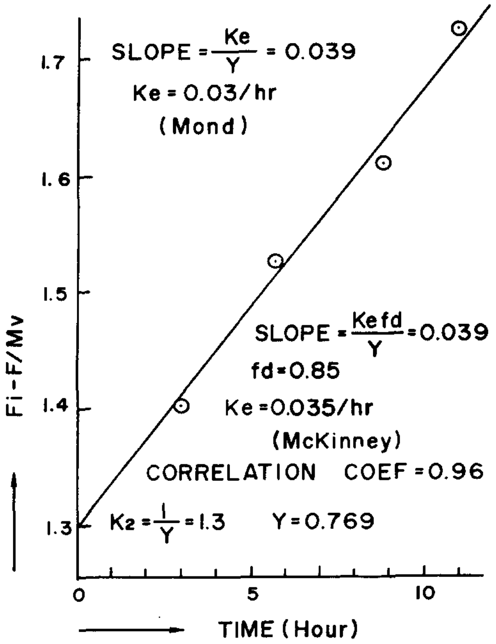

Fig. 6. Determination of Y and Ke.

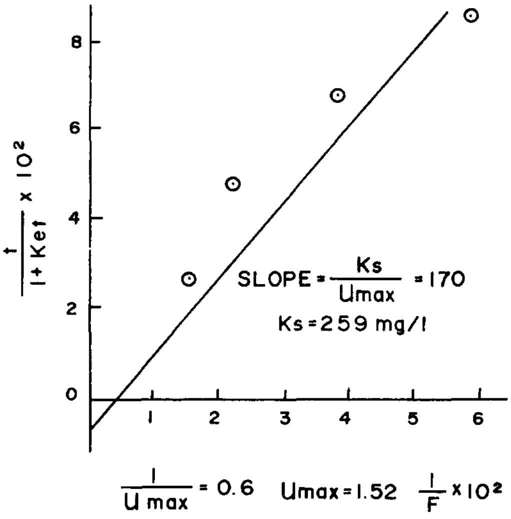

Fig. 7. Determinationof U_{max} and K_s.

ectively 0.769/hr, 0.03/hr, 1.52/hr, 259*mg/l* as shown in Fig. 6 and 7.

The values of unmetabolized substrate concentration can be caculated by eq. (1d), (2j), (3c), (4c) in each models and the caculated and observed values were compared in Table. 5.

Table 5. Comparison of unmetablolized BOD concentration in models

t	Measured *F(mg/l)*	Michaslis-Menton	$\triangle F$	Monod	$\triangle$F	Ecken-felder	$\triangle$F	Mckinney	$\triangle$F
2.9	59	60	1	85	+26	60	1	60	−1
5.7	35	32	−3	40	+ 5	32	−3	32	−3
8.9	22	21	−1	27	+ 5	21	−1	21	−1
11	17	17	0	22	+ 5	17	0	17	0

In Table 5 the nomenclature of $\triangle F$ is a difference from the caculated and observed values in each models. It can be seen that very good agreement exists between the caculated and observed values except Monod model as shown Table 5.

The constants of the endogenous respiration(Ke) and synthesis factor(Ks)

In Monod model, the value of *Ke* had been previously caculated in Fig 6. In Eckenfelder model, the determination of *Ke* and *a* is shown Fig. 8 plotted with Table 2 data, Thus the values of *Ke*, *a* and *Ks* were computed as 0.035/hr, 0.77 and 2.36/hr respectively.

In McKinney model, the values of *Ke* and *Y* was shown Fig 6 as 0.035hr^{-1}, 0.769. Also, the mean synthesis/total energy ratio(K_1) was 0.63

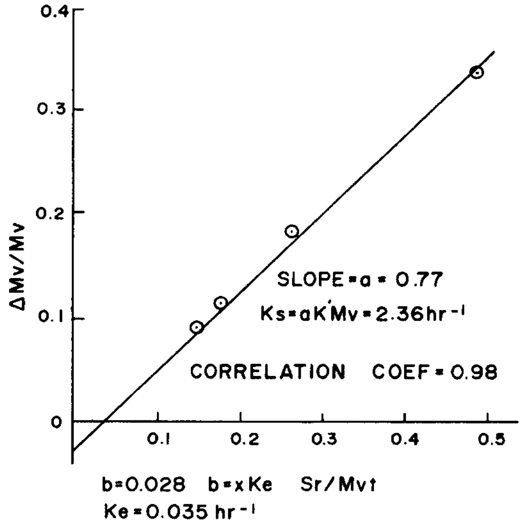

Fig. 8. Mass accumulation rate coefficients.

in Table 2. and $BODuit/BOD_5$ ratio(K_3) was 1.45 as shown Fig. 3. Therefore, the value of Ks can be caculated as 2.26hr^{-1}. The summary of the various constants in each models were shown in Table 6.

The values of a have been published as 0.6—0.7[16], 0.7[4] 0.5[17] respectively, In this study, the value of a is computed as 0.77.

The values of K_2 had been reported as 0.56—0.93[18] by the Burkhead and McKinney. Also, Hoover and Porges reported as the reasonable valule of 0.7.[19] In this study the value of 0.769 is with a good agreement in comparision of the above mentioned researchers.

Volatile suspended Solids and Total *BOD* in effluent

Table. 6 Comoarison of the *Ke*, *Ks* in models

Monod	Eskenfelder			Mckinney				
Ke(hr^{-1})	Ke(hr^{-1})	a	$Ks=aK'M_v$	Ke(hr^{-1})	K_1	K_2	K_3	K_s
0.03	0.035	0.77	2.36	0.035	0.63	0.769	1.45	2.26

Table 7. shows the comparison between measured and computed values for the volatile suspended solids(Mtv) in each model equations. The measured values are similar to the computed in each models shown Table 7.

Table 7. Comparison of the measured and the computed values

t	Measured	Monod			Eckenfelder				McKinney			
		Mv	Mi	M_{tv}	Ma	Me	Mi	M_{tv}	Ma	Me	Mi	M_{tv}
2.9	402	385	58	443	404	14	58	476	354	7	58	419
5.7	387	388	58	446	373	14	58	446	366	15	58	439
8.9	378	367	58	425	361	18	58	424	341	21	58	420
11	353	352	58	410	333	19	58	410	307	24	58	389

4. Total *BOD* in effluent and oxygen requirement.

Table 8 is shown the comparison of the measured and caculated values by using the two models of Eckenfelder and McKinney

The values caculated by McKinney equation is smaller than by Eckenfelder model because both Ma and Me yield BOD in Eckenfelder model, but the only Ma yields BOD in McKinney model. Thus, the values obtained from McKinney model is considered to be reasonable to the meaured values rather than those of Eckenfelder model.

Table 8. comparison of the measured and calculated values in effluent total *BOD*

t	Measured(mg/l)	Eckenfelder(mg/l)	McKinney(mg/l)
2. 9	367	393	342
5. 7	322	344	328
8. 9	265	325	295
11	255	300	263

Table 9 Comparison of the oxygen requirement in two models

Unit O_2 $mg/l/hr$

t	Eckenfeder	McKinney
2. 9	114	120
5. 7	64	62
8. 9	45	45
11	38	40

The amount of the total oxygen requirement by using the eq. (9b) and (9c) proposed by Eckenfelder and McKinney is indicated in Table 2.

Conclusions

The conclusions derived from this study are as folllows:

1. The mathematical for aeration only activated sludge process advanced by McKinney and Eckenfelder could be interpretted completly for the each parameters and found to be identical but the mathematical models proposed by Michalis—Menton and Monod-could be interpretted the only effluent soluble *BOD* and/or volume of sludge.

   It is hoped that this comparison and equalities presented will be enough to reduce and eliminate confusions caused by using other model in the same wastewater treatment process.
2. The substrate removal rate coefficients used by Michalis—Menton, Eckenfelder and McKinney except Monod model are equal as 3. 26hr^{-1} in glutamic acid wastewater even though the nomenclatures used by them is not presented equally. And those values were lower than that of the municipal wastewater.
3. The values of unmetabolized substrate concentration cacluated by Michaelis—Menton, Eckenfelder and McKinney equation appeared to be the same as shown Table 5. and similar to the measured values. However, the values caculated by Monod equation is higher than that of other researchers.
4. The endogenous respiration rate appeared to be the same as 0. 035/hr by Eckenfelder and McKinney model and to be as 0. 03/hr by Monod model. The synthesis factor appeared as 2. 36/hr and 2. 26/hr by Eckenfelder and McKinney model respectively.
5. The vlues of the sludge production computed by Monod, Eckenfeder and McKinney-equation is a little difference from the measured values.
6. The effluent total*BOD* caculated by McKinney model is smaller than that of Eckenfelder model.

The values obtained from McKinney model is considered to be reasonable to the measured value. It may be more useful to predict the effluent quailty by using McKinney model.

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活性슬럿지工法의 數學的 解析에 關한 硏究

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生物學的 廢水處理 工法中 活性슬럿지 工法의 數學的 解法에 關한 硏究는 Michaelis-Menton, Monod, Eckenfelder, McKinney 等에 의하여 開發되어왔다. 이들에 의해서 開發된 數學的 모델은 各記 硏究되어온 測面이 서로 다르고 使用된 記號가 相異하기 때문에 使用者로 하여금 많은 混亂을 주고있다.

本硏究는 各記 數學的 모델을 理論的, 段階的으로 分析하고, 各己 모델에서 使用된 記號의 相互關係를 比較하여 各己모델을 通一시켜 使用者가 理解하기 쉽도록 노력하였다.

實際로 模型實驗에는 Aeration Only Activated Sludge 工法을 採擇하고 시료는 Glutamic acid 廢水로 行하였다. 實際로 處理하여 얻어진 結果値와 各己모델의 計算値를 1) 有機物除去速度 2) 슬럿지生産率 3) 流出水에 處理되지 않고 남은 有機物濃度 4) 曝氣槽內 酸素要求量에 對하여 比較 檢討하였다.

本硏究에서 施行한 運轉結果와 그分析으로부터 McKinney와 Eckenfelder의 모델은 活性슬럿지工法의 生物學的 處理施設의 設計要素들을 모두 求할수 있으나 Michaelis-Menton과 Monod의 모델은 모든 要素들을 求할수 없다.