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BEYOND LINEAR PROGRAMMING*

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Abstract

Decision models are an attempt to reduce uncertainty in the decision making process. The models describe the relationships of variables and given proper input data generate solutions to managerial problems. These solutions may not be answers to the problems for one of two reasons. First, the data input into the model may not be consistant with the underlying assumptions of the model being used. Frequently parameters are assumed to be deterministic when in fact they are probabilistic in nature. The second failure is that often the decision maker recognizes that the data available are not appropriate for the model being used and begins to collect the required data. By the time these data has been compiled the solution is no longer an answer to the problem. This relates to the timeliness of decision making.

The authors point out through the use of an illustrative problem that stocastic models are well developed and that they do not suffer from any lack of mathematical exactiness. The primary problem is that generally accepted procedures for data generation are historical in nature and not relevant for probabilistic decision models. The authors advocate that management information system designers and accountants must become more familiar with these decision models and the input data required for their effective implementation. This will provide these professionals with the background necessary to generate data in a form that makes it relevant and timely for the decision making process.

1. Introduction

There is little doubt that the rapid growth in the use of operations research methods in managerial decision making has been the impetus for redefining the body of knowledge for management information systems. Over a decade ago, R.M. Trueblood recognized the impact of operations research on auditiong, inventory management, and forecasting. New decision models under development and those of the future will require infoɹmation which is not a part of the traditional information system [11, pp. 47~49].

In 1969, the American Accounting Association Committee on Managerial Decision Models was

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formed for the purpose of recommending new methods of preparing and reporting financial information needed to implement decision models. The Committee's report states that if the management information system (MIS) does not consider the assumptions of the models, erroneous or irrelevant data might be furnished the management scientist. In his article *Why Should the Management Scientist Grapple with Information Systems* [13, pp. 1~14], Andrew Vazsonyi argues that the use of decision models without regard to the data input into the model may provide solutions that are not answers to the problems at hand. Vazsonyi argues for a data based information which supports the managerial decision making process. Thus, the need to obtain an adequate understanding of decision models is critical in designing and implementing management information systems [1, p. 44]. C.T. Horngren expands upon this thought in stating that the entire decision process must be known to design the best information system [3. p. 777].

Management information systems play a critical role in obtaining maximum benefit from decision models. Without relevant and meaningful data available as input, the maxim, "garbage-in equals garbage-out," holds true for the simplest or most sophisticated decision model. Continuing increases in the complexity and use of decision models have generated a demand for more information as well as information of a different nature and form. This growth will bring about new demands on the resourcefulness and ingenuity of MIS designers to provide needed data. This paper explores two areas of concern: one, data requirements for one type of decision model and two, a demonstration of how these data may enhance the usefulness of the decision model through the incorporation of risk.

2. Decision Models and Realistic Assumptions

The manager's *raison detre* is to make decisions and managerial success is measured to a large degree by the number of "correct" decisions made. These decisions include sales forecasting, product distribution, selection of warehouse locations, inventory policies, capacity allocations, portfolio selections, lease or buy decisions, new capital expenditures and product mixes. Stated simply, the problem in making the right decision involves collecting and examining an almost infinite volume of information, developing alternative courses of action and then selecting and implementing the "best" alternative, compatible with other decisions made within the firm. Many of these decisions are characterized by complexity, conflict and uncertainty. Decision models help to reduce uncertainty by increasing the knowledge available to the decision maker. Decision models have become a prime tool for determining the interactions between pertinent variables, for quantifying these interactions, and determining their effect on resulting solutions.

Often unrealistic assumptions of certainty have to be made to obtain explicity solutions. The importance of understanding these assumptions cannot be overemphasized when making value judgements about solutions and the weight that should be given each alternative in making a decision. These decision models are by no means a cure-all for decision making, but they are a means of reducing uncertainty.

Economic order quantity models, the transportation model and the now famous linear programming model are examples of models in wide use today which require that input data be known with certainty. This assumption of perfect knowledge can produce a misleading solution, which, if not viewed with reservations, may have undesirable results. Generally speaking, there are few optimal solutions available in actual situations, or better stated, "Never forget—a solution is not necessarily an answer" [13, p. 5].

Common practices, in the face of uncertainty, include providing plenty of fat during the formulation of deterministic models such as those named above. This provides a contingency for unseen events or replaces random variables with an anticipated value based on experience [6, p. 487]. In addition to this practice, managers usually conduct sensitivity analyses to determine the possible consequences of a variable taking on a value within specified limits [9, pp. 441~56].

While the above do provide a cushion against the unknown, they do not allow the direct incorporation of risk into the decision model. The inclusion of risk is a relaxation of the certainty assumptions and increases the reality a model depicts. The cost of additional reality is the geometric growth in the complexity of the model. This increase in complexity may be illustrated by showing the incorporation of risk into the general linear programming problem. Consider the classical formulation:

$$\text{MAX or MIN } Z=CX \tag{1.1}$$

$$\text{Subject to } AX\leq b \text{ and,} \tag{1.2}$$

$$X\geq 0 \tag{1.3}$$

where

C is a row vector with N elements that are known constants

X is acolumn vector with N elements

A is a $M \times N$ matrix of constants

b is a column vector with M elements that are known constants

0 is a column vector with N elements that are zero

The mathematical expressions (1.1) through (1.3) are read as follows: Find non-negative values for the elements of X which maximize or minimize (1.1) and satisfy the constraints of (1.2). The elements of C are costs or profit contributions, the elements of b are measurements of anticipated demand or resources and the elements of A are called technological coefficients which may, for example, relate the production of goods to available resources or more basically may comprise the recipe for the product. The elements of A, b and C are treated as known constants in the general linear programming problem. Since future demand, resources for next year, costs, and profits are random variables, using this model may result in inaccurate conclusions becaus of the chance variation of these elements.

Now consider a stochastic linear programming model, one which incorporates risk by treating the vectors A, b and C of the general model as random[1)] rather than constant vectors. One such model is called the chance constrained programming model. It is formulated as follows:

$$\text{MAX or MIN } Z=f(C, X) \tag{2.1}$$

$$\text{Subject to } P(AX\leq b)\geq\alpha \tag{2.2}$$

where one or more of the elements of A, b and C are now random variables, "P" means probability, and α is a vector whose elements are constants between zero and one, representing risk. Each constraint has a probability$\geq(1-\alpha)$ of being violated, which says, for example, that a decision maker accepts a percent risk that production will not satisfy demand. In general, the problem is to find the "best" nonnegative values of X, the vector of decision variables, which satisfy most of the constraints of equation (2.2).

To date there exists no adequate solution for this general model which depicts greater reality by treating one or all elements as random variables. However, progress has been made to the extent that solutions can be found for certain cases. The move toward reality then is not lost in its entirety. The approach generally used to solve any stochastic programming problem is to first select a decision rule and some rational objective such as expected value optimization. Then, with certain assumptions, a deterministic equivalent is derived in which there exists no random variables. If this deterministic equivalent is linear in all respects and can be formulated as a linear programming problem, the powerful simplex algorithm may be used to generate a solution.

One type of objective that allows the transformation of a chance constrained programming problem to a suitable deterministic equivalent is the optimization of the expected value of a linear objective function subject to a class of linear decision rules. This is the so-called 'E-Model' [2, p. 25] :

$$\text{Maximize} \quad E(Z)=E(CX) \tag{3.1}$$

$$\text{Subject to} \quad P(AX\leq b)\geq\alpha \tag{3.2}$$

$$\text{and} \quad X=Db \tag{3.3}$$

where A, b, C and X are defined as in expressions (2.1) and (2, 2), E is the expected value operator and D is an $N\times M$ matrix. Equation (3.3) is the linear decision rule where the elements of D are to be determined with reference to a model such as defined by equations (2.1) and (2.2). The decision variables in X are related to the random variable of b by the elements of D. Thus, the vector X consists of random variables.

Assuming that the elements of b and C are normally distributed random variables and are uncorrelated, that the α are greater than one-half,[2] and that the elements of A are constants, the resulting deterministic equivalent for (3.1)—(3.3) is developed by Charnes and Cooper [2, p. 28] and appears as follows:

$$\text{Minimize} \quad -\mu_c D\mu_b \tag{4.1}$$

$$\text{Subject to} \quad v_i\geq 0 \quad i=1, \cdots\cdots, M \tag{4.2}$$

$$\mu_i(D)-v_i\geq 0 \tag{4.3}$$

$$-K^2_{\alpha i}\sigma^2_i(D)+K_{\alpha i}\mu^2{}_i(D)+v^2{}_i\geq 0 \tag{4.4}$$

$$\text{where} \quad \alpha^2{}_i(D)=E(a_iDb-b_i)^2 \tag{4.5}$$

$$\text{and} \quad \mu^2{}_i(D)=(\mu_{b_i}-a_iD\mu_b)^2 \tag{4.6}$$

The μ_c and μ_b are expected values of the elements of b and C; the v_i's are new variables (to be treated as slack variables) introduced to create a convex programming problem. The $\alpha^2{}_i$'s are the variances defined by (4.5) ; the a_i's are the i^{th} row of the matrix A; and the K_{α_i}'s are the upper α_i percentage points (the normal deviate) corresponding to α_i.

The mathematical program defined by equations (4.1)-(4.4) is a deterministic programming problem where the values of all variables except D are known, and the random variables are related through the estimated mean and variance of their distribution functions.[3] The increase in complexity over the linear programming problem defined by (1.1)-(1.3) is apparent.

3. Management Information Systems' For Probabilistic Models

The use of stochastic programming models, as is true for all models, depends upon the availability of data. Decision models such as the above example are not being used because of the inability to obtain real time solutions from the models and/or the inaccessability of data. Obtaining real time solutions is essentially a function of the availability of data in the form needed for the models. This leads to the conclusion that both the use and value of either deterministic or stochastic models pivot on the data element of the decision process. Without having found a solution to this problem, more sophisticated models are continually being developed which require further innovative refinements in the information system. Given both the concept of a data based information system and access to computerized information systems for extracting and manipulating data; there still remains the essential question of identifying basic data elements and their formulation.

Traditionally, the accountant is the source of financial data. And he is largely responsible for collecting, formating, processing, and analyzing data to be utilized in the decision process. These traditional accounting procedures and practices of data generation are not attuned to providing data for decision models and especially those which include probabilistic concepts utilized in stochastic programming models. Many MIS designers have begun to recognize the need for altering these practices and procedures in light of the demands of decision models. To date these changes have been directed toward satisfying the needs of that group of models known as deterministic decision models. Given the inputs, these models can produce an opaimum solution to the problem at hand.

The perfect knowledge assumption of deterministic models is bold and often a misrepresentation of the real world. Some decision theorists argue, quite correctly, that the perfect knowledge assumption provides a means of getting at the problem and that a solution based on the assumption is at least a starting point. The more nearly the certainty assumption is to the actual situation the better the solution. Also, this is often better than making no attempt to quantify the problem.

While these arguments may be true, there are models which do not depend on certainty assumptions and if data were available could be employed in the decision process. The proper consideration of uncertainty in the decision making process is of utmost importance. Incorporating risks into the decision model is an attempt to accomplish this task. Utilization of probabilistic models requires that data be available in usable form. This means that the MIS manager must begin to collect data and formulate distributions which depict the nature of costs, revenues, expenditures, performances, sales, and pertinent information areas.

4. The Product Mix Case

To lilustrate that probabilistic data supplied by the accountant are of significant value and that stochastic programming models, which require these data as inputs, can increase the effectiveness of the decision maker, consider the following simple example. A small manufacturing firm produces only two products for which demand exceeds supply. The firm's management has been able to expand existing facilities and increase production but not to a point where demand is completely satisfied. The current problem facing management is to determine the production mix which will maximize the total contribution to overhead and profit subject to the new production capacity. The following data are provided.

	Product	
	X_1	X_2
Selling Price	\$60	\$45
Variable Cost	30	24

Pre-Unit Contribution to Overhead and Profit	\$30	\$21
Number of Hours Needed to Produce One Unit		
Machine M	10	6
Machine N	5	4
Total Machine Hours Available		
Machine M	10,000	
Machine N	6,000	

The following linear program may be constructed from the data. Let X_1 and X_2 represent the number of units of their respective products to be produced.

$$\text{Maximize Profit Contribution} = \$30X_1 + \$21X_2 \tag{5.1}$$

$$\text{Subject to} \quad 10X_1 + 6X_2 \leq 10,000$$

$$5X_1 + 4X_2 \leq 6,000 \tag{5.2}$$

$$X_1, X_2 \geq 0 \tag{5.3}$$

After solving this problem, management's decision was to produce 400 units of Product X_1 and 1,000 units of Product X_2 with a resulting profit contribution of \$33,000. Management noted, however, that these values may be subject to variation.

Further analysis of the problem revealed that the process times, profit contributions and machine hours available were based upon forecasts, experiences, judgements and historical data. These values are not constant nor known with certainty. Management then asked if the problem could be solved by treating these values as random variables to in corporate uncertainty and increase the reality of the firm's decision model.

The firm's information system manager, using historical data and working closely with decision analysts, was able to determine that the M and N machine hour availability could be approximated by normal distributions with means of 10,000 and 6,000 hours and variances of 6,400 and 3,600 hours respectively. Likewise, the profit contributions of products X_1 and X_2 were found to be distributed normally with means of \$30 and \$21 and variance of \$4 and \$2, respectively. Because machines M and N were operated by computers, the number of hours required to process each product was found to have such a small variance that this variable could be treated as a constant with no loss in reality. The analysts decided to reformulate the program as a chance constrained program. In considering inventory requirements and the possible consequences of drastic variations in the number of machine hours available during each period, the analysts were able to obtain agreement from management that the production mix should be such that the probability of violating machine capacity constraints will not exceed approximately 2.3 percent. The analysts working within the information system constructed the following revised data table.

	Product	
	X_1	X_2
Profit Contribution	$N(30, 4)$	$N(21, 2)$
Number of Hours Required to Produce One Unit		
Machine M	10	6
Machine N	5	4
Total Machine Hours Available		

Machine M	$N(10000, 6400)$
Machine N	$N(6000, 3600)$
Covariance	$\mathrm{Cov}(M, N)=4800$

From the risk factor agreed upon, the revised data table and the 'E-Model' formulation of equations (3.1)−(3.3), the chance constrained program is:

$$\text{Maximize } E(Z)=E[30X_1+21X_2] \tag{6.1}$$

$$\text{Subject to } P(10X_1+6X_2\leq 10000)\geq 0.977$$

$$P(5X_1+4X_2\leq 6000)\geq 0.977 \tag{6.2}$$

and

$$X=Db \tag{6.3}$$

To solve the program of (6.1)−(6.3) it is necessary to transform it into a deterministic equivalent of the form (4.1)−(4.4). For the objective function (6.1). expanding and substituting into (4.1) gives:

$$(\mu_{c_1},\ \mu_{c2})\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}\begin{pmatrix} \mu_{b_1} \\ \mu_{b_2} \end{pmatrix}=$$

$$(30, 21)\quad \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}\begin{pmatrix} 10000 \\ 6000 \end{pmatrix}=$$

$$(30d_{11}+21d_{21}),\ (30d_{12}+21d_{22})\ \begin{matrix} 1000 \\ 6000 \end{matrix}=$$

$$\text{Maximize } 300000d_{11}+210000d_{21}+180000d_{12}+126000d_{22} \tag{7.1}$$

For the constraints (6.2). First, using (4.6) and expanding gives:

$$\mu_1(D)=(\mu_{b1}-a_1D\mu_b)$$

$$=10000-\left\{(10, 6)\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}\begin{pmatrix} 10000 \\ 6000 \end{pmatrix}\right\}$$

$$=10000-(1000000d_{11}+60000d_{21}+60000d_{12}+36000d_{22})$$

$$\mu_2(D)=\mu_{b2}-a_2D\mu_b)$$

$$=6000-\left\{(5, 4)\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}\begin{pmatrix} 10000 \\ 6000 \end{pmatrix}\right\}$$

$$=6000-50000d_{11}+40000d_{21}+30000d_{12}+24000d_{22})$$

Then, rearranging and substituting into (4.3) gives:

$$100000d_{11}+60000d_{21}+60000d_{12}+36000d_{22}+v_1\leq 10000 \tag{7.2}$$

$$50000d_{11}+40000d_{21}+30000d_{12}+24000d_{22}+v_2\leq 6000 \tag{7.3}$$

Secondly, expanding (4.5) and substituting for the a_i values gives:

$$\hat{\alpha}^2{}_i(D)=E(-\hat{b}_i+a_iD\hat{b})^2$$

$$\hat{\alpha}^2{}_1(D)=E(-\hat{b}_1+a_1D\hat{b})^2$$

$$=E\left\{-\hat{b}_1+(10, 6)\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \end{bmatrix}^2\right\}$$

$$=E(-\hat{b}_1+(10d_{11}+6d_{21})\hat{b}_1+(10d_{12}+6d_{22})\hat{b}_2)^2$$

$$=E(\hat{b}_1(10d_{11}+6d_{21}-1)+\hat{b}_2(10d_{12}+6d_{22}))^2$$

Further expansion and taking the expected value gives:

$$\hat{\alpha}^2{}_1(D)=\sigma^2{}_{b_1}(10d_{11}+6d_{21}-1)^2+\sigma^2{}_{b_2}(10d_{12}+6d_{22})^2$$

$$+2\rho(b_1, b_2)\sigma_{b1}\sigma_{b_2}(10d_{11}+6d_{21}-1)(10d_{12}+6d_{22})$$

$$\rho(b_1, b_2)=\frac{\mathrm{Cov}(b_1, b_2)}{\sigma_{b_1}\ \sigma_{b_2}}=\frac{4800}{(80)(60)}=1, \text{ therefore}$$

regrouping gives the following:

$$\hat{\alpha}^2{}_1(D)=(\sigma_{b_1}(10d_{11}+6d_{21}-1)+\sigma_{b_2}(10d_{12}+6d_{22}))^2$$

Using the same procedure, it follows that:

$$\hat{\alpha}^2{}_2(D)=(\sigma_{b_1}(d_{11}+4d_{21})+\sigma_{b_2}(5d_{12}+4d_{22}-1))^2$$

The constraint in (4.4) becomes:

$$v_i{}^2\geq K_\alpha{}^2{}_i\hat{\alpha}^2{}_i(D)=4\hat{\alpha}^2{}_i(D)$$

Taking the positive square root of (7.6) using (7.4) and (7.5) and the values for σ_{b_1} and σ_{b_2}, the last two constraints of the deterministic equivalent are written as:

$$1600d_{11}+960d_{21}+1200d_{12}+720d_{22}-v_1\leq 160 \quad (7.4a)$$

$$800d_{11}+640d_{21}+600d_{12}+480d_{22}-v_2\leq 120 \quad (7.5a)$$

Rearranging and collecting equations (4.2), (7.1), (7.2), (7.3), (7.4a) and (7.5a), the deterministic equivalent program becomes:

$$\begin{aligned}&\text{Maximize } 300000d_{11}+180000d_{12}+210000d_{21}+126000d_{22}\\&\text{Subject to}\\&100000d_{11}+60000d_{12}+60000d_{21}+36000d_{22}+v_1\leq 10000\\&50000d_{11}+30000d_{12}+40000d_{21}+24000d_{22}+v_2\leq 6000\\&1600d_{11}+1200d_{12}+960d_{21}+720d_{22}-v_1\leq 160\\&800d_{11}+600d_{12}+640d_{21}+480d_{22}-v_2\leq 120\\&v_1, v_2\geq 0\end{aligned} \quad (8.1)$$

Using the simplex algorithm, an optimum solution to the deterministic equivalent in (8.1) is $d^*_{11}=1/25$, $d^*_{12}=0$, $d^*_{21}=1/10$ and $d^*_{22}=0$. For the optimal policy, the expected values of X_1 and X_2 can be found by using this solution and the linear decision rule given by (6.3), $X=Db$.

$$E(X)=E(Db)=DE(b)$$

$$E(X)=\begin{bmatrix}1/25, & 0\\1/10, & 0\end{bmatrix}\begin{pmatrix}10000\\6000\end{pmatrix}$$

$$E(X_1)=400 \text{ units}$$

$$E(X_2)=1000 \text{ units}$$

The variance of X_1 and X_2 are given by:

$$\mathrm{Var}(X)=D\ \mathrm{Var}(b)$$

$$\mathrm{Var}(X)=\begin{bmatrix}1/25, & 0\\1/10, & 0\end{bmatrix}\begin{pmatrix}6400\\3600\end{pmatrix}=\begin{pmatrix}256\\640\end{pmatrix}$$

The respective variance of X_1 and X_2 are 256 and 640 and X_1 is $N(400, 256)$ and X_2 is $N(1000, 640)$.

Though the optimal solution to the stochastic program is not *the answer*, it is valuable because the uncertainties or profit margins and available machine time have been transformed into estimated variances for the production mix. Improved information is now available and should be used in conjunction with other economic, social and behavioral data so that more knowledgeable decisions may be made by the firm. For example, these results may be useful in managing inventories of the raw materials required to produce products X_1 and X_2.[4)] model assumptions. For a full appreciation of this phenomenon, one need only recall that several assumptions were made for this presentation.

5. Probabilistic Information Systems and New Decision Horizons

Stochastic programming methods may be used for increasing reality in other major areas of the decision process. Consider, for example, the capital budgeting problem. Deterministic models such as linear, integer and dynamic programming models have been used to assist managers in selecting an alternative from a set of mutually exclusive projects [7, p. 21] or in selecting an optimum set of proposals froma large number of alternatives, given that certain resources are fixed [12, pp. 552~61]. Howeve, in each case the cash inflows of each period and, thus, the net present value of a project or proposal is considered constant and known. William H. Jean, in his two recent texts [4] [5], presents excellent discussions on techniques which incorporate chance variation into capital budgeting decisions. His presentations illustrate the use of expected valnes and variances to determine the expected net present value and its variance for the selection of various investment proposals. However, Jean's presentation assumes that the expected value and variance of the cash inflow at each time period is known with certainty.

Another illustration is Bertil Näslund's use of the chance constrained programming model for finding an approximate solution to a capital budgeting problem under risk [8]. Näslund treats the b and C vectors of the problem on page 5 as constants but allows the elements of the matrix A to be random variables. His deterministic equivalent is quadratic in the constraints rather than linear and the formulation requires approximation techniques in the solution process.

The application for stochastic models in the decision process are numerous but there are stumbling blocks which must be overcome in the application of these models. For example, Jean discusses the excessive use of the normality assumpiton [5, pp. 115~117]. He points out that this assumption may be totally inappropriate for cash flow in a given period. The assumption for the most part is made for two reasons: (1) to allow simplification of the model so that it may be solved, and (2) simply because of the nonavailability of data or the excessive cost of massaging historical data so that it is usable in probabilistic decision models.

Given that relevant data are available and in usable form, another weakness of stochastic models is the requirement of management input in the form of risk decisions, such as selecting an α in the example problem. While decision makers indirectly and consistently think in terms of and make decisions utilizing probability concepts, they are hesitant to commit this process to numbers. Clearly, the education of managers with respect to the usefulness of stochastic programming solutions as aids in the decision making process is an area of concern in future management education [10, p. 140~41].

Having provided reasons for disillusionment regarding the use of stochastic programming models, it is apropos to try and answer the question, why bother? First, the potential payoff of the additional information provided by these models may prove to be very valuable for long-run one-time decisions involving large capital expenditures. Information which reduces uncertainty should be welcomed by decision makers. Secondly, and probably the more important reason for advocating the use of these models is that decisions are going to have to be made with or without this additional

This example demonstrates the additional complexity which accompanies the relaying of decision information and managerial effectiveness is improved through the use of these models.

6. Conclusions

Because of the expanding role of MIS in all decision processes, it is quite clear that MIS designers must be acquainted with the available decision tools. They must know the capabilities and limitations and must temper decisions with experience and subjective judgements about the intangibles which defy quantification.

But, even more important is the role of MIS in the decision science. This role is one of expanding research into methods of data cellection, manipulating and formating decision models and, in particular, stochastic models. Decisions will have to be made cencerning the trade-off between the cost of obaining and storing data versus the benefits of more effective management through the utilization of advanced decision models. This involves an insight into the "risk" involved in considering solutions to nonstchastic problems. The MIS designer's knowledge and experience is indispensable in the development of the system. Because data collection and storage can be, and should be, treated as an inventory cost, the efficiency of the design and use of the information system is a major consideration. Only in this way might it be practical to query and search a large data base for the purpose of developing probability distributions for variables used in models.

The need for the personal involvement of MIS designers and managers in recognizing and defining the future data requirements for advance decision models is clear to the authors. Of no less importance is the assistance that the MIS managers may provide in the education of managers. Hopefully, however, because of the current trend of exposing potential managers to decision models and operations research in our universities this problem will be somewhat alleviated in the future.

Stochastic programming models promise to provide future mangement with information for the decision process that is not now available. While the misuse of these programming techniques or a belief that they are ***the answer*** to eliminating uncertainty can be and usually is disastrous, it may be equally disastrous to dismiss almost thirty years of research because of a lack of knowledge, denial that such a tool exists, or the fear of using the results of the models. The incorporation of risk into decision models is a relatively new concept with much to be accomplished before it is accepted and used to maximum advantage. The challenging role of MIS is clear-develop a system that is supportive of the developing decision models.

1) A random vector is one which contains one or more random variables as elements.

2) This assumption is not unrealistic because if it is permissible for a constraint to be violated more than fifty percent of the time, then it probably should not be considered a constraint in the problem.

3) Note that this transformation has a corresponding dual solution. The duality relationships may be used to investigate the sensitivity of any decision vector X chosen to the risk factors, the α_i, and the variability of the vectors C and b before their values become known in the future.

4) There are other obvious advantages to having such data available. The reader might also recognize that the dual for the deterministic equivalent canbe written and various levels of risk, the α_i, can be examined for their impact on the optimal policy, X.

5) As mentioned previously, chance constrained programming is only one of several stochastic programming

models. And the 'E Model' is only one of three types of optimizing objectives. See Charnes and Cooper for a presentation of the 'V Model' objective, minimizing a generalized mean square error, and the 'P Model' objective, a satisficing approach. Also, see Hellier for a presentation on bounded or 0-1 decision variables in chance constrained programming.

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