Martin J. Beckmann 敎授의 강연 (2)

3. ALLOCATION AND PRICING IN PUBLIC TRANSPORTATION AND THE FREE RIDER THEOREM*

Martin J. Beckmann.**

Abstract

Consider a time interval during which the demand for trips is fixed (e.g. the rush hour period). The traveller has a choice between various public modes, whose travel times and fares are fixed, and the automobile mode, for which travel time and cost depend on the volume of traffic flow on those roads, which are subject to congestion. We consider the equilibrium in terms of a representative traveller, who choses for any trip the mode and route with the least combined money and time cost. When several (parallel) modes or routes are chosen, then the combined cost of money and time must be equal among these.

Our problem is first, to find the optimal flows of cars and of public mode carriers on the various links of their networks and second the optimal fares for trips by the variousmodes. The object is to minimize the total operating costs of the carriers and cars plus the total time costs to travellers.

The optimal fares are related to, but not identical with the dual variables of the underlying Nonlinear Program. They are equal to these dual variables only in the case, when congestion tolls on trips or on the use of specific roads are collected from automobile users. When such tolls are not collected, they must be passed on as subsidies to travellers using competing modes. The optimal fares of public modes are then reduced by the amounts of these subsidies. Note that subsidies are not a flat payment to public carriers, but are calculated on the basis of tickets sold.

Fares and subsidies depend in general on the period considered. They will be higher during periods of higher demand. When the assumption of fixed trip demand is relaxed, this fare system is no longer best, but only second best since too much traffic will, in general, be generated. The Free Rider Theorem states the following: Suppose road tolls can be charged, so that a best pricing system for public modes is possible. Then there may exist free rides on some routes and modes, but never on a complete round trip.

*I have benefited from discussion with William Spreitzer and Thomas Golob of the Department of Transportation Research and Urban Analysis, General Motors Research Laboratories, and I should like to express my sincere thanks for their generous critcism. The economic imlpications developed here are, of course, entirely my own.

**Brown University and Technical University of Munich

1. Introduction

The object of this paper is the proper allocation and pricing of resources in a public transportation system. For convenience this system will be specified as a bus system operating on its own roads or road lanes. Alternatively it may be a rail system. The important assumption is that delays to automobile traffic from congestion will not affect the public mode under consideration.

Its resources are a given number of busses and their drivers. The problem is thus a short run one of optimal utilization rather than a long run one of optimal investment. The period considered is also a short one: the daily rush hour period when most of the home to work trips or work to home trips must be made. The demand for trips in that period will be considered fixed. Thus q_k^k trips are demanded between origin i and destination k regardless of the state of a system *i.e.* of cost and delays.

Travellers have, however, a choice of using their cars or the public transportation system. If the frequency is given with which the public mode operates on the various links ij of its network-and this may include a frequency of zero on routes that are not operated-then the problem is an equilibrium problem of predicting the number of travellers by car and by bus on the various links of their networks. In addition however, we face an optimization problem: how to set the frequency of trips of the public mode so as to achieve an overall optimum. In view of the fixity of demand such an optimum can only mean a minimum of total transporttaion cost. Again the cost under consideration is a short run one consisting of the operating cost of the public mode and of the cars and of the time cost to travellers. All fixed costs of the public transportation system can be safely disregarded. The entire problem can be viewed as a generalization of that studied for automobile traffic in networks in *Studies in the Economics of Transportation* (1). Although we have fixed total demand for trips, the demand for automobile trips between pairs of points is now a function of auto travel times and costs in relation to transportation times and costs by the public mode.

Therefore an important aspect of this problem is pricing. For travellers' decisions are critically dependent on the fares they are charged for trips by the public mode and on congestion tolls that may (or may not) be charged for automobile travel on various roads, and on parking charges aswell.

A solution to the problem sketched is of far reaching economic significants. It has a bearing on the question of subsidies to public transportation systems and/or tolls to automobile users. This question is made more difficult when a first best solution involving road tolls is not available. Then we must look for a second best solution without road tolls. Does this imply subsidies to the public mode? If so, how large should they be and in what form should they be distributed?

2. The formal model

We consider a network consisting of

nodes i,j,k and

links or roads ij,jk

defined by the pair of nodes that are connected by this link. It is necessary to distinguish roads or lanes in either direction. The direction is indicated by the order of the nodes: a road ij has a direction from i to j.

In the following we ignore the fact that public mode users may have to travel by car from their homes to a station. We assume in fact that all origins of trips have been aggregated so as to be identified with stations of the public mode. (Each station comprises its hinterland).

The possibility that some public mode travellers may perfer to go more than a minimum distance by car is ignored. It is useful to consider the problem first without congestion. Let

x_i^k number of cars travelling from i to k

r^k number of riders per car with destination k

y_i^k number of travellers by public mode from i to k

z_{ij} number of busses travelling link fj per unit of time

q_i^k demand for trips from i to k

a capacity of a bus

t_{ij} travel time on link ij by car

s_{ij} travel time on link ij by bus

b_{ij} money cost of operating a bus on link ij

c_{ij} money cost of operating a car on link ij

d money equivalent of the value of time

p^k cost of parking a vehicle at destination k

w_i^k waiting time at origin i for busses when travelling to destination k.

This waiting time is assumed independent of the frequency of busses since bus users know the schedule. It merely reflects waiting time due to random delays. It includes waiting at transfers which is also independent of requency, due to scheduling.

The following constraints describe the interactons of flows

(1) $q_i^k = y_i^k + r^k\ x_i^k$ demand distribution among modes

(2) $x_i^k = \sum_j x_{ij}^k - x_{ji}^k \quad i \neq k$ flow of carsthrough point i

(3) $y_i^k = \sum_j {}_{ij}^k - y_{ji}^k \quad i \neq k$ flow of bus riders through intermediate ooint i

(4) $0 = \sum_j z_{ij} - z_{ij}$ bus arrivals = bus departures

(5) $az_{ij} \geq \sum_k y_{ij}^k$ bus capacity must be adequate

We sometimes use the aggregate flows on each link

(6) $x_{ij} = \sum_k x_{ij}^k$ cars travelling on road ij

(7) $Y_{ij} = \sum_k y_{if}^k$ bus riders on link ij

(8) $r_{ij} = \sum_k x_{ij}^k\ r^k$ automobile travellers on ij

The object is to minimize total cost. This consists of the operating cost of busses

$$\sum_{ij} b_{ij}\ z_{ij},$$

the operating cost of cars

$$\sum_{ij} c_{ij}\ x_{ij},$$

the time cost to travellers

$$d \sum_{ij} s_{ij}\ y_{ij} + d \sum_{ijk} t_{ij}\ x_{ij}\ r^k,$$

plus the waiting costs for bus travellers

$$d \sum_{i,k} w_i^k\ Y_i^k,$$

and parking fees

$$\sum_{i,k} p_k\ x_i^k$$

An optimum utilization of the transportation system is achieved when total cost is minimized subject to the network flow constraints

(9) $x_{ij}^k,\ Y_{ij}^k,\ z_{ij} \geqq O,$

$$\text{Mxn} \sum_{ij} ds_{ij}\ Y_{ij} + \sum_{ij,k} (c_{ij} + dt_{ij})\ r^k\ x_{ij}^k +$$

$$+ \sum_{i,k} dw_i^k\ Y_i^k + \sum_{ij} b_{ij}\ z_{ij} + \sum_{i,k} p^k\ x_i^k$$

subject to (1)...(9).

This is a Linear Program. In the absence of capacity constraints (the number of busses can be chosen as large as desired but at a cost) this LP is clearly feasible. Its optimal solution will be discussed in terms of the efficiency conditions as given by Koopmans' price theorem(2).

3. Discussion of efficiency conditions

The dual variables or efficiency prices to be associated with the various constraints are, respectively,

$$\alpha_i^k,\ \lambda_i^k,\ \mu_i^k,\ \omega_i,\ \beta_{ij}$$

The Langrangean of the linear program may then be written

$$- \sum_{ijk} ds_{ij}\ Y_{ij}^k - \sum_{ij,k} (c_{ij} + r^k\ dt_{ij}) x_{ij}^k - \sum_{i,k} dw_i^k\ Y_i^k -$$

$$- \sum_{ij} b_{ij}\ z_{ij} - \sum_{i,k} p^k\ x_i^k$$

$$+ \sum_{i,k} \alpha_i^k [Y_i^k + r^k\ x_i^k - q_i^k]$$

$$+ \sum_{i,k} \lambda_i^k\ [\sum_j (x_{ij}^k - x_{ji}^k) - x_i^k]$$

$$+ \sum_{i,k} \mu_i^k\ [\sum_i (Y_{ij}^k - Y_{ji}^k) - Y_i^k]$$

$$+ \sum_i \omega_i \sum_j (z_{ij} - z_{ji})$$

$$+ \sum_{ij} \beta_{ij} [a z_{ij} - \sum_k Y_{ij}^k]$$

The efficiency conditions are now as follows

(10) $x_{ij}^k \{\substack{=\\ \geqq}\} 0$ according as $-(c_{ij} + r^k\ dt_{ij} + \lambda_i^k - \lambda_j^k \{\substack{\leqq\\ =}\} 0$

(11) $Y_{ij}^k \{\substack{=\\ \geqq}\} 0$ according as $-ds_{ij} + \mu_i^k - \mu_j^k - \beta_{ij} \{\substack{\leqq\\ =}\} 0$

(12) $z_{ij} \{\substack{=\\ \geqq}\} 0$ according as $-b_{ij} + \omega_i - \omega_j + a\beta_{ij} \{\substack{\leqq\\ =}\} 0$

(13) $x_i^k \{\substack{=\\ \geqq}\} 0$ according as $-p^k+\alpha_i^k r^k-\lambda_i^k \{\substack{\leq\\ =}\} 0$

(14) $Y_i^k \{\substack{=\\ \geqq}\} 0$ according as $\alpha_i^k-\mu_i^k-w_i^k \{\substack{\leq\\ =}\} 0$

Since all constraints but (5) are eouations, the signs of the α, λ, μ and ω are indeterminate. Forβ e have however,

(15) $\beta_{ij} \{\substack{=\\ \geqq}\} 0$ according as $az_{ij} \{\substack{\geq\\ =}\} \sum_k Y_{ij}^k$

The economic interpretation of the efficiency conditions and of the Lagrangean multipliers is as follows.

Let there be a positive amount of travel between origin i and each destination $k, i \neq K$ Then for each i and k one of thl variables x_{ij}^k must be positive so that the equal sign is taken on in the right half of (10) for some j.

$$\lambda_i^k \leqq c_{ij}+r^k dt_{ij}+\lambda_j^k \qquad \begin{array}{l}\text{all } j,\\ =\text{for some } j.\end{array}$$

This is equivalent to

(16) $\lambda_i^k=\operatorname*{Min}_j[c_{ij}+r^k dt_{ij}+\lambda_j^k]$

Since in constraint (10) $i \neq K$ we have automatically

(17) $\lambda_k^k=0$

Now the statements (16) and (17) together determine$_i^k$ to be the length of a shortest path from i to k in a network whose links have lengths

(18) $c_{ij}+r_i^k\ dt_{ij}$

Now (18) are the actual costs of travel to the riders of a car when traversing link ij.

The efficiency condition (10) says therefore no more than that efficiency requires that auto users should choose shortest paths (in terms of combined money and time costs).

To summarize

(19) $x_{ij}^k \{\substack{=\\ \geqq}\} 0$ according as $c_{ij}+r^k\ dt_{ij}+i_j^k \{\substack{\geq\\ =}\}\ \lambda_i^k=$

$\operatorname*{Min}_h[c_{ih}+r^k\ dt_{ih}+\lambda_h^k]$

The same may once more be derived for bus riders

(20) $Y_i^k \{\substack{=\\ \geqq}\} 0$ according as $\beta_{ij}+ds_{ij}+\mu_j^k \{\substack{\geq\\ =}\} \mu_i^k=$

$=\operatorname*{Min}_h[\beta_{ih}+ds_{jh}+\mu_h^k]$

Overall efficiency requires that bus riders, too, should use shortest paths in a network whose link have lengths

(21) $\beta_{ij}+ds_{ij}$

To induce that behaviour, β_{ij} must then be the fare to be charged for riding the bus on link ij.

We turn to equations (11) and (12) which determine the mode choice of travellers. Again if q_i^k 0 all i and $i \neq K$, at least one of x_i^k and y_i^k must be positive. Combining the right halves of (13) and (14),

(22) $\alpha_i^k \leqq \frac{1}{r^k}[\lambda_i^k+p^k]$

(23) $\alpha_i^k \leqq \mu^k+w_i^k$

and then "plus" sign applies in at least one of (22), (23). This may be written as

(24) $\alpha_i^k=\mathrm{Min}[\frac{\lambda_i^k+p^k}{r^k},\ w_i^k+\mu_i^k]$

The message of (24) is that travellers will choose the least costly mode (or either one if both cost the same). The first term in (24) is the cost of riding than car plus parking calculated per rider; The second is the cost of riding the bus plus expected waiting time. Summarizing we may say that system efficiency requires users to be rational: always chose the cheaper mode and the shortest route on that mode.

The efficiency implications for the operators of the public transportation system are contained in equation (12).

This is best comprehended when the right hand half is sumeed over the entire round trip of a bus

$$a\sum_{\sigma}\beta_{ij}\{\leqq\}\sum_{\sigma}b_{ij}+\sum_{\sigma}(\omega_j-\omega_i)$$

On any round trip the terms cancel and we have

(25) $a\sum_{\sigma}\beta_{ij}\leqq\sum_{\sigma}b_{ij}$ and "="

on road trips that should be made *i.e.* for which $z_{ij}>0$ on all links. Condition (25) then shows that on no round trip will fares collected $a\sum_{\sigma}\beta_{ij}$ exceed operating cost, but on all efficient routes fares will actually-cover this cost. (Recall that operating costs are short run costs excluding all fixed charges).

Thus system efficiency requires a pricing system that forces bus operators to select rational routes and frequencies. For the penalty for irrationality is economic loss. The frequencies must be such that busses are filled on at least one segment of each round trip: Condition (15) states that otherwise fares would have to be zero in violation of the second half of condition (25).

4. The Free Rider Theorem

One implication of (15) is that there may be free rides available on the bus (in the sense of zero fares). Every ride is free on a bus not filled to capacity. But is there then available also a free ride back to the origin? This would imply that a vehicle round trip could exist with excess capacity. On such a round trip the condition (25), second half, would be violated. In any cases it is easy to see that a reduction of bus frecuency on this circular route could be achieved with a resulting reduction in overall costs. It follows that "There may be free rides but never free round trips". (Free Rider Theorem). In other words if they don't catch you going there they will catch you on the way back.

5. Extension to the Case of Congestion

Having laid the foundations, the extension to the more relevant and interesting case of congestion is straightforward. Congestion means that travel times for cars are a function of traffic flows

(26) $t_{ij}=t_{ij}(x_{ij})$

The congestion function t_{ij} has the following properties

$t_{ij}(x_{ij})=$constant for small flows x_{ij}

(27) $\frac{dt_{ij}}{dx_{ij}}>0$ for large x_{ij}

$$\frac{d^2t_{ij}}{dx_{ij}^2}>0 \text{ for large } x_{ij}$$

These properties imply that the t_{ij} are monotone nondecreasing convex functions. Upon substitution in the objective function this becomes a convex rather than a linear function to be minimized subject to linear constraints as before. The Kuhn-Tucker conditions apply and replace the Koopmans efficiency conditions. The only change is in quation (10) as follows:

(10a) $x^k_{ij}\{\gtreqless\}0$ according as $-c_{ij}-r^k\ t_{ij}-t'_{ij}\sum_k r^k\ x^k_{ij}+$

$+\lambda^k_i-\lambda^k_j\{\lesseqgtr\}0$

In words, the expression (18) for the length of the links has been increased by an additional term

(28) $t'_{ij}\sum_k r_k\ x^k_{ij}$

The (28) measures the incremental time delay to all travellers on link ij caused by an additional traffic unit on link ij.

This is the well known result (1) that efficiency of road utilization requires the implimentation of suitable road user tolls on congested links. The first best solution is therefore to impose tolls and thereafter let travellers choose cheapest modes and cheapest roads in the light of these tolls. The induced shifts in car use can only decrease automobile traffic flows on congested roads and increase the use of the public mode.

6. Pricing and Allocation without Road Tolls

The imposition of specific road user charges may not be practical or politic [cf, however, 3]. In the situation assumed here, are alternative methods of pricing available that can achieve the same efficient result? Such a pricing system must be consistent with the efficiency conditions (10a), (11)... (15). The only difference must therefore lie in the implementation of the Lagrangean multipliers α, λ, μ and β.

Supression of the term (28) in each equation (10a) means economicaly speaking a subsidy of that amount to each vehicle or of amount

(29) $\sigma\ \Sigma^k_{ij}=\dfrac{t'_{ij}\sum_h r^h\ x^h_{ij}}{r^k}$

to automobile riders individually on link ij. If the mode choice is not to be affected by this subsidy, then bus riders must be allowed the same deduction (29) from their fares. The new fares to be charged to bus travellers with destination k on link ij must therefore be

(30) $f^k_{ij}=\beta_{ij}-\underset{\sigma}{\Sigma^k_{ij}}$

Notice that link fares new depend on ultimate destination k. This expression may actually be negative. If the total fare for a trip turns out to be null or negative, this is to be implemented as a zero free round trips on the public system.

With an efficient public transportation system being efficiently operated along the lines developed here, it is unlikely, however, that the congestion tolls should reach a general level that would cause fares to drop to zero. The result is then a subsidization of bus fares to the extent of preserving an (near) optimal allocation of trips between public and private mode. Nothing will be said here about the source of the subsidies. It may be assumed that these are taken out of general taxes.

This subsidization scheme cannot remedy, however, the inefficiency in the utilization of congested roads by the travellers that choose to go by car. For in the absence of road tolls, the choice of routes is, in general, at least as the incidence of congestion is unequal among different links. The second best falls therefore, shor of the optimum.

This is reinforced when the assumption of a fixed total demand for trips is dropped, for then the subsidization of trips during congested times does not encourage sufficiently a shift of shiftable trips to non-congested times.

7. Conclusion

If a full fledged economic theory of traffic flow in networks will some day exists, then it must treat allocation and pricing as its contral problems. The challenge of traffic flow to economic theory lies in the fact that traffic allocation is usually solved not by pricing or that prices do not reflect directly and adequately the true social cost incurred. This was the central thene of "*Studies in the Economies of Transportation*"(1). This paper has tried to include in this issue the old economic problem of the optimal allocation between public and private modes. But it is no more then a first step in the direction of a full treatment of this timely and challenging problem, to wit, the Economic of Traffic Flows in Networks.

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4. MANPOWER PLANNING IN A HIERARCHICAL ORGANISATION*

M.J. Beckmann** J. Fischer**
E. Marwitz** J. Rabe***
G. Uebe**

Abstract

The paper gives a summary description of a linear programming system modeling the manpower flow within a major Federal department

1. The hierarchical network representation of the manpower problm

The hierarchical organization under analysis can be considered like most other bureaucratic organizations as a large directed network of jobs (see e.g. Uebe 1976).

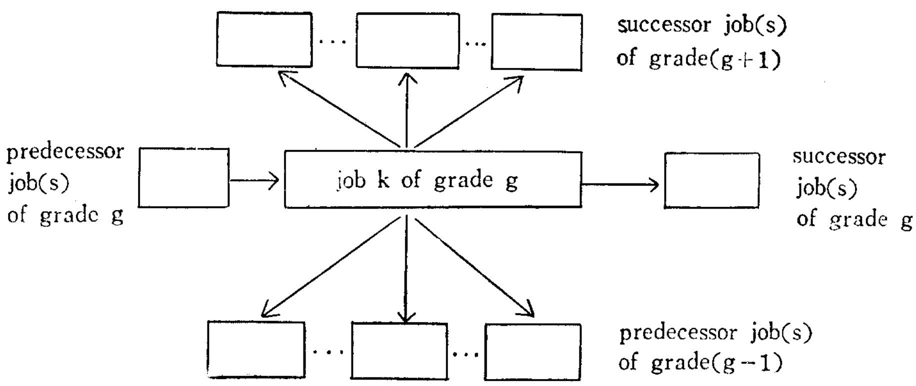
Each job is specified by

(i) a set of requirements such as (listing the most important) number of qualified persons necessary in order to obtain a satisficing or even "optimal" performance of the organization.

(ii) a set of predcessor jobs and
a set of successor jobs

(iii) a fixed relation to the grades (I.e. each job is tied to a specific grade).

The network representation is the usual one of figure 1.



Fgiure 1: The embedding of jobk

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** Technical University Munchen.

*** Fubrungsstab der Streitkräfite Bonn

A box denotes a job and an arrow denotes the flow of people from one job to another.

The jobs are grouped together into networks of offices and the offices are grouped into networks of divisions, which finally make up the department.

For illustration two examples are given, office K20 and office K34:

Independently of these institutional lines of responsibility a 'natural' decomposition can be found by following the predecessor (respectively successor) relation of the individual jobs across the offices (e.g. in K20 to K15, in K34 to K30, etc.). By aggregation of these offices, which have simultaneously successor-and predeceessor relations, a hierarchy of ordered subnetworks can be obtained, each comprising up to 6 offices and up to 100 jobs and 100 job relations. Using a decimal notation for the offices the decomposition of the department can be summarized by a tree like figure 3.

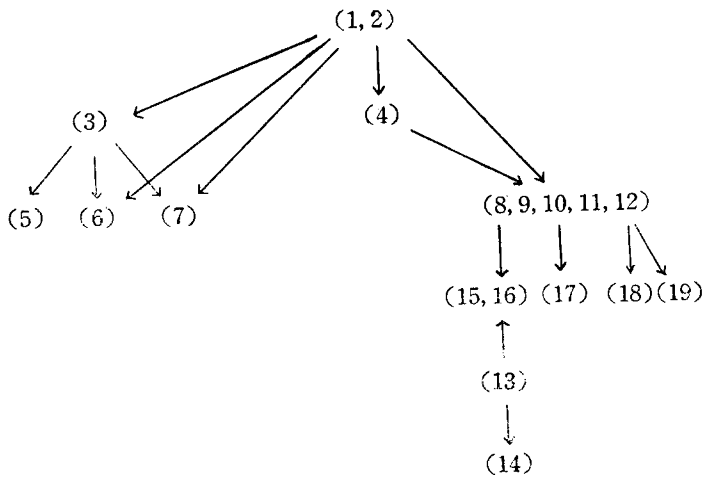


Figure 3: Diagram of Hierarchy of Networks

Each of these subnetworks of offices, as given by the brackets are interdependent subsystems. Consider e.g. the subnetwork (8, 9, 10, 11, 12). Given a decision on the manpower flow in the predecessor systems (1, 2) and (4) a decision in (8, 9, 10, 11, 12) can be obtained. There is no feedback from the subnetworks (15, 16), (17), (18), (19). This natural decomposition permits to solve a really large problem by solving a sequence of identically structured medium sized linear programs.

2. The linear program of the typical (sub) problem

The linear program can be summarized by the following three sets of relations:

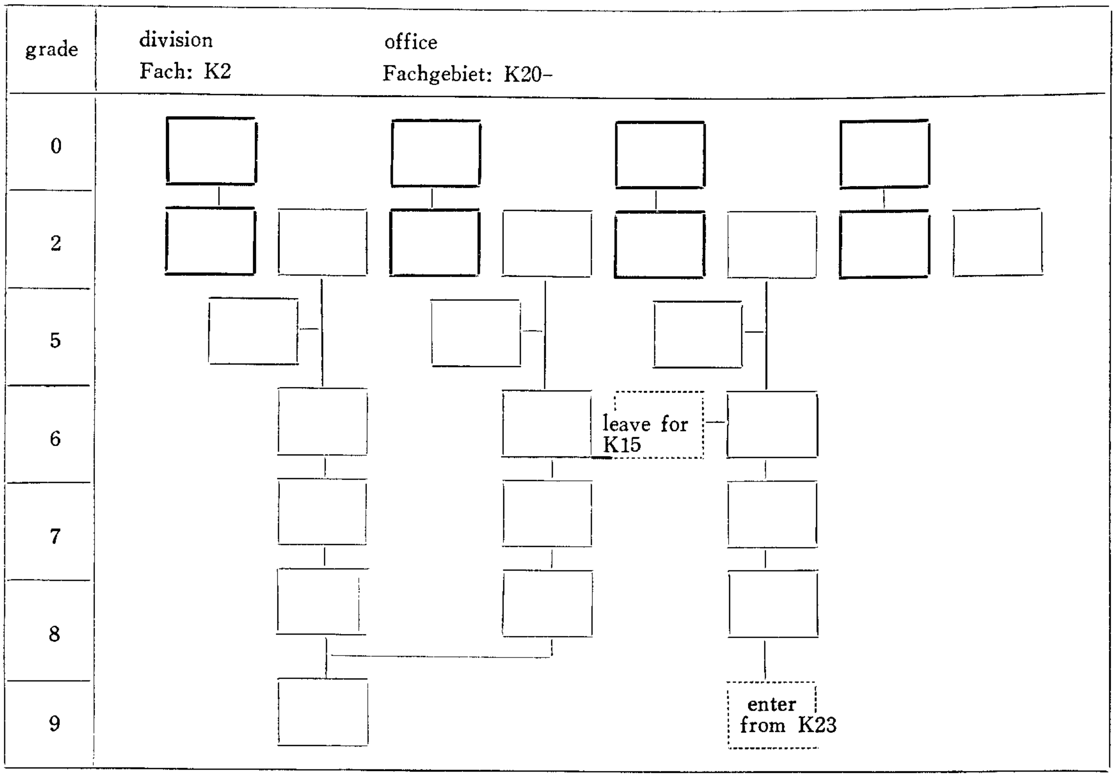
2.1 The conservation of flow

A first set of relations is grouped around the balance equations:

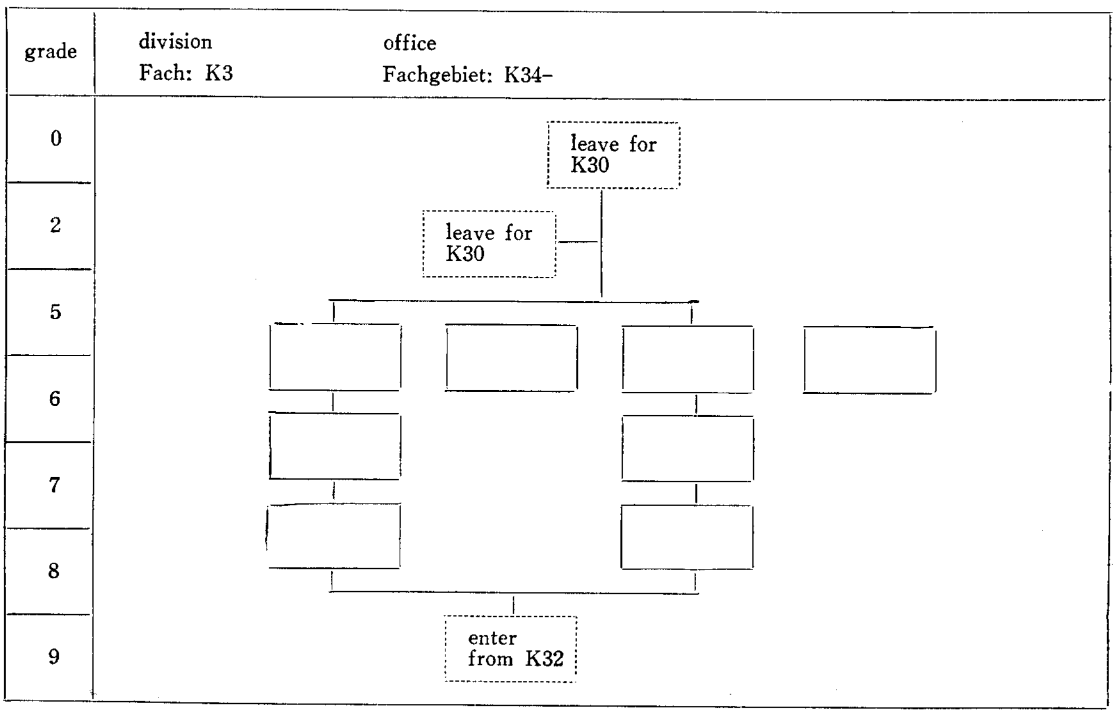
For each job k there is a conservation of flow:

(1) $YE_K = YA_K + ZU_K - AB_K + \sum_{l \in V_K} Z_{lK} - \sum_{l \in N_K} Z_{Kl}$

K20


grade
division
Fach: K2
office
Fachgebiet: K20-
0
2
5
6
7
8
9
leave for
K15
enter
from K23

K34


grade
division
Fach: K3
office
Fachgebiet: K34-
0
2
5
6
7
8
9
leave for
K30
leave for
K30
enter
from K32

Given the initial stock of persons filling job k, YA_K, the final stock of persons, YE_K, is made up by those entering from outside, ZU_K, leaving for outside, AB_K, and those being moved inside the network, Z_{lK}, V_K and N_K denote the set of predecessors, respectively successors.

The leavers are constrained to be a percentage of the initial stock.

(2) $AB_K \leqq SQ_K * YA_K, \quad 0<SQ \leqslant 1$

The entries from outside are either fixed by a preceding network, in which they play the role of leavers.

(3.1) $ZU_K = UE_K$

or they are inked to the final stock, i.e.

(3.2) $ZU_K \leqq ZQ_K * YE_K, \quad 0<ZQ_K \leqslant 1$

The rationale for (2) and (3) is obvious.

Sinee this version of the LP is one shot problem-the transition from an initial situation to a final one-the temporal distributtion of the manpower flow is bounded by a maximum time in job k, $\bar{M}_K$ and by a minimum time in job k, $0<\underline{M}_K<\bar{M}_K$. Linking these bounds to the initial stock we have the inequalities

(4) $YA_K \leqq \bar{M}_k * \sum_{l \epsilon N k} Z^k{}_l$

(5) $YA_K \geqq \underline{M}_K * \sum_{l \epsilon N_k} Z_{Kl}$

2.2 The appropriation of positions

A second set of relations is obtained with respect to the number of positions assigned to each job. Alternatively two versions can be used. The difference lies in the treatment of vacancies. For each job an "optimal" assignment is assumed quite to be known. By long experience the user of the LP has definite ideas about those numbers, SO_k.

For the final assignment YE_k each person has to be either on a duty appropriated position, SO_k, or on a position which has been left free somewhere else and has been transferred temporarily.

In version 1 vacant positions can only be transferred to the immediate predecessors below, while in version 2 vacant positions are collected across the grade and transferred into down below. Figure 4 illustrates the different transfermechanisms:

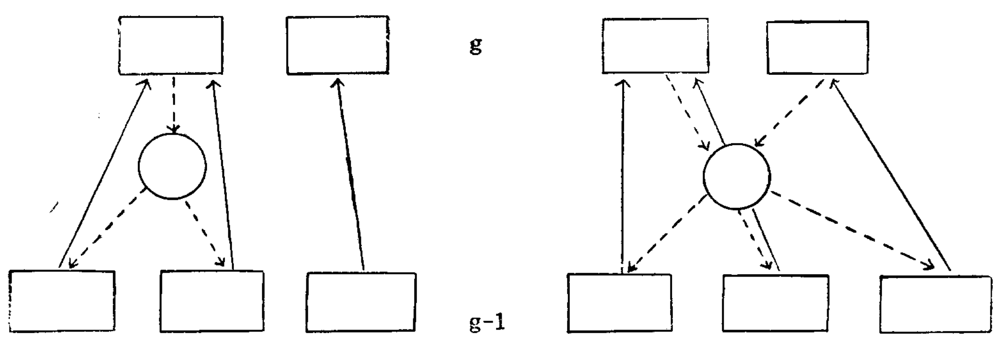


Figure 4

Trarsfer to the immeaiase predecssor

Transfer to the grade below

The corresponding relation in both cases are

$YE_K + YN_K = SO_K$

(6) or

$YE_K + YN_K = SO_K + ZO_K$

YN_e denoting vacances, and ZO_k denoting positions transferred

(7) $OU(I) = \sum_e YN_e$

OU denoting the total number of vacancies

(8) $OU(I) = \sum_e ZO_e + RU(I)$

$RU(I)$ denoting the vacancies, which are left transferred The difference lies in the summation in (7) and (8).

2.3 The obiective funciton(s)

A third set of relations arises from the objective function. Due to the essentially nonlinear nature of the problem and due to the diffuseness with respect to the objectives of the organization, our formulation permits alternatively various linear objective functions. There are different approximations to the basically unknown "true" objective function.

objective Function 1 (01)

Minimize the costs of the final stock. Costs are the actual money costs per period.

(9.1) Min $\sum_K ZL_K * YE_K$

objective Function 2 (02)

Minimize the deviation from the desired assignment (see above (6))

(9.2) Min $\sum_K YN_K$

objective Function 3 (03)

Minimize the costs of the deviations from the desired assignment (see about (6)). Costs are those of 01.

(9.3) Min $\sum_K ZLK_K * YN_K$

objective Funotion 4 (04)

Minimize the costs of the deviations from the desired assignment (see above (6)). Costs are arbitrary weights, different from 01.

(9.4) Min $\sum_K CKMI_K * YN_K$

Objectjve Function 5 (05)

Minimize the largest deviation from the desired assignment (see abov (6))-first alternative.

(9.5.1) $YN_K \leqq ZL$

(9.5.2) Min ZL

Objective Function 6 (06)

Minimize the largest deviation from the desired assignment (see above (6))-second alternative.

(9.6.1) $YN_K \leqq SD$

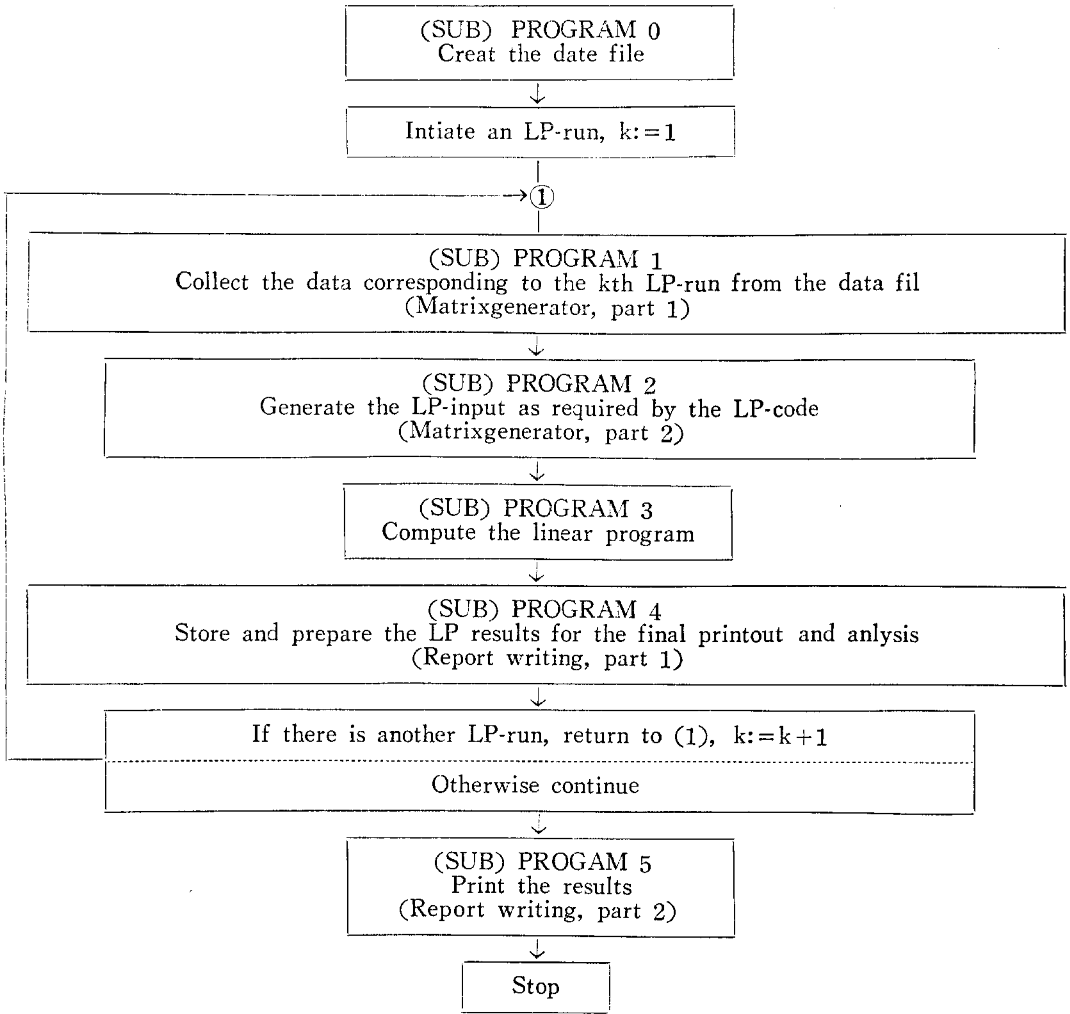
(9.6.2) Min $M * SD + \sum_K YN_K$

(M a prohibitively large number)

3. The implementation-general outline

3.1 The flow diagram

The implementation has been done by a program of six segments residing on two disks and working with 4 tape units as is summarized by the following flow diagram.



3.2 Some remarks on the program

For each of the segments ((sub) programs) a few remarks should suffice here.*

SUB 0 is the creation and maintenance program of all the data as required by the Lp-manpower system. It is a small data bank residing on a disk.

SUB 1 is basically a search procedure. Corresponding to the hierarchy of networks and/or

* A most detailed description is being perpared to the Project Sponsor, to whom the interested reader is refered to.

corresponding to an alternative LP-run of one particular subnetwork (see above section 2) the required data is being searched for from the data bank and prepared for SUB 2.

SUB 2 is an usual even though large matrix generator. Depending on the specification for the LP (see above 2) all coefficients, bounds, etc. of the linear program generated as required by the input format of the LP-code.

SUB 3 is the LP code. We use the LPS-code of IBM, which is relatively old. It handles up to 1500 rows (equations) and 2000 columns (variables) and does suffice for our problem due to the above decomposition (see section 2).

A minor difficulty has been, that LPS has been designed for card input-we use tapes respectively disks-a less trivial impediment has been, that the code cannot be called as a subprogram.

SUB 4 and SUB 5 are large and sophisticated report writing programs. Like in other LP-systems a significant amount of work has been put into input-output programs. Our guiding principle has been to minimize paper printouts and to avoid as far as possible any cards to be punched or to be stored.

SUB 4 is a housekeeping routine, which stores and maintains the intermediate results for an eventual analysis by SUB 5.

SUB 5 finally is the main report writing program. Since it is always tedious to digest large arrays of numbers and since the problem input is of a graphical nature (see above section 1) our intention has been to present the results of the LP as compact as possible and graphically to the utmost extent. Except for some summary measures all results are written into the job network. The results are basically a re-pjot of the originally manually drawn network of jobs. Depending on various options the plotting includes either no information from the LP, i.e. the network jobs per se, or various amounts of the LP-results, e.g. initial and final stock and transitions. For illustration consider the sample printout of figure 5.

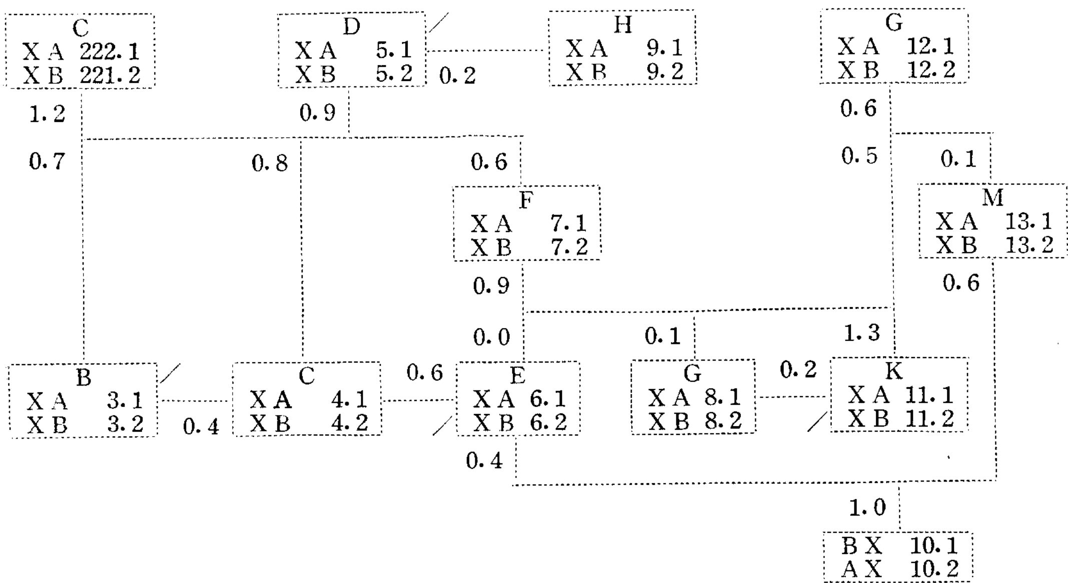


Figure5: Sample Printout

4. Conclusion

The operational status is of introduction into use. The programs have been tested and linked and the first operational data is fed into the databank of SUB 0.

REFERENCES

IBM Linear programming system/360 (LPS/360) (360A-CO-18X)

G. Uebe Manpower Planning, contribution to "Handwörterbuch der mathematischen Wirtschaftswissenschaften, forthcoming 1976.