The Sigmoid Kinetics of Mass-action and Photosynthesis based on Influx and Efflux in a Plant Bio-system

Ⅰ. The Theoretical Models

Nam Kee Chang

(Dept. of Biology, College of Education, Seoul National University)

流出入의 原理에 依한 物質代謝와 光合成能에 關한 動力學的 研究

Ⅰ. 理 論 모델

張 楠 基

(서울大學校 師範大學 生物敎育科)

ABSTRACT

The sigmiod kinetics of mass-action in a biosystem have been studied by theoretical bases on the carrier hypothesis of influx and efflux of substrates.

The sigmoid kinetic equations of assimilation and dissimilation rates indicate that each trophicfactor and each bio-factor behave according to the sigmoid kinetic equation and the bell shape case, and all of them are multiplicative. The general sigmoid kinetics of mass-action is given by the equation (30) which is determined by the total of the equation (28) of the assimilation rate and the equation (29) of the dissimilation rate. The sigmoid kinetic model of photosynthesis has been derived from the general equation of the sigmoid kinetics of mass-action.

In theoretical studies on the kinetic models of mass-action and photosynthesis in a plant bio-system, Chang(1975) has proposed an improved model of mass-action which is described by the Michaelis-Menten equation and the relationship with the product of all the environmental factors. Whereas influx and efflux reactions according to carriers exhibit a hyperbolic relationship between the rate and the substrate concentration as represented by the Michaelis-Menten equation, there are a number of exceptional types of behaviour. Sometimes, there is a sigmoid curve of which the slope of the plot, instead of decreasing steadily as in the Michaelis-Menten case, first increases and then decreases.

Therefore, in this paper, the general model of the sigmoid kinetics of mass-action in a bio-system was derived from the carrier hypothesis of influx and efflux of substrates and the kinetic equation of photosynthetic activity was given by this general model of mass-action, too.

Methods

Early in the development of theoretical models on the kinetics of mass-action and photosythesis in a plant bio-system, it became apparent that a wholesystem diagram for a concept of the environmental factor affecting the amount of photosynthesis was needed with an intensive site specifically in mind.

The mathematical expression of the basic model was found in a number of reports in the literature and own conception, and then was constructed as an original concept. Expansion of this basic concept was carried out in order to obtain other new derivative models. Any results of those methods are conditional on the validity

of its assumption. A major part of model validation is the determination of which variables should be introduced and should be deleted. Therefore, the sigmoid kinetics of mass-action and photosynthesis was abstracted as a theoretical model expressed in the language of mathematics.

Assumption

The procedure requires (1) that according to the basic principle of the input-output relationship for a black boxes, the rate of mass-action of a bio-system can be formulated by the kinetic equation of influx and efflux rates of substrates. It is also assumed (2) that the rate of mass-action in a bio-system is the bio-function of pH, temperature, time, etc., and the trophic-function of light, CO_2, H_2O, N, P, K, Ca, Mg, S, etc., and (3) that the rate of mass-action is changed by holding all the environmental factors of the bio- and trophic-functions throughout a series of the bio-system.

The kinetic model of photosynthesis in a plant bio-system can be derived from the carrier kinetics such as a community, a plant, a leaf, a mesophyll cell or a chloroplast, used to the following equation:

$$CO_2 + H_2O + light \rightarrow (CH_2O)_n + H_2O + O_2$$
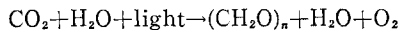
Procedure

1. The sigmoid kinetics of absorption by carriers

The absorption of substrates by an organism, whereas a source of energy or of material, can be regarded as the carrier hypothesis. The influx kinetics that substrates are transported across biological membranes by carriers can be expressed in the following equation:

$$v_a = \frac{V S_a}{K_a + S_a} \quad \cdots \cdots (1)$$

where v_a, K_a, S_a, and V represent, respectively, the rate of absorption, the dissociation constant of the carrier-substrate complex, the substrate concentration, and the capacity factor. This is the Michaelis-Menten equation relating the rate of enzymic catalysis to the concentration of the substrate(Epstein and Hagen 1952).

Under the condition of a certain state of the carrier which is changed by any environmental factor, the influx rate of carrier-mediated transport must be shown to be dependent on the equation (1). If this state of the carriers is changed into the other state, the influx velocity of carrier mediated absorption may be fitted by the equation (1), too. Therefore, the influx ratio between substrate concentrations in these two states of the carrier continuously changes, but the influx specific change rates of the two dimensions of substrate concentrations remain constant. This principle of constant specific change of substrate concentrations is exhibited in terms of

$$\frac{1}{y_a} \frac{dy_a}{dt} = C \frac{1}{S_a} \frac{dS_a}{dt} \quad \cdots \cdots (2)$$

where y_a, S_a, t and C are the external substrate concentration in a certain of the carrier, the external substrate concentration in the other state, time and Chang's influx specific change index of the carrier, respectively.

If we eliminate time in the equation (2) and integrate, we obtain

$$y_a = a S_a^c \quad \cdots \cdots (3)$$

where a expresses a integrate constant.

The change rate of carrier-mediated absorption under the various conditions of the carrier is

$$v_a = \frac{V a S_a^c}{K_a + a S_a^c} \quad \cdots \cdots (4)$$

As in the equation(4), Va is the product, V_a, of V and a, this equation is discribed by

$$v_a = \frac{V_a S_a^c}{K_a + a S_a^c} \quad \cdots \cdots (5)$$

The equation (5) is a still more general model expressing the varous mechanisms for hyperbolic and sigmoid kinetics of the uptake of the substrate by carriers.

When c and a are equal to 1, respectively, the Michaelis-Menten equaton is obtained by the equations (4) and (5). This equation exhibits a hyperbolic relationship between the rate of absorption and the substrate concentrations.

If Chang's influx index, c, is equal to 2, the equation (5) becomes the simplest sigmoid equation:

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$$v_a = \frac{V_a S_a{}^2}{K_a + a S_a{}^2} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (6)$$

As shown in Fig. 1, schematic representation of the sigmoid kinetic equation (6) instead of decreasing steadily as in the equations (4) and (5) substituted for Chang's influx index, $c=1$, first increases and then decreases.
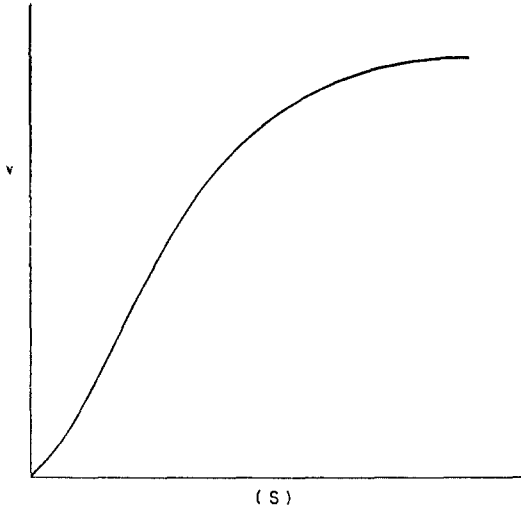

Fig. 1. Schematic representation of the sigmoid kinetics.

2. Sigmoid kinetics of efflux by carriers

If the efflux of substrates is considered to be due to carriers, the efflux rate, ve_e of substrates can be represented by the following kinetics:

$$v_e = -\frac{V b S_e{}^h}{K_e + b S_e{}^h} = -\frac{V_b S_e{}^h}{K_e + b S_e{}^h} \quad \cdots\cdots\cdots (7)$$

where S_e, K_e, b and h are the internal substrate concentration, the dissociation constant of the efflux carrier-substrate complex, the integrate constant and Chang's efflux specific change index, respectively.

When h is equal to 1, the efflux kinetics is given by

$$v_e = -\frac{V_h S_e}{K_e + b S_e} = -\frac{V S_e}{K_{me} + S_e} \quad \cdots\cdots\cdots\cdots (8)$$

where V_b and K_{me} represent the efflux capacity factor and the dissociation constant of efflux carrier-substrate complex.

If Chang's efflux index, h, is equal to 2, the equation (7) becomes the following simplest sigmoid:

$$v_e = -\frac{V_b S_e{}^2}{K_e + b S_e{}^2} \quad \cdots\cdots\cdots\cdots\cdots\cdots (9)$$

3. Basic concept for the sigmoid kinetics by a bio-system

An organism can be considered as one bio-molecular system which has a very multiple structure. This biological system of mass-action which is different from simple carriers is defined as a bio-system. From this point of view, the influx and efflux action of substrates of an organism can not only be interpreted by the mass-action of independent catalystic sites of the carriers but also may be depended on relationships among the carriers in a bio-system. By observing the influx and efflux relationship for a black boxes, the basic diagram of this concept from bio-molecular systems to an ecosystem is shown in Fig. 2.
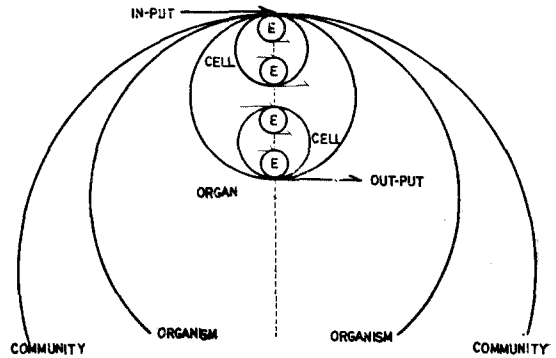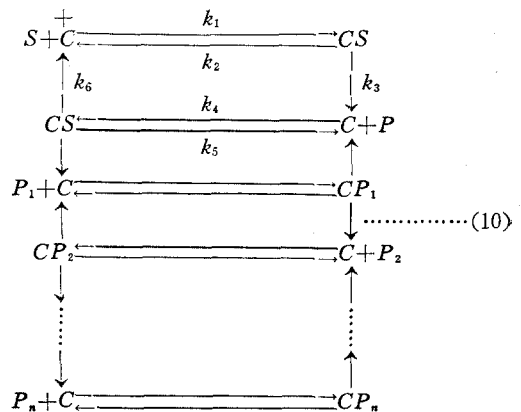

Fig. 2. The diagram of the basic concept of influx and efflux substrates from carrier molecules (E) to bio-systems.

The total of the influx and efflux rates of substrates is the net productivity and trophic growth of a bio-system. The mechanism of this

$$\cdots\cdots\cdots (10)$$

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action into the convenient closed-loop type of geometric array may be arranged by the King's method. Therefore, the sum of the in flux and efflux substrates by a bio-system can be written by where $P, P_1, P_2, \cdots\cdots, P_n$ express products of each reaction. The basic figure in the chemical quation (10) is a square form. This equation suggests that the net productivity and trophic-growth do not allow for any reverse reaction between the assimilation and dissimilations in a bio-system.

(1) The assimilation kinetics of substrates in a bio-system

According to the basic concept for the influx kinetics of substrates, the assimilation rate, v_i, of external substrates is determined by the concentrations of substrates such as light, carbon, N, P, Ca, and Mg which serve as trophic independent variables in the following equation:

$$v_i = V_{am} \prod_{i=1}^{m} \frac{S_{ai}^{ci}}{K_{ai} + a_i S_{ai}^{ci}} \quad\cdots\cdots\cdots\cdots\cdots\cdots (11)$$

where S_{ai} is the kinds of influx trophic variables.

The assimilation equation(11) can be applied to the kinetics of absorption, photosynthesis, and productivity.

(2) The dissimilation kinetics of substrates in a bio-system

The dissimilation rate, v_e, of internal substrates in a bio-system is expressed by

$$v_e = -V_{bm} \prod_{i=1}^{m} \frac{S_{ei}^{hi}}{K_{ei} + b_i S_{ei}^{hi}} \quad\cdots\cdots\cdots\cdots\cdots (12)$$

where S_{ei} is the kinetics of efflux trophic variables. This equation can be applied to respiration of a bio-system.

(3) The accumulation kinetics of substrates in a bio-system

The sum of the influx and efflux concentrations of substrates is the accumulation rate, v, in the bio-system. This kinetics is

$$v = V_{am} \prod_{i=1}^{m} \frac{S_{ai}^{ci}}{K_{ai} + aS_{ai}^{ci}} - V_{em} \prod_{i=1}^{m} \frac{S_{ei}^{hi}}{K_{ei} + b_i S_{ei}^{hi}} \cdots (13)$$

The equation (13) is applied to net photosynthesis and net productivity. Where a single substrate is a limiting factor, trophic growth can be dete-

rmined by this equation.

(4) Influx and efflux rates affected by the bio-variables

When the trophic-factors are acted upon by the constant concentrations, the influx and efflux rates of substrates of a bio-system depend on the bio-variables. In terms of the sigmoid kinetics of influx and efflux, S_a^c and S_e^h instead of K_a and K_e take a certain value, respectively.

The equations (4) and (7) are formulated as follows:

$$v_a = \frac{V_i}{1 + \frac{K_a}{aS_a^c}} = \frac{V_i}{1 + K_{Sa}} \quad\cdots\cdots\cdots\cdots\cdots (14)$$

and

$$v_e = -\frac{V_e}{1 + \frac{K_e}{bS_e^h}} = -\frac{V_e}{1 + K_{Se}} \quad\cdots\cdots\cdots\cdots (15)$$

where $K_a/aS_a^c = K_{Sa}$ and $K_e/bS_e^h = K_{Se}$ are specific rate constants of influx and efflux, respectively.

When the influx and efflux rates of substrates are the function of the bio-variables, extremely valuable information about reaction mechanisms of a bio-system is provided by studies of the effect of time, temperature and pH on kinetic constants. The law that applies almost universally to these time(Robortson's growth equation), temperature(Arrhenius law) and pH effects are as follows:

$$K_{st} = Q_t e^{-gt} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (16)$$
$$K_{ste} = Q_{te} e^{-E/RT} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots (17)$$

and

$$K_{spH} = Q_{pH} e^{-(l_n 10)pH} = Q_{pH} e^{-2.303pH} \quad\cdots\cdots\cdots (18)$$

where te is temperature. The equations (16), (17) and (18) are divided into influx and efflux, respectively, and can be expressed by a general formular:

$$K_s = Qe^{-dx} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (19)$$

where x and d represent the bio-variables and a certain constant such as g, E/R and 2.303. Therefore, the bio-function of time, temperature and pH for influx and efflux rates are given by

$$v_a(x) = \frac{V_a}{1 + Q_a e^{-d_a x}} \quad\cdots\cdots\cdots\cdots\cdots\cdots (20)$$

and

$$v_e(x) = -\frac{V_e}{1 + Q_e e^{-d_e x}} \quad\cdots\cdots\cdots\cdots\cdots\cdots (21)$$

The mean change rate per unit change of the

bio-variable becomes

$$v_a = \frac{dv_a(x)}{dx} = \frac{V_a Q_a d_a e^{-d_a x}}{(1+Q_a e^{-d_a x})^2} \quad \cdots\cdots\cdots\cdots (22)$$

and

$$v_e = -\frac{dv_e(x)}{dx} = -\frac{V_e Q_e d_e e^{-d_e x}}{(1+Q_e e^{-d_e x})^2} \quad \cdots\cdots (23)$$

As shown in Fig. 3, the equation (22) exhibits the bell shape of a limiting factor profiles.
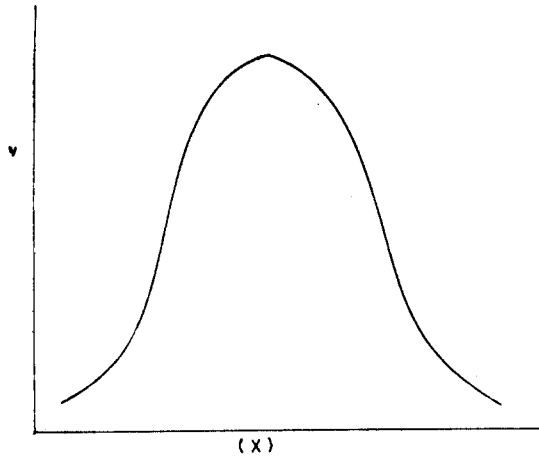

(X)

Fig. 3. Schematic representation of the bell shape kinetics.

When the bio-variables are independent and dependent, the equation of influx rate becomes

$$v_a = V_{an} \prod_{j=1}^{n} \frac{Q_{aj} d_{aj} e^{-d_{aj} x_j}}{(1+Q_{aj} e^{-d_{aj} x_j})^2} \cdots\cdots\cdots\cdots\cdots (24)$$

and the equation of efflux rate is

$$v_e = -V_{en} \prod_{j=1}^{n} \frac{Q_{ej} d_{ej} e^{-d_{ej} x_j}}{(1+Q_{ej} e^{-d_{ej} x_j})^2} \cdot \cdots\cdots\cdots\cdots (25)$$

The biotic accumulation rate in a bio-system is given by

$$v = v_a + v_e = V_{an} \prod_{j=1}^{n} \frac{Q_{aj} d_{aj} e^{-d_{aj} x_j}}{(1+Q_{aj} e^{-d_{aj} x_j})^2}$$
$$-V_{en} \prod_{j=1}^{n} \frac{Q_{ej} d_{ej} e^{-d_{ej} x_j}}{(1+Q_{ej} e^{-d_{ej} x_j})^2} \cdots\cdots\cdots\cdots (26)$$

The biotic accumulation of substrates in the bio-system, G, is expressed by

$$G = \int_0^x (v_a + v_e) dx = V_{an} \prod_{j=1}^{n} \frac{1}{1+Q_{aj} e^{-d_{aj} x_j}}$$
$$-V_{en} \prod_{j=1}^{n} \frac{1}{1+Q_{ej} e^{-d_{ej} x_j}} \quad \cdots\cdots\cdots\cdots (27)$$

This equation exhibits the biotic growth of an organism and standing crop of a community.

4. The sigmoid kinetic equation of the rate of mass-action in a bio-system

Since the bio- and trophic-factors are independent and dependent in a bio-system, the general equation of influx rate is

$$v_a = V_{amn} \prod_{j=1}^{n} \frac{Q_{aj} d_{aj} e^{-d^x j a j}}{(1+Q_{aj} e^{-d_{aj} x_j})^2} \prod_{i=1}^{m} \frac{S^{ci}_{ai}}{K_{ai} + S^{ci}_{ai}} \cdots (28)$$

and the general equation of efflux rate is given by

$$v_e = -V_{emn} \prod_{j=1}^{n} \frac{Q_{ej} d_{ej} e^{-d_{ej} x_j}}{(1+Q_{ej} e^{-d_{ej} x_j})^2} \prod_{i=1}^{m} \frac{S^{hi}_{ei}}{K_{ei} + S^{hi}_{ei}}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (29)$$

The equations (28) and (29) demonstrate that each bio-factor and each trophic-factor of all the environmental factors behave according to the bell shape kinetic equation and the sigmoid case, and all of them are multiplicative.

Therefore, the accumulation rate, v, of substrates in a bio-system

$$v = V_{amn} \prod_{j=1}^{n} \frac{Q_{aj} d_{aj} e^{-d_{aj} x_j}}{(1+Q_{aj} e^{-d_{aj} x_j})^2} \prod_{i=1}^{m} \frac{S^{ci}_{ai}}{K_{ai} + S^{ci}_{ai}}$$
$$-V_{emn} \prod_{j=1}^{n} \frac{Q_{ej} d_{ej} e^{-d_{ej} x_j}}{(1+Q_{ej} e^{-d_{ej} x_j})^2} \prod_{i=1}^{m} \frac{S_h{}^i_{ei}}{K_{ei} + S^{hi}_{ei}}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (30)$$

The equation (30) exhibits the general kinetic model of the mass-action rate such as the metabolism rate, net photosynthetic rate, net productivity, and growth rate of a bio-system under all environmental conditions of the bio- and trophic-factors.

5. Sigmoid kinetics of photosynthesis in a bio-system

When the bio-variables affecting the photosynthetic activity in a plant bio-system have each constant value, the sigmoid kinetic equation of the rate of photosynthesis can be obtained by the equation (11):

$$p_h = \frac{P_h I C^I C_o{}^{co} C_o H_o{}^{c_{HO}}}{(K_{aI} + a_I I^{c_I})(K_{aC_o} + a_{C_o} C_o{}^{c_{i}})(K_{aH_o} + a_{H_o} H_o{}^{c_{HO}})}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (31)$$

where p_h is the photosynthetic rate, and P_h expresses the asymptotic value of photosynthesis in a plant bio-system. In this equaction, I, C_o, and H_o represent light, CO_2, and H_2O concentrations as the trophic variables of photosynthesis,

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respectively.

When the Chang's influx indices of C_I, C_{C_o}, and C_{H_o} are respectively 1, the equation(31) is

$$p_h = \frac{P_h I C_0 H_o}{(K_{aI} + a_I I)(K_{aC_o} + a_{c_o} C_o)(K_{aH_o} + a_{H_o} H_o)} \cdots (32)$$

If a_I, a_{c_o}, and a_{H_o} in the equation (32) are respectively 1, the kinetic equation of the photosythetic rate is simplified by

$$p_h = \frac{P_h I C_o H_o}{(K_{aI} + I)(K_{aC_o} + C_o)(K_{aH_o} + H_o)} \cdots (33)$$

When the bio-variables affecting the respiration in a plant bio-system have each constant value, the kinetic equation of the rate of respiration can be given by the equation (12):

$$r_p = -\frac{R_p M^{h_M} O^{h_o} H_o^{h_{HO}}}{(K_{eM} + b_M M^{h_M})(K_{eo} + b_0 O^{h_o})(K_{eH_o} + b_{H_o} H_o^{h_{HO}})} \cdots (34)$$

where r_p represents the rate of respiration, and R_p is the asymptotic value of respiration. In the equation (34), M and O express organic matter, and oxygen concentrations as the trophic-variables of respiration, repectively.

When h_M, h_O, and h_{H_o} are 1, respectively, the ebuation (34) is given by

$$r_p = -\frac{R_p M O H_o}{(K_{eM} + b_M M)(K_{eo} + b_o O)(K_{eH_o} + b_{H_o} H_o)} \cdots (35)$$

and when in the equation (35), b_M, b_o, and b_{HO} are respectively 1, the equation of the respiration rate is

$$r_p = -\frac{R_p M O H_o}{(K_{eM} + M)(K_{eo} + O)(K_{eH_o} + H_o)} \cdots (36)$$

When the trophic variables affecting the photosynthetic activity are constants, the rate of photosynthesis is expressed by the equation (24) as follows:

$$p_h = P_{hn} \prod_{j=1}^{n} \frac{Q_{aj} d_{aj} e^{-d_{aj} x_j}}{(1 + Q_{aj} e^{-d_{aj} x_j})^2} \cdots (37)$$

and the respiration rate are modified by the equation (25) as follows:

$$r_p = -R_{pn} \prod_{j=1}^{n} \frac{Q_{ej} d_{ej} e^{-d_{ej} x_j}}{(1 + Q_{ej} e^{-d_{ej} x_j})^2} \cdots (38)$$

Therefore, the general model of the photosynthetic activity is given by

$$p_h = P_{hn} \prod_{j=1}^{n} \frac{Q_{aj} d_{aj} e^{-d_{aj} x_j}}{(1 + Q_{aj} e^{-d_{aj} x_j})^2}$$

$$\times \frac{I^c I C_0{}^{c_{co}} H_0{}^{c_{H_o}}}{(K_{aI} + a_I I^{c_I})(K_{aC_o} + a_{c_o} C_o{}^{c_{co}})(K_{aH_o} + a_{H_o} H_o{}^{c_{H_o}})} \cdots (39)$$

and the general model of the respiration rate is

$$r_p = -R_{pn} \prod_{j=1}^{n} \frac{Q_{ej} d_{ej} e^{-d_{ej} x_j}}{(1 + Q_{ej} e^{-d_{ej} x_j})^2}$$

$$\times \frac{M^{h_M} O^{h_o} H_o^{h_{HO}}}{(K_{eM} + b_M M^{h_M})(K_{eo} + b_o O^{h_o})(K_{eH_o} + b_{H_o} H_o^{h_{HO}})} \cdots (40)$$

In the equations (39) and (40), P_{hn} and R_{pn} are the products of P_h and P_n, and R_p and R_n, respectively.

According to the equations (39) and (40), the amount of net photosynthesis per unit time, p_{hn}, is given by

$$p_{hn} = P_{hn} \prod_{j=1}^{n} \frac{Q_{aj} d_{aj} e^{-d_{aj} x_j}}{(1 + Q_{aj} e^{-b_{aj} x_j})^2}$$

$$\times \frac{I^c I C_0{}^{c_{co}} H_o{}^c{}_0}{(K_{aI} + a_I I^{c_I})(K_{aC_o} + a_{c_o} C_o{}^{c_{co}})(K_{aH_o} + a_{H_o} H_o{}^{c_{H_o}})}$$

$$- R_{pn} \prod_{j=1}^{n} \frac{Q_{ej} d_{ej} e^{-d_{ej} x_j}}{(1 + Q_{ej} e^{-d_{ej} x_j})^2}$$

$$\times \frac{M^{h_M} O^{h_o} H_o^{h_{HO}}}{(K_{e}{}^M + b_M M^{h_M})(K_{eo} + b_o O^{h_o})(K_{eH_o} + b_{H_o} H_o^{h_{HO}})} \cdots (41)$$

When the bio-variables affecting the photosynthetic activity are the constant values, the equation (41) is simplified as follows:

$$P_{hn} = \frac{P_h I^{c_I} C_o{}^{c_{co}} H_o{}^{c_{H_o}}}{(K_{aI} + a_I I^{c_I})(K_{aC_o} + a_{c_o} C_o{}^{c_{co}})(K_{aH_o} + a_{H_o} H_o{}^{c_{H_o}})}$$

$$- \frac{R_p M^{h_M} O^{h_o} H_o{}^{h_{HO}}}{(K_{eM} + b_M M^{h_M})(K_{eo} + b_o O^{h_{HO}})(K_{eH_o} + b_{H_o} H_o^{h_{HO}})} \cdots (42)$$

If C_I, C_{C_o}, C_{H_o}, a_I, a_{c_o}, a_{H_o}, h_M, h_o, h_{H_o}, b_M, b_o, and b_{H_o} are respectively 1, the equation (42) becomes

$$p_{hn} = \frac{P_h I C_o H}{(K_{aI} + I)(K_{aC_o} + C_o)(K_{aH_o} + H_o)}$$

$$- \frac{R_p M O H_o}{(K_{eM} + M)(K_{eo} + O)(K_{eH_o} + H_o)} \cdots (43)$$

Discussion

Since accumulation of substrates in a bio-system is positively related to transport across biological membranes by carriers, the kinetics of mass-action must be based on influx and efflux rates of substrates which depend upon the first reaction of assimilation and the final products of dissimilation.

Most ecological and physiological descriptions of the growth of phytoplankton have used Monod's(1942) kinetic model for steady state growth of microbes:

$$\mu = \frac{\mu_{max}(S)}{K_s + (S)}$$

where μ is the specific growth rate at limiting substrate concentration S, μ_{max} is the maximum specific growth rate at concentrations of substrate that are saturating, and K_s is the concentration of substrate at which the specific growth rate is one-half its maximum value. A rigorous approach to the interaction of several rate-limiting substrates or processes has been presented by Lockhart (1965a, b). Empirically the interactions of day length, light intensity, and temperature have been determined for phototrophic growth in the alga chlorella(Tamiya et al. 1955). According to Chang(1975) and Canale(1976), the trophic growth rate of plants was determined by the growth limiting factors such as light, carbon, N and P, which serve as independent variables. It is noted that those kinetic models is described by the Michaelis-Menten equation and the relationship with the product of all the trophic factors.

Whereas influx and efflux reactions according to carriers exhibit a hyperbolic relationship between the rate and the substrate concentration as represented by the Michaelis-Menten equation (Epstein & Hagen, 1952; Chang, 1975), there are a number of exceptional types of behaviour. Sometimes, as illustrated schematically in Fig. 2, there is a sigmoid relationship: the slope of the plot, instead of decreasing steadily as in the Michaelis-Menten case, first increases and then decreases. It seems reasonable to validate that the sigmoid kinetics includes the Michaelis-Menten equation. Hence, in this paper, the sigmoid kinetic equations of assimilation and dissimilation affected by the substrates were represented by the equations (5) and (7) and the relationship with the product of all the trophic factors.

Considering the relation between mass-action and the bio-variables, the kinetic equations of assimilation and dissimilation were descrived by the equations (20) and (21) and the relationship with the product of all the bio-factors. According to Goldman & Carpenter(1974), the Arrhenius equation was inserted into the Monod model for the maximum growth rate, so that the growth rate was described by the product of temperature and nutrient expressions. Roon et al.(1975) showed that methylamine uptake of *Saccharromyces cerevisiae* x2180-A by means of a specific active transport system was pH and temperature dependent and exhibited the bell shape of pH and temperature activity profiles. Therefore, the equations of the bio-function of assimilation and dissimilation which were derived from Chang (1975) were generalized by the equations (20) and (21), respectively.

Since accumulation rate of the substrates is the sum of influx and efflux rates, the general kinetic model of mass-action in a bio-system is expressed by the equation(30). Moreover, it seems reasonable to be that the general kinetic equation of photosynthetic activity can be derived from the equation (30).

摘 要

生物系에 있어서 物質代謝能의 動力學的 研究가 基質의 流出入에 關한 擔體假說의 原理에 입각하여 理論的으로 行하여 졌다.

物質의 同化와 異化率을 나타내는 動力學方程式은 S字型動力學式으로 表示되는 各 營養條件要素函數와 鍾型動力學式으로 表示되는 各 生物條件要素函數의 곱으로 表現되었다. 따라서 物質代謝能의 一般動力學方程式은 同化率方程式인 (28)式과 異化率方程式인 (29)式의 合으로 (30)式에 依하여 주어졌다.

光合成能에 關한 動力學方程式은 (30)式을 適用하여 誘導하였다.

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