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NOTE ON COMPACT SUBMANIFOLDS OF CODIMENSION 2 WITH TRIVIAL NORMAL BUNDLE IN AN EVEN-DIMENSIONAL EUCLIDEAN SPACE

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§ 0. Introduction

It is now well known that submanifolds of codimension 2 in an even-dimensional Euclidean space admit an (f, g, u, v, λ)-structure. The submanifolds of codimension 2 in an even-dimensional Euclidean space in terms of this structure have been studied by many authors (cf. [1], [2], [7], [10] and [11] etc.).

Yano and Ki [10] have proved

THEOREM A. *Let M be a complete submanifold of codimension 2 in an even-dimensional Euclidean space E^{2n+2} such that the scalar curvature of M is constant and there are global unit normals C and D to M which are parallel in the normal bundle. If $fH=Hf$ and $fK=-Kf$ hold, where H and K are the second fundamental tensors of M respectively with respect to C and D, f being the tensor field of type $(1, 1)$ appearing in the induced structure (f, g, u, v, λ) of M, then M is in E^{2n+2}, provided that $\lambda(1-\lambda^2)$ is non-zero almsot everywhere in M, congruent to one of the following submanifolds:*

$$E^{2n},\ S^{2n}(r),\ S^n(r)\times S^n(r),\ S^l(r)\times E^{2n-l}(l=1,2,\cdots,2n-1),$$

$$S^k(r)\times S^k(r)\times E^{2n-2k}\ (k=1,2,\cdots,n-1),$$

where $S^k(r)$ denotes a k-dimensional sphere of radius r (>0) imbedded naturally in E^{2n+2}.

Moreover, Ki and Pak [2] studied a compact submanifold in Theorem A without the constancy of the scalar curvature.

On the other hand, Simons has given a formula for the Laplacian of the norm of the second fundamental form of a submanifold in a Riemannian manifold. Nakagawa and Yokote [3], [4] proved a compact hypersurface of nonegative curvature and with constant scalar curvature in a Euclidean space is a sphere.

In the present paper, using the similar formula to that of Simons, we investigate that a compact submanifold of codimension 2 in an even-dimensional Euclidean space such that $Hf=fH$ holds as stated in Theorem A.

§ 1. Induced structures of submanifolds of codimension 2 of E^{2n+2}

Let E^{2n+2} be a $(2n+2)$-dimensional Euclidean space and X the position vector starting from the origin of E^{2n+2} and ending at a point of E^{2n+2}. The E^{2n+2} being even-dimensional it can be regarded as a flat Kaehlerian manifold with numerical structure tensor F:

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$F^2=-I$, where I denotes the unit tensor and $FY\cdot FZ=Y\cdot Z$ for arbirtray vector fields Y and Z, where the dot denotes the inner product of vectors of E^{2n+2}.

Let M be a $2n$-dimensional orientable manifold covered by a system of coordinate neighborhoods $\{U, x^h\}$, where here in the sequel the indices $h, j, i, \cdots$ run over the range $\{1, 2, \cdots, 2n\}$ and the summation convention will be used with respect to these indices.

We assume that M is immersed in E^{2n+2} by $X: M\rightarrow E^{2n+2}$ and put $X_i=\partial_i X$, $\partial_i=\partial/\partial x^i$. Then X_j are $2n$-linearly independent local vector fields tangent to $X(M)$, and $g_{ji}=X_j\cdot X_i$ are local components of the tensor representing the Riemannian metric induced on M from that of E^{2n+2}.

We assume that we can take two globally defined mutually orthogonal unit normals C and D to $X(M)$ in such a way that X_1, $X_2, \cdots, X_{2n}$, C and D give the positive orientation of E^{2n+2}. In the sequel we identify $X(M)$ with M itself.

The transforms FX_i by F can be expressed as linear combinations of X_i, C and D, that is, we have equations of the form

$$FX_i=f_i{}^hX_h+u_iC+v_iD, \tag{1.1}$$

where $f_i{}^h$ are components of a tensor field of type (1, 1) and u_i, v_i are those of 1-forms of M. The transforms FC and FD of C and D by F can be expressed as

$$FC=-u^hX_h+\lambda D, \tag{1.2}$$

$$FD=-v^hX_h-\lambda C, \tag{1.3}$$

respectively, where $(g^{ji})=(g_{ji})^{-1}$, $u^h=u_ig^{ih}$, $v^h=v_ig^{ih}$ and λ is a function on M.

Applying F to (1.1), (1.2) and (1.3), and using $F^2=-I$, (1.1), (1.2) and (1.3), we find

$$\begin{aligned} &f_i{}^tf_t{}^h=\delta_i^h+u_iu^h+v_iv^h,\\ &u_tf_i{}^t=\lambda v_i, \quad f_i{}^hu^i=-\lambda v^h,\\ &v_tf_i{}^t=-\lambda u_i, \quad f_i{}^hv^i=\lambda u^h,\\ &u_tu^t=v_tv^t=1-\lambda^2, \quad u_tv^t=0. \end{aligned} \tag{1.4}$$

We also have, from (1.1),

$$g_{ts}f_j{}^tf_i{}^s=g_{ji}-u_ju_i-v_jv_i, \tag{1.5}$$

that is, M admits an (f, g, u, v, λ)-structure (cf. [7], [10], [11]). We can easily see that $f_{ji}=f_j{}^tg_{ti}$ is skew-symmetric in lower indices j and i.

We denote by $\left\{{h \atop j\ i}\right\}$ the Christoffel symbols formed with g_{ji} and by ∇_i the operator of covariant differentiation with respect to $\left\{{h \atop j\ i}\right\}$. Then equations of Gauss are

$$\nabla_jX_i=h_{ji}C+k_{ji}D, \tag{1.6}$$

h_{ji} and k_{ji} being the components of the second fundamental tensors with respect to the normals C and D respectively, and equations of Weingarten are

$$\nabla_j C=\partial_j C=-h_j{}^h X_h+l_j D, \tag{1.7}$$

$$\nabla_j D=\partial_j D=-k_j{}^h X_h-l_j C,$$

where $h_j{}^h$ and $k_j{}^h$ are given $h_j{}^h= h_{ji}g^{ih}$ and $k_j{}^h=k_{ji}g^{ih}$ respectively and l_j are components of th third fundamental tensor, i. e., components of the connection induced on the normal bundle.

Now, differentiating (1. 1) covariantly and taking account of $\nabla_j F=0$ and of eqatuions of Gauss and Weingarten, we find (cf. [7], [10], [11])

$$\nabla_j f_i{}^h=-h_{ji}u^h+h_j{}^h u_i-k_{ji}v^h+k_j{}^h v_i, \tag{1.8}$$

$$\nabla_j u_i=-h_{jt}f_i{}^t-\lambda k_{ji}+l_j v_i, \tag{1.9}$$

$$\nabla_j v_i=-k_{tt}f_i{}^t+\lambda h_{ji}-l_j u_i. \tag{1.10}$$

Similarly we have, from (1. 2),

$$\nabla_j\lambda=-h_{ji}v^i+k_{ji}u^i. \tag{1.11}$$

In the sequel, we need the structure equations of the submanifold M, that is, the following equations of Gauss

$$K_{kjih}=h_{kh}h_{ji}-h_{jh}h_{ki}+k_{kh}k_{ji}-k_{jh}k_{ki}, \tag{1.12}$$

where K_{kjih} are covariant components of the curvature tensor of M, and equations of Codazzi and Ricci

$$\nabla_k h_{ji}-\nabla_j h_{ki}-l_k k_{ji}+l_j k_{ki}=0, \tag{1.13}$$

$$\nabla_k k_{ji}-\nabla_j k_{ki}+l_k h_{ji}-l_j h_{ki}=0, \tag{1.14}$$

$$\nabla_j l_i-\nabla_i l_j+h_{jt}k_i{}^t-h_{it}k_j{}^t=0. \tag{1.15}$$

§ 2. Submanifolds of codimension 2 with certain commutative conditions

Throughout the present paper, we suppose on M that the global unit normals C and D are parallel in the normal bundle, the function $\lambda(1-\lambda^2)$ is non-zero almost everywhere and satisfies

$$f_j{}^t h_t{}^h-h_j{}^t f_t{}^h=0, \tag{2.1}$$

which is equivalent to

$$h_{jt}f_i{}^t+h_{it}f_j{}^t=0. \tag{2.2}$$

We first prepare (cf. [10])

PROPOSITION 2. 1. *Let M be a submanifold of codimension 2 of E^{2n+2} such that the global unit normals C and D are parallel in the normal bundle. Assume that* (2. 1) *satisfied and the function $\lambda(1-\lambda^2)$ is almost everywhere non-zero on M. Then we have*

(2.3) $$h_{ji}u^i = pu_j, \quad h_{ji}v^i = pv_j,$$

(2.4) $$h_{jt}h_i^t = ph_{ji}, \quad p = constant,$$

p being given by

$$p = \frac{h_{st}u^s u^t}{1-\lambda^2} = \frac{h_{st}v^s v^t}{1-\lambda^2}.$$

Proof. (2.3) can be derived from (1.4) and (2.1). Now we prove the relationships (2.4). Since C and D are parallel in the normal bundle, the third fundamental tensor l_j vanishes identically. Then differentiating the first equation of (2.3) covariantly, we find by using (1.9) with $l_j=0$,

$$(\nabla_j h_i^t)u_t + h_i^t(-h_{js}f_t^s - \lambda k_{jt})$$
$$= (\nabla_j p)u_i + p(-h_{jt}f_i^t - \lambda k_{ji}),$$

from which, using equation (1.15) of Codazzi with $l_j=0$,

(2.5) $$2h_j^t h_{is} f_t^s = (\nabla_j p)u_i - (\nabla_i p)u_j - 2ph_{jt}f_i^t.$$

Transvecting this with u^i, we find

(2.6) $$(1-\lambda^2)\nabla_j p = (u^t \nabla_t p)u_j.$$

In the same way, we can prove, from the the second equation of (2.3)

$$h_t^s(k_{is}f_j^t - k_{js}f_i^t) + p(k_{it}f_i^t - k_{it}f_j^t)$$
$$= (\nabla_j p)v_i - (\nabla_i p)v_j.$$

Transvecting this with $u^i v^j$, we find $(u^t \nabla_t p)(1-\lambda^2) = 0$. This fact and (2.6) imply that $\nabla_j p = 0$, i.e., p is constant because $\lambda(1-\lambda^2)$ does not vanish almost everywhere. Thus (2.5) becomes $h_j^t h_i^s f_{ts} = ph_{jt}f_i^t$, or $h_j^t h_t^s f_{is} = ph_{jt}f_i^t$, from which, transvecting with f_k^i,

$$h_j^t h_t^s(-g_{sk} + u_s u_k + v_s v_k) = ph_j^t(-g_{tk} + u_t u_k + v_t v_k).$$

Therefore, using (2.3), we have

$$h_{jt}h_k^t = ph_{jk}.$$

This completes the proof of the proposition.

PROPOSITION 2.2. *Under the same assumption as those stated in Proposition* 2.1, *we have*

(2.7) $$h = mp, \quad \nabla_k h_{ji} = 0,$$

(2.8) $$h_{jt}k_i^t = pk_{ji},$$

m being the multiplicity of the eigenvalue p of (h_j^h), where h and k are defined by

$h=h_{ji}g^{ji}=h_t^t$, $k=k_{ji}g^{ji}=k_t^t$ *respectively.*

Proof. (2.7) resulted from (2.4) (cf. [10]). Now we prove the relationships of (2.8). Differentiating (2.2) covariantly and taking account of (1.8) and (2.7), we obtain

$$h_{jt}(-h_{ki}u^t+h_k^t u_i-k_{ki}v^t+k_k^t v_i)$$
$$+h_{it}(-h_{kj}u^t+h_k^t u_j-k_{kj}v^t+k_k^t v_j)=0,$$

from which,

$$(h_{jt}k_k^t-pk_{jk})v_i+(h_{it}k_k^t-pk_{ik})v_j=0$$

by virtue of (2.3) and (2.4). Transvection this with v^j gives

$$h_{it}k_k^t-pk_{ik}=0.$$

Thus Proposition 2.2 is proved. We are going to prove formulas (2.12) which will be useful in the sequel.

Transvecting (1.12) with g^{kh} and taking account of (2.4) and (2.7), we have

$$K_{ji}=(m-1)ph_{ji}+kk_{ji}-k_{jt}k_i^t, \tag{2.9}$$

from which,

$$\tilde{K}\equiv g^{ji}K_{ji}=m(m-1)p^2+k^2-k_{ji}k^{ji}, \tag{2.10}$$

where K_{ji} is the components of Ricci tensor of M. Since $\nabla_j\nabla_i k_{kh}-\nabla_i\nabla_j k_{kh}=-K_{jih}{}^t k_{tk}-K_{jik}{}^t k_{th}$, transvection g^{ih} yields

$$\nabla_j\nabla_i k=\nabla^k\nabla_k k_{ji}-K_{jt}k_i^t-K_{jki}{}^t k_t k$$

by virtue of (1.14) with $l_j=0$, where $\nabla^k=g^{ki}\nabla_i$.

Let $\Delta=\nabla^k\nabla_k$ be differential operator of Laplace and Beltrami. Then we have from (1.12), (2.4) and (2.9)

$$\Delta k_{ji}=\nabla_j\nabla_i k+mp^2k_{ji}-pkh_{ji}+kk_{jt}k_i^t-(k_{st}k^{st})k_{ji}.$$

This leads to the equations

$$k^{ji}\Delta k_{ji}=k^{ji}\nabla_j\nabla_i k+p^2(mk_{ji}k^{ji}-k^2)+kk_{ji}k_i^t k^{ji}-(k_{ji}k^{ji})^2$$

by virtue of (2.8).

Making use of the eigenvalues $\lambda_1, \lambda_2, \cdots\lambda_{2n}$ of $(k_j{}^h)$, we get the Nomizu and Smyth formulas (cf. [3], [5]):

$$k^{ji}\Delta k_{ji}=k^{ji}\nabla_j\nabla_i k+\sum_{j>i}^{m}(p^2+\lambda_i\lambda_j)(\lambda_i-\lambda_j)^2. \tag{2.11}$$

Where we have used the relations of Proposition 2.1 and 2.2.

By a simple computation, equation (2.11) can be rewritten as follow:

(2.12)
$$\frac{1}{2}\Delta(k_{ji}k^{ji})-\nabla_j(k^{ji}\nabla_i k)$$
$$=\nabla_k k_{ji}\nabla^k k^{ji}-\nabla_k k\nabla^k k+\sum_{j>i}^{m}(p^2+\lambda_i\lambda_j)(\lambda_i-\lambda_j)^2.$$

§ 3. Compact submanifold of codimension 2 with certain conditions

We suppose in this section that M is of a compact submanifold of codimension 2 of E^{2n+2} such that all assumptions as those stated in Proposition 2.1 are satisfied and the product of the mean curvature $\tilde{H}$ and the scalar curvature $\tilde{K}$ of M is constant, that is, $\tilde{H}\tilde{K}=constant$, where $\tilde{H}$ defined bv $\tilde{H}^2=\frac{1}{4n^2}(h^2+k^2)$. We shall investigate the sign of the right hand side of (2.12). First of all, we consider the first and the second terms of the right hand side of equation (2.12). By the calculating the square of the norm of $k(\nabla_k k_{ji})-(\nabla_k k)k_{ji}$, we get

(3.1)
$$\|k(\nabla_k k_{ji})-(\nabla_k k)k_{ji}\|^2$$
$$=k^2\nabla_k k_{ji}\nabla^k k^{ji}-2k(\nabla^k k)k^{ji}\nabla_k k_{ji}+(\nabla_k k\nabla^k k)k_{ji}k^{ji}.$$

Differentiation of (2.10) covariantly yields

(3.2)
$$k^{ji}\nabla_k k_{ji}=\frac{1}{2}\nabla_k(k_{ji}k^{ji})=k(\nabla_k k)-\frac{1}{2}\nabla_k\tilde{K}.$$

Eliminating $k_{ji}k^{ji}$ and $k^{ji}\nabla_k k_{ji}$ from these equations, we obtain

(3.3)
$$\|k(\nabla_k k_{ji})-(\nabla_k k)k_{ji}\|^2$$
$$=k^2(\nabla_k k_{ji}\nabla^k k^{ji}-\nabla_k k\nabla^k k)-\{\tilde{K}-m(m-1)p^2\}\nabla_k k\nabla^k k+k\nabla^k k\nabla\tilde{K}.$$

Since $\tilde{H}\tilde{K}=constant$, differentiating this covariantly, we have

(3.4)
$$\nabla_k\tilde{K}=-\frac{1}{h^2+k^2}\tilde{K}\nabla_k k.$$

Substituing (3.4) into (3.3), we find

(3.5)
$$\|k(\nabla_k k_{ji})-(\nabla_k k)k_{ji}\|^2=k^2(\nabla_k k_{ji}\nabla^k k^{ji}-\nabla_k k\nabla^k k)$$
$$-\frac{1}{h^2+k^2}[\{2\tilde{K}-m(m-1)p^2\}k^2+\{\tilde{K}-m(m-1)p^2\}h^2]$$
$$\times\nabla_k k\nabla^k k,$$

because $\tilde{H}$ is non-zero on compact submanifolds of E^{2n+2}.

If we suppose that $\nabla_k k_{ji}\nabla^k k^{ji}-\nabla_k k\nabla^k k<0$ and $\tilde{K}\geqq m(m-1)p^2$, it follows from (3.5) and these relationships

$$0\geqq k^2(\nabla_k k_{ji}\nabla^k k^{ji}-\nabla_k k\nabla^k k)$$

$$\geqq \frac{1}{h^2+k^2}[\{2\tilde{K}-m(m-1)p^2\}k^2+\{\tilde{K}-m(m-1)p^2\}]$$

$$\times \nabla_k k \nabla^k k \geqq 0.$$

Thus $k^2(\nabla_k k_{ji}\nabla^k k^{ji}-\nabla_k k\nabla^k k=0$, i.e., $k^2=0$.

It leads that $\nabla_k k_{ji}\nabla^k k^{ji}-\nabla_k k\nabla^k k\geqq 0$, provided that $\tilde{K}\geqq m(m-1)p^2$.

For each point x in M, let $X_1, X_2, \cdots X_{2n}$ be an orthonormal frame of the tangent space M_x such that any X_j is an eigenvector of the second fundamental tensor K corresponding to an eigenvalue λ_j. Then by remembering equation (1.12) of Gauss, the sectional curvature $K(X_i, X_j)$ of the plane section spanned by X_i and X_j is given by

$$K(X_i, X_j) = -\frac{K(X_i, X_j, X_i, X_j)}{\|X_i\|^2\|X_j\|^2-\langle X_i, X_j\rangle^2} \tag{3.6}$$

$$=p^2+\lambda_i\lambda_j.$$

If M is of non-negative curvature, by the Green's theorem and equation (2.12) obtain

$$\int_M \{\nabla_k k_{ji}\nabla^k k^{ji}\nabla - {}_k k\nabla^k k+\sum_{i<j}^{m}(p^2+\lambda_i\lambda_j)(\lambda_i-\lambda_j)^2\}\,dM=0,$$

dM being volume element of M. $\nabla_k k_{ji}\nabla^k k^{ji}-\nabla_k k\nabla^k k$ and $p^2+\lambda_i\lambda_j$ being both non-negative, we find

$$\nabla_k k_{ji}\nabla^k k^{ji}-\nabla_k k\nabla^k k=0, \tag{3.7}$$

$$(p^2+\lambda_i\lambda_j)(\lambda_i-\lambda_j)^2=0 \tag{3.8}$$

for any indices i and $j\leqq m$ at each point in M.

Equation (3.8) shows that for distinct eigenvalues λ_i and λ_j, $p^2+\lambda_i\lambda_j=0$. Thus we have

LEMMA 3.1. *Let M be a compact orientable submanifold of codimension 2 in an even-dimensional Euclidean space E^{2n+2} such that the global unit normals C and D are parallel in the normal bundle and the function $\lambda(1-\lambda^2)$ is non-zero almost everywhere. Suppose that M is of non-negative curvatures and $Hf=fH$, $\tilde{H}\tilde{K}=$constant. If $\tilde{K}\geqq m(m-1)p^2$, then there exist at most two eigenvalues of $(k_j{}^h)$, say λ, μ such that*

$$p^2+\lambda\mu=0. \tag{3.9}$$

Finally we prove

THEOREM 3.2. *Let M be a compact orientable submanifold of codimension 2 in an even dimensional Euclidean space E^{2n+2} such that M is of non-negative curvatures and there are global unit normals C and D to M which are parallel in the normal bundle. If the product of the mean curvature $\tilde{H}$ and the scalar curvature $\tilde{K}$ is constant and $fH=Hf$ hold, where H are the second fundamental tensor of M with respect to C, f being the tensor field of type $(1,1)$ appearing in the induced structure. (f, g, u, v, λ) of M, then M is in E^{2n+2}, provided that $\lambda(1-\lambda^2)$ is non-zero almost everywhere in M and $\tilde{K}\geqq$*

$m(m-1)p^2$, congruent to a pythagorian product $S^r(a)\times S^{2n-r}(a)$, where $S^r(a)$ denotes a r-dimensional sphere of radius $a(>0)$ imbedded naturally in E^{2n+2}.

Proof. Taking account of (3.5), (3.7) and the assumption $\tilde{K}\geqq m(m-1)p^2$, we see that

$$[\{2\tilde{K}-m(m-1)p^2\}k^2+\{\tilde{K}-m(m-1)p^2\}h^2]\nabla_k k\nabla^k k=0.$$

If we assume that there exists a set $M(x)$ such that

$$M(x)=\{x\,|\,(\nabla_k k)(x)\neq 0\} \text{ in } M,$$

then

$$\{2\tilde{K}-m(m-1)p^2\}k^2+\{\tilde{K}-m(m-1)p^2\}h^2=0 \tag{3.10}$$

holds on $M(x)$.

Differentiating (3.10) covariantly along $M(x)$ and using (3.4) and definition of $M(x)$,

$$2k^2\tilde{K}+h^2\tilde{K}=2m(m-1)p^2(h^2+k^2) \tag{3.11}$$

on $M(x)$. Substituting (3.11) into (3.10),

$$m(m-1)p^2(h^2+k^2)=0 \tag{3.12}$$

on $M(x)$.

On the other hand, if $p=0$, then $h_{ji}=0$. Then M can be regarded as hypersurface of E^{2n+2} because $l_j=0$. From (3,9) we have that the type number $t(x)$ at each point x of M is equal to 0 or 1. Since M is compact, it is seen that there exists a point in M at which all eigenvalues of $(k_j{}^h)$ are positive or negative. This contradicts the fact that the typer number $t(x)$ is equal to 0 or 1. Thus the constant p must be non-zero. Secondly we consider the case $m=0$. Then $h=0$, and consequently $h_{ji}h^{ji}=ph=0$, from which, $h_{ji}=0$. It contradicts the fact that p must be non-zero. Finally we consider the case $m-1=0$. Then $h=p$, and consequently $h_{ji}h^{ji}=p^2$. By the calculating the square norm of $(1-\lambda^2)h_{ji}-pu_ju_i$ and $(1-\lambda^2)h_{ji}-pv_jv_i$ respectively, we have

$$(1-\lambda^2)h_{ji}=pu_ju_i,\quad (1-\lambda^2)h_{ji}=pv_jv_i,$$

from which, $pu_ju_i=pv_jv_i$ hods and it shows that $p=0$ because $1-\lambda^2$ is non-zero almost everywhere. Thus $m-1$ must be non-zero. Hence we obtain from (3.12) that $M(x)$ is empty. Therefore $\nabla_k k=0$ on the whole space M and consequently k is constant. From (3.7), we have $\nabla_k k_{ji}=0$.

On the other hand, there exist exactly two eigenvalues of $(k_j{}^h)$ λ,μ satisfying (3.9), where λ and μ have coonstant multiplicity r and $2n-r$ respcctively. It follows that $k=r\lambda+(2n-r)\mu$ is constant.

From this and (3.9) imply that eigenvalues λ and μ are both constant.

Summing up the arguements developed above, $(k_j{}^h)$ has exactly two distinct and non-zero constant eigenvalues λ and μ and M satisfies $\nabla_k k_{ji}=0$. The subspace of the tangent space $T_p(M)$ at P defined by

$$D_\lambda(P)=\{w^h\in T_P(M)\mid k_t{}^h w^t=\lambda w^h\}$$

and

$$D_\mu(P)=\{w^h\in T_P(M)\mid k_t{}^h w^t=\mu w^h\}$$

difine two mutually orthogonal distributions of dimension r and $2n-r$ respectively. It follows that the both distributions are integrable and the integral submanifolds are totally geodesic in M. Thus in the usual way (cf. [6], [9]), using de Rham's decomposition theorem, we have the conclusions as required.

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