EFFICIENT RESOURCE ALLOCATION MECHANISMS FOR LARGE ORGANIZATIONS

Hyung Kee Kim*

Abstract

Economists have become interested in the centralization and decentralization of planning, in the linking up of models into a homogenous model system, and in multi-level planning.

In this study, Input/Output techniques used for explaining the resource allocation mechanisms to be more rational through detailed specifications of a large organization's objectives and explicit linking of centralization and decentralization to such objectives.

Also the application of mathematical methods to the higher levels of planning in the optimal allocation resources can't fully describe the actual practice of planning.

On the other hand, 1-0 techiques are standard in economic analysis and planning. However, the application procedures to the armed forces hold only when their assumptions are met and when their solutions are convergent. So, It is of limited applicability.

1. Introduction

This study of a large organization's resoure allocation decision-making process is concerned with how large organizations may make decisions or reach positions that satisfy some conditions with reference to the organizations environment. The importance of planning decisions themselves are the existance of the external planning environment have long been recognized. There is, however, increasing controversy over the relative importance of internal processes and procedures used by organizations in arriving at decisions. Are the processes and procedures themselves important determinants of outcome, or is it sufficient to know that if the choices satisfy some externally determined condition, the organizations will somehow respond appropriately?

The increasing size and complexity that characterize large organizations that are found in almost every facet of our economy make it imperative to understand the manner in which they function. One may be faced with the question of whether or not to allow the organizational levels to retain their current functions more or less unchanged, or to reallocate the functions among the levels, or to strengthen, or to weaken the power of the central unit. So, the process of resource allocations in a large organization more generally seems to be one characterized by the use of simple internal decision procedures and rules-of-thumb.

* Ministry of Defense

While in economics one deals with goal conflicts due to multiplicity of consumers, linear and non-linear programming models usually presuppose a simple well-defined objectives function to be, say, maximized, i.e., a situation corresponding to an economy with a single consumer. So, it is not surprising that the mechanisms designed to a large extent with one-objective function problem that thus failed to face the crucial issue of goal conflict. Nevertheless, one should not underrate their usefulness as a necessary step on the road to the harder as multi-objective problem, since the difficulties of the simpler situations don't disappear when goal conflicts are introduced.

However objectives do not remain fixed, but evolves through external effects and experience. In the real world, it is necessary to divide the problems into components that are meaningful according to relevant criteria. "Necessary and sufficient conditions for optimum planning system characteristics" are:

a. Monotonic
b. Convergence
c. Always feasibility
d. Finiteness

The decision maker may approach a given problem more effectively in terms of subproblems because of lack of information about objective costs and technologies. To apply the general equilibrium approach the entire system of the models would be solved simultaneously. A decision based on this solution would then be optimal in the sense that it would take account of all possible interactions within the system. This optimal decision would be interpreted as outcome which would prevail in a perfectly competitive market-making the usual assumptions on the absence of externalities economy of scale, etc., alternatively, it could be interpreted as centralized decision making, if the central part has perfect knowledge of the entire economy.

In the actual practice of plannig, it is of course rather difficult to talk about planning in general, since there are the following factors:

a. Political and economic
b. Timing
c. Organizational and bureaucratic
d. Individual and personality

The purpose of this study is to apply 1-0 models and procedures to determine an optimal resource allocation.

2. A simple Input/Output analysis

An economy consists of a large number of consumers and producers who conduct among themselves transactions-sales and purchases of goods and services. Statistical measure of these transactions will clear in what manner indivdual economic units are dependent on one another. The assumptions of this I/O analysis are basically linear in economic models and no technical change; that is, there exists commodities that are used in further production of commodities and linear activities for each industry; assume that one industry produces only one commodity, so, there is no joint production. Hence, the number of industries is the same as the number of commodities; assume further that no changes occur in inventories of the commodities. The Leontief models assume constant proportions among inputs and outputs for each industry. This production relationship rules out factor substitutability.

An I/O table I fulfills two separate functions. First, it is a descriptive framework for showing the relations between industries and sectors and between the nature of production functions. It is an analytical tool for measuring the impact of autonomous disturbances on an economic outcome and income.

(1) The mathematical I/O models

Consider the following extensive of a Leontief (table 1) I/O system where the table I$[d(i,j)\,Y(j)]$ specifies the commodity requirements of the output vector Y. Notice that the intermediate production (commodity) vector X appears in two places. This is in consonance with the Leontief expression for this tableau.

The tableau I is defined to be display of the components of a matrix multiplication. Thus, the tableau of the vector SX is $[s(i,j)X(j)]$

$$X=SX+DY$$

from which

$$X=(I-S)^{-1}DY$$

The primary input (resources) vector is

$$W=UX+VY$$

Table I

from \ to	Commodity purchaser $X(j)$	Final user $Y(j)$	
Commodity producer	$s(i,j)X(j)$	$d(i,j)Y(j)$	Total output $X(i)$
Primary input (resource) $W(i)$	$u(i,j)X(j)$	$v(i,j)Y(j)$	

then yields

$$W=U(I-S)^{-1}DY+VY$$

Define a primary input matrix R for the output vector Y such that

$$W=RY$$

hence, we see that

$$R=U(I-S)^{-1}D+V$$

or $$R=F+V$$

where

V= direct input matrix

$F=U(I-S)^{-1}D=$ indirect input matrix

To obtain final output cost, we only need a vector of the resource cost coefficients, K, with a component for each type of primary input.

The cost of commodities is then

$C=KU$

Defining a vector of commodity prices P, the value of output is the

PDY

and for equilibrium we have

$PDY=KUX$

$PDY=KU(I-S)^{-1}DY$

$P=KU(I-S)^{-1}$

$P=C(I-S)^{-1}$

Letting total final output cost be represented by \$, we have

$\$=KW=KRY$

$\$=KW=KRY=KU(I-S)^{-1}DY+KVY$

Thus, we see that

KVY is the direct cost of output vector Y

$PDY=KU(I-S)^{-1}DY$ is the indirect cost of output vector Y

To view the Leontief model of the support establishment in a linear programming context,

let $A=(I-S)$

$b=DY$

and $c=KUC$

The objective function

Minimize $\$=KUX+KVY$

is composed of a fixed part, KVY which for purposes of optimization are constant and therefore can be ignored, and a variable part,

$KUX=cx$

where we have replaced capital X with a lower case x. Thus, the linear programming formulation is

Minimized $Z=cx$

subject to $Ax\geq b$

Notice that the vector of dual variables for the L.P. problem will be the P vefctor defined earlier.

(2) The application to the service (Navy)

The model calculates ship and aircraft operating costs and manpower (direct) and the related support operating costs and manpower (indirect) of support resources, only those in the "general support" fiscal guidance categories that vary with the size or operating policies of the forces are estimated. Direct and support resources not estimated are throughput, i.e., their output values are equal to their input values.

Fiscal guidance categories

I. Major mission

A. Force mission

1. Land forces
2. TAC Air forces
3. Naval forces

B. Other mission

1. Intelligence (Security)
2. Communication
3. R&D

II. General support

A. Base & individual support

B. Trainning

C. Command

D. Logistics

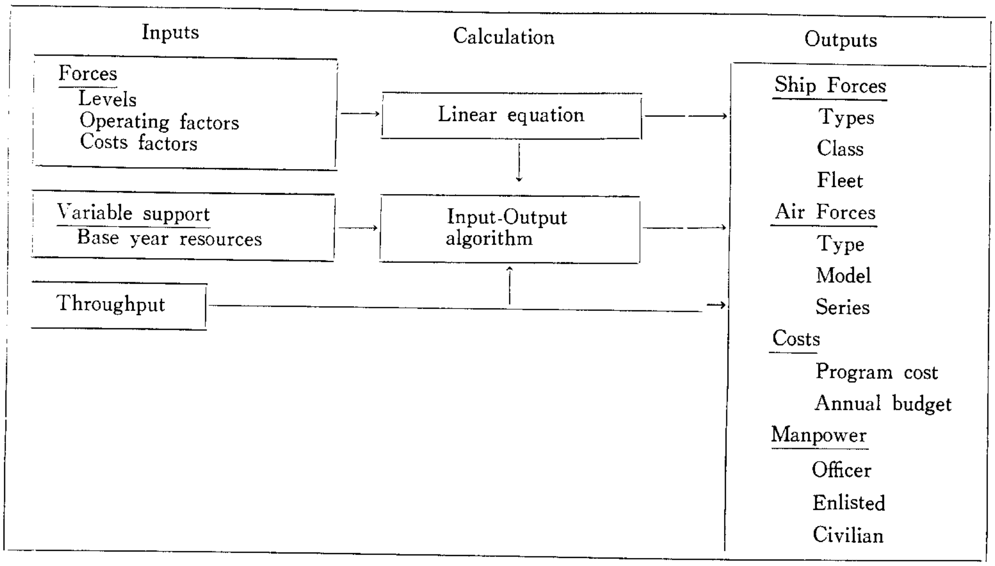

Figure I

The first step in using input-output analysis for the service is to devide the components of the system being modeled into "Sectors". A sector represents one organization or function. In models of the economy, a whole industry such as steel becomes one sector. In this model of the navy sectors represent functions such as recruiting training, ship maintenance and operation, and sea warfare forces operation.

These sectors are; inturn, divide into two groups: those that support other sectors and those that don't.

Sectors that support other sectors produce goods and services and in turn goods and services; such sectors are called support sectors. Those that don't support others but who do consume the out-put of the support sectors, consisting mainly of the operating forces, are called final users: final in the sense that they are the final step in the system.

The resource flows that bind the system together are captured by measuring each sectors output provide to other support sectors and to final users (operating forces). These output measures, or

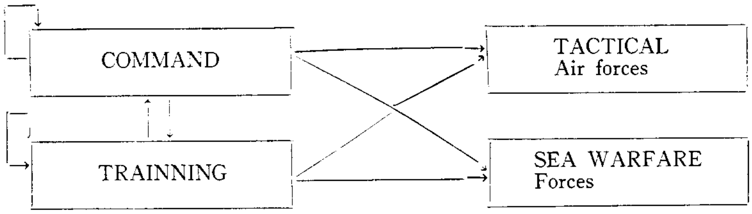

Figure II

workload indicators, relate changes in the final users to changes in the workloads and the resource consumed by the support sector. It in not necessary that those output measures be actual output.

In this paper, proxy variables are utilized for output measures. Proxy variables are data which do not directly measures. Proxy variables are data which do not directly measure the workload of the support activity but which are assumed to vary roughly in proportion to a direct measure. For example, the actual output of recruit training is trained recruits. The proxy variable is the number of Navy enlisted men in each sector. The rational is that on the average the number of trained recruits required by each sector will vary in proportion to the number of men in that sector.

This workload information is then organized into a matrix (called the transaction matrix) which has one row for each support sector and as many columns as there are support sectors and final users. Each row represents the output of that support sector are showed how that sector's output is allocated to each of the consuming sectors, including itself. One sector's out-put becomes another's sector input so that each entry in a row is an input into the sector represented by the column. Thus, this matrix is techique feature, each element in the matrix is simultaneously an output of the sector represented by row and an input to the sector represented by the column (the name input-output comes from this layout).

In addition, all sectors, both support sectors and final users, needs inputs not only from other sectors within the system, but also inputs from outside the system, such as labor and equipment. These primary inputs can be measured in any appropriate unit and are added to the above relationships by expanding the matrix to include more rows representing the additional inputs.

Table II shows the five support sectors and three final users, their abbreviations used in late Tables, the proxy variables for the support sectors, and the primary inputs for this example; Table III is the transactions.

Table II

Sectors and proxy variables used in sample problem

Support sectors			Proxy variables
1. Base operating support	(BOS)	:	Military pay, operations and maintenance (MPN+OMN)
2. Medical support	(MED)	:	Military Personel (MILPER)
3. Trainning	(TRA)	:	Military personal (MPN)
4. Command	(CMD)	:	Total obligational authority (TOA)
5. Logistic	(LOG)	:	Operations and Maintenance (O&M)
Final User Sectors			**Primary inputs**
1. Tactical air (TAC AIR)			1. Operations & Maintenance (O&M)
2. Sea warfare forces (SWF)			2. Military personel (MPN)

3. All other navy (Other)

3. Total obligational authority (TOA)
4. Navy personel (NAVPER)
5. Marine Corps personel (MCPER)
6. Civilian personel (CIVPER)

The following transaction Matrix can be functionally divided into four quadrants as shown below:

	m columns	n columns
m rows	U	V
k rows	W	Z

The rows of the U and V matrices represent the flow of support from the support sectors to both support sectors and final users. The columns of W and Z are the inputs to each sector from outside. Assuming there are m support sectors, n final users and k resource inputs, U is m×m, V is m×n, W is k×m and Z is k×n.

Table III

Transactions Matrix

	Support Sectors					Final users		
	BOS	MED	TRA	CMD	LOG	TACAIR	SWF	Other
BOS	858	362	1123	784	2032	682	1237	2102
MED	71	18	136	40	9	65	128	458
TRA	395	150	864	328	74	406	797	1019
CMD	972	365	1542	815	2370	2920	3937	5989
LOG	463	212	259	456	1958	286	440	1083
OMN	463	212	259	456	1958	286	440	1083
MPN	395	150	864	328	74	406	797	1019
TOA	972	365	1542	815	2370	2920	3937	5989
NAVPER	61	18	136	39	9	64	127	177
MCPER	10	0	0	1	0	1	1	281
CIVPER	53	9	11	24	202	0	0	105

In terms of Table III, the matrix U is a 6×6 square matrix, V is a 6×3 matrix, W is a 6×6 matrix of resource inputs, and Z is a 6×3 matrix of resource inputs for the final users.

This matrix of inputs and outputs in either dollars or units of real output is just a form of double entry bookkeeping. The usefulness of input-output analysis is that this matrix can be transformed into other matrices which can be manupulated to show structual interdependence and to predict the impact on support of force level changes.

Several steps are involved in transforming the subdivided transactions matrix into a predicting device. In step I, each row of U and V is summed to create a column vector X of total output.

1)

$$\sum_{j=1}^{m} U_{ij}+\sum_{j=1}^{n} V_{ij}=X_i \qquad i=1,\cdots,\mathrm{m}$$

in the example of table III, $X=9932$

Each row of V is summed to create a column vector Y, representing that part of total output being consumed by the final users.

2)

$$\sum_{j=1}^{n} V_{ij} = Y_i \qquad i=1, \cdots, m$$

in the example Y:

4031
651
368
2222
12846
1809

Each element of the jth column of U is divided by the jth element of X to create the m×m matrix A.

3)

$$\frac{U_{ij}}{X_j} = A_{ij} \qquad \begin{array}{l} i=1, \cdots, m \\ j=1, \cdots, m \end{array}$$

Step 3 converts the U matrix, which contains gross data into a matrix of proportional coefficients, commonly called the A matrix. The A matrix is a square matrix with as many rows and colnmns as there are support sectors. In word, A is formed by first summing across each row; this sum gives the total ontput of that sector. Then each element in the corresponding column of U is divided by that total. The results for each support sector, is the number of units of inputs required from each of the support sectors including itself for each unit of its output.

For example, Column 1 of U from Table III is;

857
71
81
395
972
463

Each element is divided by $X_1=9932$. These results form column of A which;

.086
.007
.006
.040
.098
.067

Table IV shows the A matrix for example. The entries in Table IV are interpreted as follows: .040 (at column 1, row 4) means that for every unit of BOS output, TRA must supply .040 of a unit of its output. This model is built upon these proportional relationships.

Table IV

This is the A matrix

	BOS	MED	TRA	CMD	LOG
BOS	0.086	0.367	0.237	0.040	0.392
MED	0.007	0.018	0.029	0.002	0.002
TRA	0.040	0.152	0.182	0.017	0.014
CMD	0.098	0.370	0.325	0.041	0.457
LOG	0.049	0.215	0.055	0.023	0.377

The matrix A is subtracted from the identity matrix and then inverted. The logic behind its derivation and use can be understood by considering the following series of equations:

4)

$$X=AX+Y$$
$$X-AX=Y$$
$$(I-A)X=Y$$
$$X=(I-A)^{-1}Y$$

As before, X is a column vector representing total output from the support establishment and Y is a column vector representing that part of total otuput being consumed by the final users:

Table V

This is a (I−A) inverse matrix

	BOS	MED	TRA	CMD	LOG
BOS	1.191	0.784	0.540	0.085	0.832
MED	0.013	1.039	0.049	0.004	0.016
TRA	0.081	0.305	1.336	0.033	0.112
CMD	0.219	0.856	0.676	1.094	0.965
LOG	0.110	0.479	0.204	0.052	1.720

AX is a column vector representing output consumed by support sectors. The $(I-A)^{-1}$ matrix has a very important property; each of its element r_{ij} shows the total support (as measured by the proxy variables) sector i must provide to enable sector j to provide one unit of its output. This total support includes all indirect support.

The usefulness of the $(I-A)^{-1}$ matrix is that once it is developed for one set of demands it can be multiplied times a new level of demands by final users (e.g., a new force level), denoted Y, to obtain an estimate of the total output requirements from each of the support sectors. Given Y′ the required output X′ is estimated by;

$$X'=(I-A)^{-1}Y'$$

where

$$Y_i'=\sum_{j=1}^{n}V_{ij} \qquad i=1,\cdots,m$$

The inverse matrix, $(I-A)^{-1}$, can also be used to estimate the next final input of a force level change. Defining the force level change as $\Delta Y'$, then the marginal impact $\Delta X'$, is computed by;

$$\Delta X'=(I-A)^{-1}\Delta Y'$$

The resources required to produce the new output, X' must now be calculated. We assume that all resource requirements vary proportionaly to output. Our resource estimates are arrived at in the following way; Each element in the jth element of X;

5)

$$\frac{W_{ij}}{X_j}=B_{ij} \qquad \begin{matrix} i=1,\cdots,k \\ j=1,\cdots,m \end{matrix}$$

The new W' is calculated by multiplying each element in the jth column of B by the j^{th} element of X'.

6)

$$W'_{ij}=B_{ij}X'_j \qquad \begin{matrix} i=1,\cdots,k \\ j=1,\cdots,m \end{matrix}$$

The matrix W' contains the resource estimate for each support sector for the forces contained in Y'. The matrix W' can contain as much detail about resources as desired. Each desired element becomes a row in B.

The inverse matrix can also be used to allocate support resources to forces (final users). This is done by calculating shadow prices (which can be defined as the cost of all resources used, including those used indirectly, to produce each unit of support output) and multiplying these shadow prices times the units of support output used by each final user.

Shadow prices P are generated by the following operations. The appropriate elements of each column of B are summed to create a row vector B_j;

7)

$$\sum_{i=a}^{b} B_{ij}=\bar{B}_j \qquad j=1,\cdots,m$$

8)

$$P=\bar{B}(I-A)^{-1} \qquad \text{(P of cource is a row vector)}$$

Shadow prices for the example are listed in Table VI; the support (dollar) resources allocated to each final user by using these shadow prices are shown in Table VII.

Table VI

Listing of shadow prices

BOS:	0.1908
MED:	0.7840
TRA:	0.5400
CMD:	0.0854
LOG:	0.8320

Table VII

Soppors dollar allocated to final users

	TACATR	SWF	OTHER
BOS	132	236	401
MED	51	100	359
TRA	210	430	550
CMD	238	366	901
LOG	889	1468	2722
Total	1569	2500	9933

The total support dollars used by the j^{th} final user; S_j are determined by:

$$S_j=\sum_{i=1}^{m}P_iV_{ij} \qquad j=1,\cdots,n$$

Also, of cource

$$\sum_{j=1}^{n}S_j=\sum_{j=1}^{n}\sum_{i=1}^{m}P_iV_{ij}$$

is equal to

$$\sum_{i=\alpha}^{\beta}\sum_{j=1}^{m}W_{ij}$$

This tells us that the total support dollars have been completely allocated. In Table VII; the support dollars are allocated as follows: $ 1, 032 to TACAIR, $ 1, 751 to SWF, and $ 3, 116 to OTHER. Their sum ($ 5, 899) agrees with the total support operating cash in Table III (OMN+MPN= $ 5, 900)

3. Conclusion

Traditionally, economic analysis treats the economic system as one of the givens and the interdependence in planning is the central concern of study. Experiments are proceeding in two directions: these has been the use of mathematical programming in several sectors of industry, to form a basis for their plans; the other direction has been the use of I/0 tables, static open Leontief models in national planning.

I/0 matrix of the economy is used to check the inner coordination of the long-range plans. It is, however, as is general known, not suitable for optimization and thus only useful in achieving the correct proportions among sectors.

The model is open in the sense that not all of its variables are mutually and simultaneously determined by the operation of the model. It is a static model in the sense that capital requirements are explicitly excluded from the technical requirements represented in the coefficient matrix. The static-open model is purely a technical model which determine production levels once the final demand objectives are specified. Other important properties of the model explained above are crucial to the nature of the model. These are assumptions of constancy and proportionality in the inputs coefficients.

The assumption that each input to an industry is exactly proportional to the industries level of output means that constant return scales are assumed. The assumption that the input coefficient remain constant overtime means that there is no substitution among the inputs of any sector.

On the other hand, this is a sharp distinction between an I/O and L.P. matrix. In I/O analysis, the activities have multiusers i.e., each may be used as both a final product and an intermeadiate product. In L.P., the resources or raws, have no other use, but, are merely inputs that may enter into one or more outputs and whose supply is constrained. In I/O only the primary inputs are constrained. Therefore, I/O is a special case of L.P.

Finally, in the application to the Navy, the model does not allow for trade-offs between resources or program elements within a support sector, so that different resources within a sector are distributed similarly in all years and present a framework for using computerized system of I/O model both for (step 6) prediction and for (step 8) allocation.

References

1. Nathan Keyfits, "Elements of a Theory of Planning", *Rao's Essays on Economics and Planning.*
2. Harry W. Richardson, *Input/Output and Regional Economics,* 1972.
3. E Malinvoud, *Activity Analysis in the Theory of Growth and Planning,* New York, 1967.
4. Orchard-Hays, *Advanced Linear-Programming Computing Techniques,* 1968.
5. Lous M. Goeur and Alan s. Manne, *Multi-Level Planning; Case studies in Mexico,* North-Holland publiching Company, 1973.
6. J. Kornai, *Mathematical Planning of Structural Decision,* 1967,
7. Hurwiz, Lo, "Optimality and Informational Efficiency in resource allocation process," *Mathematical Methods in the social sciences,* 1959.
8. Marschak, T, "Centralization and Decentralization in Economic Organization, "*Econometrics,* Vol.27." July, 1957.
9. T.C.Hu, *Integer Programming and Net Works Flows,* 1969.
10. A.Browdy and A.P. Carter, *Input-Output Techniques,* 1972.
11. R. Dorfman and P.A. Samuelson and R.M.Solow, *Linear Programming and Economic Analysis,* McGraw-Hill Book Company, 1958.
12. Rand, *Program Budgeting Long-Range planning in the Department of Defense.*
13. Fremont J. Lyden and Ernest G. Miller, *Planning Programming Budgeting, A system approach to Management.*