Numerical Investigation on the Role of Maruo's Line Integral Term for the Improvement of Michell's Wave Resistance Calculations

by

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In the present paper author has investigated numerically on some properties of Maruo's line integral term associated with the wave resistance theory. The role of Maruo's line integral term for the improvement of Michell's wave resistance calculations are discussed.

By applying Green's theorem over a closed surface with linear wave elevation, it was shown in a paper of Eggers and Choi (1975, Appendix B) that within consistent second-order theory, there is *no* line integral term in the expression of a velocity potential for a surface-piercing ship.

In the work of Maruo (1966), however, a line integral term appears with source density $\lim_{z\to o}\varphi^{(1)}x/k_o$ along waterline as a part of second-order potential. This term may be considered as a necessary correction to the Michell's source potential due to change of wetted hull area. But this line integral term is already inherent in the centerplane source distribution of second-order and just the same term, but with opposite sign, has to be included in compense to transfer the free surface boundary condition to the undisturbed free surface. Thus at the end stage we get no line integrals.

Nevertheless the Maruo's line integral contains some interesting properties. The output of its source distribution balances the upward flow inside the waterplane which results from non-zero total output of sources distributed over wetted hull up to the undisturbed free surface, if the body boundary condition holds exactly and at the same time if the free surface boundary condition is made linearized (i.e. for a Neumann-Kelvin problem).

The Lagally force on this source distribution corresponds to the momentum flux in direction of ship's motion over the wave profile area and mounts to the difference between resistance from far-field waves and that from pressure integration over wetted hull up to $z=0$, if there is no flow through the hull(see Eggers, 1975).

The Maruo's line integral may be expressed in the form,

$$I_M=-\frac{2\epsilon}{4\pi k_o}\int_{-1}^{1}\varphi_x^{(1)}(x',\ o,\ o)\cdot f_x(x',\ o)\cdot G(x,y,z;x',o,o)dx'$$

where $\varphi^{(1)}$ is the first-order Michell's potential, $f(x,z)$ the hull equation, G Havelock's source potential with output -4π and ϵ is the ratio of ship's beam to its length. All length variables are made dimensionless with ship's half length as unity.

Then the contribution from this line integral to wave resistance may be obtained by Lagally theorem (i.e. minus the Lagally force on the source distribution),

$$R_M^{(1)}=-\frac{2\rho\epsilon}{k_o}\int_{-1}^{1}\varphi_x^{(1)2}(x,\epsilon f(x,o),o)\cdot f_x(x,o)dx.$$

In principle, this procedure can be performed successively by using an iteration formula for expression of I_M,

$$\varphi^{(n)}=-\frac{1}{4\pi k_o}\oint\varphi^{x(n-1)}Gdy,\ n\geq 2$$

where the line integral should be performed counterclockwise if ship moves in +x direction and $\varphi^{(1)}$ is the first-order Minchell potential.

It was also shown in Eggers (1975) that the next approximation of the contribution to wave resistance becomes

$$R_M^{(2)}=\frac{2\rho\epsilon^2}{\pi k_o^2}\int_{-1}^{1}\varphi_x^{(1)}(x,\epsilon f,o)f_x\int_{-1}^{1}\varphi_{x'}^{(1)}(x',\epsilon fo)G^x(x,\epsilon f,o;x',\epsilon f,o)dxdx'$$

Recieved on Ang. 20, 1976.
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and that the order of magnitude of these forces is

$R_M^{(1)}=0(<^2\epsilon)$, $R_M^{(2)}=0(<^2\epsilon^2\log\xi)$,……

where $<$is some proper smallness parameter and ξ is the beamlength ration.

As shown above $R_M^{(2)}$ is formally small enough that we stop the procedure at this stage and sum up as follows,

$R_M=R_M^{(1)}+R_M^{(2)}+0(<^2\xi^3)$.

A numerical example for Weinblum-Kendrick-Todd plank shows that this resistance component oscillates synchronously with that of Michell's integral and $R_M^{(2)}$ is actually small except high speed range (Eggers, 1975). The aim of this numerical investigation for other mathematical models is to confirm this trend and also to examine whether this wave resistance component improve the Michell's one with the residual resistance as reference.

For this purpose, we selected four simple models for which experimental work is already done.

Table of Models

Model	ξ=B/L	2T/L	f(x,z)
Weinblum-Kendrick Todd	0.0265	0.286	$1-(x-p)^2/(1-p)^2$ 2p=parallel middle body
Sharma	0.05	0.3	$1-x^2$
Wigley	0.1	0.125	$(1-x^2)(1-(z/T)^2)$
Shearer No. 2892	0.09375	0.125	$(1-1.2x^2+0.2x^4)$ $(1-(z/T)^2)$
Shearer No. 3012	0.09375	0.125	$(1-2x^2+x^4)(1-(z/T)^2)$

With these models we can see variance of beam-length ratio from 0.0265 to 0.1 and of draft from).125 to 0.3, of waterline-parabolic with or without parallel middle body, quadratic-and also of cross section-rectangle, parabolic.

The data of residual resistance are taken from Weinblum-Kendrick-Todd (1952), Sharma (1969), information given courteously of Kajitani and Shearer (1950), respectively. For all cases trim was not allowed.

In general, the contribution from Maruo's line integral to wave resistance narrows the discrepancy between residual and Michell result.

Michell result gives in general more values at humps and smaller at hollows than the residual resistance, it was corrected to some extent by adding R_M(especially see Figure for Weinblum-Kendrick-Todd plank at Froude numbers 0.275, 0.313 and 0.362).

One wholly different case is the Wigley model, in which the Maruo's integral gives too much contributions compared to the residual resistance. For reference the total pressure resistance taken from Shearer and Cross(1965) is also shown.

On the other hand $R_M^{(1)}$ alone should be equal to the difference between resistance from far-field waves and that from pressure integration over up to $z=0$ with error of $0(<^3)$, if the body boundary condition is satisfied exactly. Thus the differance between resistance from far-field waves and that from pressure integration up to $z=0$ plus $R_M^{(1)}$ is a measure of accuracy of boundary condition on the hull. For Weinblum-Kendrick-Todd and Sharma's model this calculation is performed.

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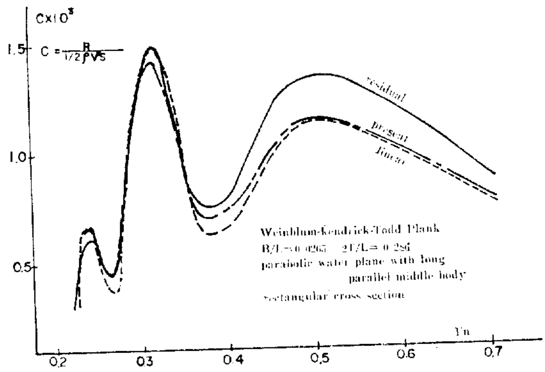
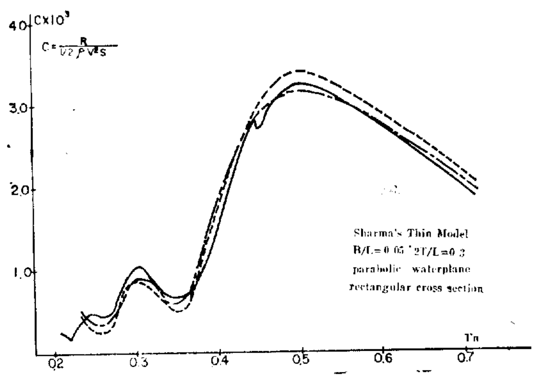
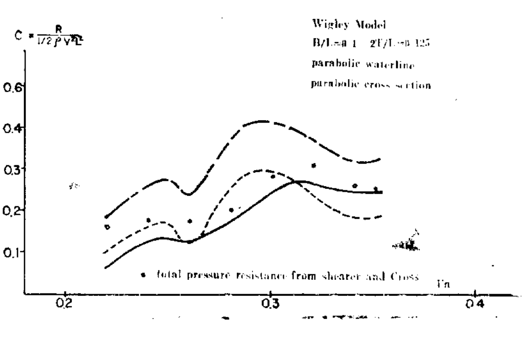
for Weinblum-Kendrick-Todd plank

Fn	0.236	0.267	0.289	0.316	0.378	0.408	0.5
①	0.957	0.899	0.997	0.986	0.925	0.950	0.985
②	0.998	1.089	1.019	0.933	1.033	1.060	0.968







①: $R^{hull}_{pressure}/R_{far\text{-}field}$

②: $\left(R^{hull}_{pressure}+R_M^{(1)}\right)/R_{far\text{-}field}$

for Sharma's thin model

Fn	0.236	0.258	0.316	0.365	0.447	0.5	0.709
①	0.903	0.790	0.865	0.961	1.020	0.982	0.983
②	1.061	1.205	1.012	1.197	0.994	0.904	0.886