Effect of a Magnetic Field on Thermal Conductivity of Partially Ionized Gases*

Hong Sik Yun

Department of Astronomy, College of Natural Sciences
Seoul National University

(Received May 30, 1976)

Abstract

The translational and reactive parts of thermal conductivity of a partially ionized solar magneto-plasma gas have been calculated based on Yun and Wyller's formulation (1972) along with Devoto's theory(1968). The computed results are presented as functions of temperature and pressure for given magnetic field strengths.

The results of the calculations show that for most photospheric conditions, the magnetic field does not play any important role in characterizing thermal properties of the ionized gas. However, when the gas pressure is low(e.g., $P<10$ dynes/cm^2), the field becomes extremely effective even if its strength is quite small (e.g., $B<0.1$ gauss). The reactive part of the thermal transport is found to be very important when the gas is undertaking active ionization.

1. Introduction

In the past the role of neutrals in the solar thermal conductivity has been largely overlooked. However, it is easy to deduce from Ulmscheider's work (1972) that their role can not be neglected even for most of the features associated with the quiet sun, sunspots or quiescent prominences. It is well known that the partially ionized solar-plasma is coupled to magnetic fields of various geometric configurations. This implies that the use of a ternary thermal conductivity is manditory to investigate their physical nature.

Recently, with the aid of OSO (Orbiting Solar Observatory) and OAO (Orbiting Astronomical Observatory) satellites, a number of observations from outside of the earth atmosphere have successfully been carried out in X-rays, extreme ultraviolet and ultraviolet regions, so that energy transport processes in the chromosphere and the corona become one of the major solar research problems. In order to tackle such problems accurate evaluations of the solar thermal conductivity are prerequisite.

Some attempts have been made to include magnetic fields in the calculation of thermal conductivity (e.g., Marshall (1957), Shkarofsky (1960) and Wyller (1963)). However, these attempts have been limited to a completely ionized gas. For a partially ionized gas, Devoto

* Work supported by the Ministry of Science and Technology, Repulic of Korea

(1967) and Ulmschneider (1970) considered a ternary system but in the absence of the magnetic field. A comprehensive generalization has been made by Yun and Wyller (1972), which allows to compute the conductivity tensor components in a partially ionized medium with a thermal gradient arbitrarily inclined to an external magnetic field.

The present work is to compute the translational thermal conductivity of the solar magneto-plasma as functions of temperature, pressure and magnetic field to investigate the magnetic effect on the translational thermal conductivity under various physical conditions.

2. Mathematical Formulation

The transport properties are described by the following equations

$$\vec{J}=\sigma\vec{E}+\alpha\nabla T \qquad (1)$$

$$\vec{q}=-\beta^*\vec{E}-K\nabla T \qquad (2)$$

where $\vec{J}$ is electric current density, $\vec{q}$ heat flow, $\vec{E}$ electric field and T temperature. The transport coefficients σ, α, β^*, K are computed from diffusion equations together with those of heat flow of each species in a gaseous mixture (c.f. Burgers (1969)).

According to Yun and Wyller (1972) these transport coefficients are given by

$$\alpha=\frac{N_e^2ekA}{2K_t}\ \frac{1}{(1-i\tilde{\eta})(\theta-i\tilde{\eta})-\dfrac{\Delta^2}{10}}$$

$$\beta^*=\frac{5N_e^2k^2T}{2K_t}\ \frac{(\theta-i\tilde{\eta})(1+\Omega_2)+\dfrac{\Delta}{5}(1+\Omega_1)}{(1-\tilde{\eta})(\theta-i\tilde{\eta})-\dfrac{\Delta^2}{10}}$$

$$\sigma=\frac{N_e^2e^2}{K_t}\ \frac{\theta-i\tilde{\eta}}{(1-i\tilde{\eta})(\theta-i\tilde{\eta})-\dfrac{\Delta^2}{10}} \qquad (3)$$

$$K=\frac{15N_e^2k^2T}{6\ K_t}\ \frac{(1-i\tilde{\eta})(1+\Omega_1)+\dfrac{\Delta}{2}(1+\Omega_2)}{(1-i\tilde{\eta})(\theta-i\tilde{\eta})-\dfrac{\Delta^2}{10}}$$

with

$$\Omega_1=\frac{\theta-i\tilde{\eta}}{\kappa+i\tilde{\eta}}(1+C_r)(1+K_r),$$

$$\Omega_2=-\frac{\Delta}{5(\kappa+i\tilde{\eta})}(1+C_r)(1+K_r),$$

$$K_r=\frac{27}{40}\ \frac{\varepsilon}{\mu}\left(\frac{1-\beta}{\beta}+\frac{27}{40}\ \frac{\varepsilon\kappa}{\kappa^2+\tilde{\eta}^2}\right),$$

$$C_r=C_\alpha+iC_\beta,$$

$$C_\alpha=\frac{1-\beta}{\beta}\ \frac{\kappa(1-\beta)+\dfrac{27}{40}\varepsilon\beta}{\mu\beta+\dfrac{27}{40}\varepsilon(1-\beta)},$$

$$C_\beta=C_\alpha\frac{(1-\beta)\tilde{\eta}}{\kappa(1-\beta)+\dfrac{27}{40}\varepsilon\beta}$$

and

$$\eta_o=\frac{\omega_e}{\gamma_{ie}},\quad \gamma_{ie}=3.63N_i\ln A/T^{3/2},\quad \omega_e=\frac{eB}{m_ec},$$

$$A=1.84\times10^4T^{3/2}/N_e^{1/2},$$

$$\tilde{\eta}=\tau\eta_o,\quad \tau=\frac{1}{1+G_2},\quad G_2=\frac{\gamma_{ne}}{\gamma_{ie}},$$

$$\gamma_{ne}=2.42\times10^{-9}N_nT^{1/2},$$

$$K_t=\frac{N_em_e\gamma_{ie}}{\tau},\quad \Delta=(3-G_2)\tau,\quad \Gamma=G_2\tau, \qquad (4)$$

$$\xi=1.865+1.3G_2,\quad \varepsilon=G_1\tau,\quad \theta=\xi\tau,$$

$$\kappa=\left(\frac{4}{5}\sqrt{\frac{m_i}{2m_e}}+\frac{59}{40}G_1\right)\tau,\quad \mu=\left(\frac{4}{5}G_3+\frac{59}{40}G_1\right)\tau,$$

$$G_1=\frac{G_2}{\sqrt{2}}\left(\frac{m_i}{m_e}\right)^{1/2}\frac{q_{in}}{q_{en}},\quad G_3=\frac{G_2}{\sqrt{2}}\left(\frac{m_n}{m_e}\right)^{1/2}\frac{1-\beta}{\beta}\frac{q_{nn}}{q_{en}}$$

where B denotes the magnetic field, N_i, N_e, N_n number densities of ions, electrons and neutrals, ω_e electron gyrofrequency, β degree of ionization, q_{in}, q_{en}, q_{nn} collisional cross-sections between various particles considered and m_e and m_i masses of electrons and ions.

The effective thermal conductivity

defined by Spitzer and Härm(1953),

$$\lambda = \mathrm{K} - \frac{\alpha\beta^*}{\sigma} \qquad (5)$$

can be rewritten as

$$\lambda = \lambda_{tr} - i\ \lambda_H, \qquad i = \sqrt{-1}$$

where λ_{tr} is the translational thermal conductivity (perpendicular to the magnetic field) and λ_H is Righi-Leduc conductivity (perpendicular both to the magnetic field and the temperature gradient).

The reactive thermal conductivity arising from the transport of internal and ionization energies is given by Devoto and Li (1968) as

$$\lambda_K = \frac{3}{8}\left(\frac{\pi}{km_i T^3}\right)^{1/2} \frac{\beta(1-\beta)}{\Omega_{in}} (\Delta h)^2$$

$$\Delta h = h_e + h_i - h_n \qquad (6)$$

where Ω_{in} is the averaged collisional cross-section between ions and neutrals, and h_e, h_i and h_n are the enthalpy per particle carried by electrons, ions and neutrals, respectively. For the solar composition Δh can be approximated by

$$\Delta h \simeq 22.3\ (1 + 0.1584 \times 10^{-4}\mathrm{T}). \qquad (7)$$

Substituting equation (7) into equation (6), the solar reactive thermal conductivity can be obtained from

$$\lambda_R = \frac{106\beta(1-\beta)(1+0.1584\times 10^{-4}\mathrm{T})^2}{\mathrm{T}\ \Omega_{in}} \qquad (8)$$

3. Results and Discussions

The results of the calculations are presented in Table 1, where the translation thermal conductivity is given as

Table 1. Computed Translational Thermal Conductivity

T(°K)	log P=−2		log P=0		log P=2		log P=4		log P=6	
	λ_{tr}	B	λ_{tr}	B	λ_{tr}	B	λ_{tr}	B	λ_{tr}	B
3,000	1.07(5)	0.0	1.07(5)	0.0	1.07(5)	0.0	1.07(5)	0.0	1.07(5)	0.0
	1.07(5)	1.00(4)	1.07(5)	1.00(4)	1.07(5)	1.00(5)	1.07(5)	1.00(4)	1.07(5)	1.00(4)
7,000	1.47(4)	0.0	1.91(4)	0.0	1.10(5)	0.0	1.93(5)	0.0	2.06(5)	0.0
	1.40(4)	2.00(−2)	1.86(4)	6.07(−1)	1.10(5)	2.80(0)	1.93(5)	0.0	2.06(5)	0.0
	1.21(4)	2.00(−1)	1.82(4)	1.0(0)	1.09(5)	1.12(1)	1.92(5)	1.72(2)		
	1.19(4)	2.02(0)	1.60(4)	1.01(1)	1.06(5)	1.40(2)	1.90(5)	2.21(3)		
	1.19(4)	2.01(1)	1.54(4)	1.01(2)	1.04(5)	1.40(3)				
	1.19(4)	1.01(4)	1.54(4)	1.00(4)	1.04(5)	1.00(4)				
13,000	1.21(4)	0.0	3.76(4)	0.0	1.28(5)	0.0	3.31(4)	0.0	9.23(4)	0.0
	9.84(3)	5.11(−3)	3.74(4)	8.39(−2)	1.28(5)	6.25(0)	3.26(4)	4.28(2)	9.18(4)	1.12(4)
	1.09(3)	5.11(−2(	3.53(4)	4.20(−1)	1.24(5)	2.50(1)	3.13(4)	1.28(3)		
	1.11(2)	5.11(−1)	2.46(4)	4.20(0)	1.11(5)	3.13(2)	3.06(4)	1.71(3)		
	5.38(1)	5.11(0)	2.30(4)	4.20(2)	1.10(5)	3.13(3)	2.96(4)	2.14(3)		
	5.32(1)	5.11(1)	2.30(4)	1.00(4)	1.10(5)	1.00(4)	2.96(4)	1.00(4)		
20,000	3.36(4)	0.0	3.98(4)	0.0	4.96(4)	0.0	6.73(4)	0.0	1.58(5)	0.0
	2.71(4)	1.84(−3)	3.64(4)	9.29(−2)	4.89(4)	2.49(0)	6.64(4)	1.87(2)	1.56(5)	7.80(3)
	2.88(3)	1.84(−2)	3.22(4)	1.55(−1)	4.29(4)	9.95(0)	5.88(4)	7.47(2)		
	1.59(2)	1.84(−1)	3.42(3)	1.55(0)	4.01(4)	1.24(1)	5.52(4)	9.33(2)		
	1.72(0)	1.84(0)	1.89(2)	1.55(1)	4.27(3)	1.24(2)	7.33(3)	9.33(3)		
	3.09(−2)	1.84(1)	2.31(0)	1.55(2)	2.50(2)	1.24(3)				

(continued)

T(°K)	log P=−2		log P=0		log P=2		log P=4		log P=6	
	λ_{tr}	B	λ_{tr}	B	λ_{tr}	B	λ_{tr}	B	λ_{tr}	B
50,000	3.29(5)	0.0	3.85(5)	0.0	4.65(5)	0.0	5.84(5)	0.0	3.69(6)	0.0
	2.67(5)	2.00(−4)	3.53(5)	1.02(−2)	4.45(5)	5.63(−1)	5.76(5)	2.24(1)	3.64(6)	3.55(2)
	2.83(4)	2.00(−4)	3.12(5)	1.70(−2)	4.02(5)	1.13(0)	5.06(1)	8.96(1)	3.38(6)	1.07(3)
	1.56(3)	2.00(−2)	3.31(4)	1.70(−1)	3.99(4)	1.41(1)	4.73(5)	1.12(2)	3.16(5)	1.78(4)
	1.70(1)	2.00(−1)	1.83(3)	1.70(0)	2.20(3)	1.41(2)	5.01(4)	1.12(3)		
	1.70(−1)	2.00(0)	1.96(1)	1.70(1)	2.38(1)	1.41(3)	2.77(3)	1.12(4)		
10^5	1.71(6)	0.0	1.98(6)	0.0	2.34(6)	0.0	2.86(6)	0.0	2.96(7)	0.0
	1.39(6)	3.82(−5)	1.71(6)	2.65(−3)	1.89(6)	2.80(−1)	2.74(6)	9.15(0)	2.84(7)	8.83(1)
	1.47(5)	3.82(−5)	1.70(5)	3.31(−2)	2.00(5)	2.80(0)	2.32(6)	2.29(1)	2.40(7)	2.21(2)
	8.12(3)	3.82(−3)	9.37(3)	3.31(−1)	1.11(4)	2.80(1)	2.46(5)	2.29(2)	2.54(6)	2.21(3)
	8.68(1)	3.82(−2)	1.00(2)	3.31(0)	1.19(2)	2.80(2)	1.36(4)	2.29(3)	1.41(5)	2.21(4)
2.5×10^5	1.53(7)	0.0	1.74(7)	0.0	2.02(7)	0.0	2.40(7)	0.0	1.47(8)	0.0
	1.40(7)	2.57(−6)	1.59(7)	2.26(−4)	1.93(7)	1.30(−2)	2.37(7)	5.45(−1)	1.41(8)	1.78(1)
	1.31(6)	4.28(−5)	1.49(6)	3.76(−3)	1.73(6)	3.24(−1)	2.08(7)	2.18(0)	1.26(7)	4.45(2)
	7.25(4)	4.28(−4)	8.25(4)	3.76(−2)	9.56(4)	3.24(0)	2.06(6)	2.72(1)	6.96(5)	4.45(3)
	7.75(2)	4.28(−3)	8.82(2)	3.76(−1)	1.02(3)	3.24(1)	1.14(5)	2.72(2)		
	7.75(0)	4.27(−2)	8.83(0)	3.76(0)	1.02(1)	3.24(2)	1.22(3)	2.72(3)		
5.0×10^5	8.07(7)	0.0	9.09(7)	0.0	1.04(8)	0.0	1.22(8)	0.0	3.77(8)	0.0
	6.93(6)	8.11(−6)	7.88(7)	5.76(−5)	9.02(7)	5.03(−3)	1.12(8)	3.22(−1)	3.27(8)	1.39(1)
	3.82(5)	8.11(−5)	7.80(6)	7.20(−4)	8.94(6)	6.28(−2)	1.05(7)	5.37(0)	3.24(7)	1.73(2)
	4.09(3)	8.11(−4)	4.31(5)	7.2(−3)	4.94(5)	6.28(−1)	5.78(5)	5.37(1)	1.79(6)	1.73(3)
	4.09(1)	8.11(−3)	4.61(3)	7.2(−2)	5.28(3)	6.28(0)	6.17(3)	5.37(2)	1.92(4)	1.73(4)
	4.09(−1)	8.11(−1)	4.61(1)	7.2(−1)	5.28(1)	6.28(1)	6.18(1)	5.37(3)		
7.5×10^5	2.14(8)	0.0	2.40(8)	0.0	2.73(8)	0.0	3.17(8)	0.0	7.39(8)	0.0
	1.84(7)	3.06(−6)	2.08(8)	2.18(−5)	2.50(8)	1.44(−3)	2.90(8)	1.24(−1)	6.78(8)	5.3(0)
	1.01(6)	3.06(−5)	2.06(7)	2.73(−4)	2.34(7)	2.40(−2)	2.72(7)	2.07(0)	5.98(8)	8.85(0)
	1.08(4)	3.06(−4)	1.14(6)	2.73(−3)	1.29(6)	2.40(−1)	1.51(6)	2.07(1)	6.35(7)	8.85(1)
	1.08(2)	3.06(−3)	1.22(4)	2.73(−2)	1.38(4)	2.40(0)	1.61(4)	2.07(2)	3.51(6)	8.85(2)
	1.08(0)	3.06(−1)	1.22(2)	2.73(−1)	1.38(2)	2.40(1)	1.61(2)	2.07(3)	3.75(4)	8.85(3)
10^6	4.27(8)	0.0	4.78(8)	0.0	5.42(8)	0.0	6.52(8)	0.0	6.38(9)	0.0
	3.67(7)	1.53(−6)	4.14(8)	1.10(−5)	4.39(8)	1.21(−3)	5.98(8)	4.18(−2)	6.11(9)	4.10(−1)
	2.03(6)	1.53(−5)	4.10(7)	1.37(−4)	4.65(7)	1.21(−2)	5.06(8)	1.05(−1)	5.17(9)	1.03(0)
	2.16(4)	1.53(−4)	2.26(6)	1.37(−3)	2.57(6)	1.21(−1)	5.37(7)	1.05(0)	5.48(8)	1.03(1)
	2.16(2)	1.53(−3)	2.42(4)	1.37(−2)	2.75(4)	1.21(0)	2.96(6)	1.05(1)	3.02(7)	1.03(2)
	2.16(0)	1.53(−1)	2.42(−1)	1.37(−1)	2.75(2)	1.21(1)	1.17(4)	1.05(2)	3.23(5)	1.03(3)

functions of temperature, pressure and magnetic field of the medium under various physical conditions of

$3000°K < T < 10^{6}°K$,

10^{-2} dyne/cm^2 $< P < 10^6$ dyne/cm^2,

10^{-5} gauss $< B < 10^4$ gauss.

The computed values of the reactive thermal conductivity are presented in Table 2, where they are expressed in terms of the ratio of the reactive to the non-magnetic translational conductivity. It is noted that when the particles are undergoing active ionizations, the

Table 2. Computed ratio of reactive to non-magnetic thermal conductivity under various physical conditions (β denotes the degree of ionization of the gas medium)

T(°K)	log P=−1		log P=3		log P=6	
	β (%)	L_R/L_{tr}	β (%)	L_R/L_{tr}	β (%)	L_R/L_{tr}
5,000	1.0	0.4	0.0	0.0	0.0	0.0
7,000	73.1	79.4	1.4	0.3	0.0	0.0
8,000	86.2	12.7	6.7	2.0	0.2	0.0
9,000	86.9	1.2	22.2	11.0	0.7	0.1
10.000	88.3	0.2	50.7	25.6	2.0	0.3
12,000	100	0.1	83.3	20.0	9.3	0.7
13,000	100	0.0	85.9	10.6	16.7	2.9
15.000	100	0.0	90.6	5.4	39.7	5.0
20,000	100	0.0	99.9	0.1	82.3	1.6
25,000	100	0.0	100	0.0	95.9	0.3

reactive part of the conductivity dominates over its corresponding translational part. When the degree of ionization reaches 40～80% the reactive part could increase up to about one order of magnitude in values of its corresponding non-magnetic conductivity. This suggests that the often neglected reactive thermal conductivity should be taken into account in the calculation of the total thermal conductivity.

Effect of the magnetic field on thermal condictivity depends on the physical conditions. The effect of the magnetic field on the translational thermal conductivity is described in Figure 1, where

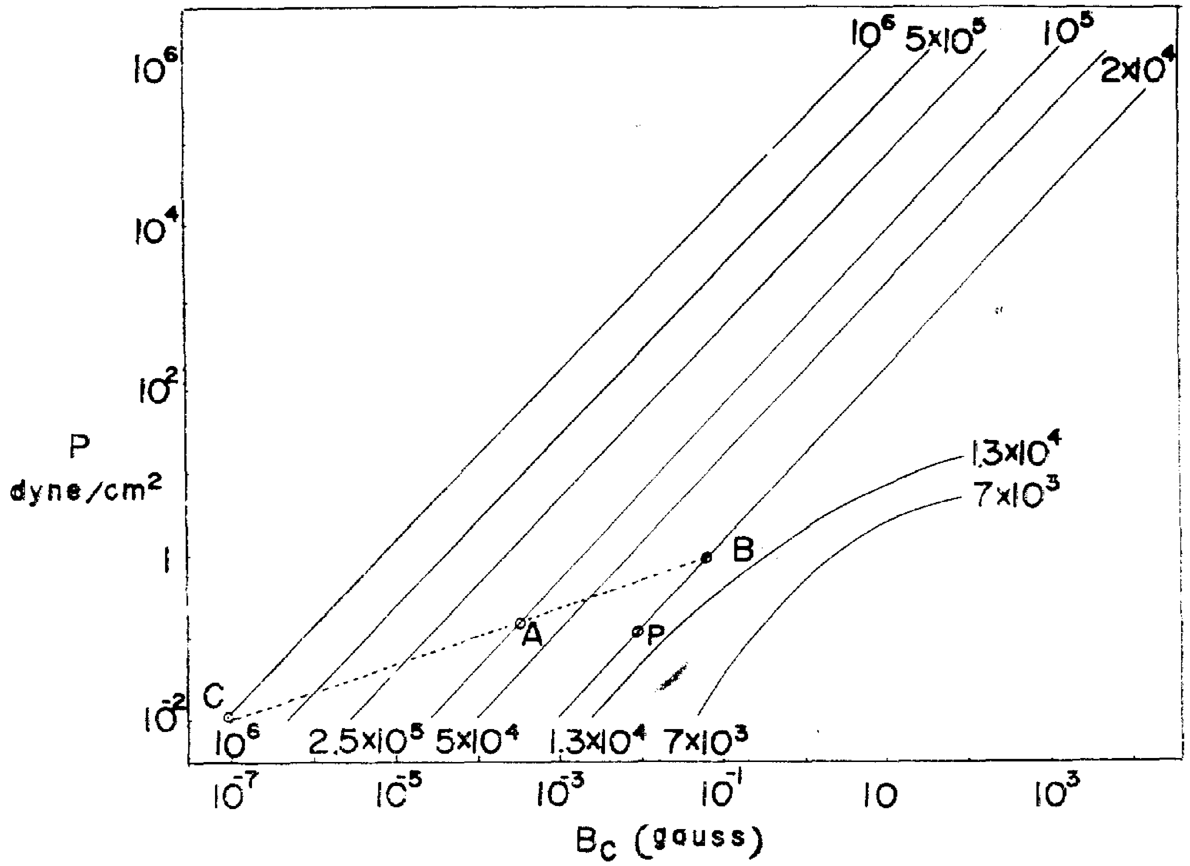

Fig. 1. Relationship between gas pressure and critical magnetic field strength for each given temperature. (Points A,B,C and P represent typical conditions for the chromosphere-corona transition region, the chromosphere, the corona and prominences)

B_c represents a critical magnetic field which is defined as a field strength capable of reducing the non-magnetic translatinal thermal conductivity by 10 %. The magnetic effect on the conductivity increases as the physical condition moves towards the left from each solid line in the figure.

An example attributed to a direct physical consequence of the magnetic effect on the thermal conductivity can be found in the long survival of prominences. It is known that the majority of the prominences extends well into the region of the corona where the temperature is of the order of 10^6 °K. In spite of the prominences being embedded into such a hot corona, they can survive for weeks without deterioration, maintaining their temperature of around 8000°K. It may appear difficult to understand their long survival, since they would be heated up to the coronal temperature within a few hours because of high coronal conductivity in the absence of the magnetic field (Rosseland, et al (1958)). The solution to this apparently puzzling phenomenon seems to lie in the proper recognition of the role played by the magnetic field in accordance in our findings that the thermal conductivity is reduced considerably in the presence of magnetic fields, thus cutting down their thermal conduction energy flowing into the prominences from their surrounding corona.

In summing up, it may be concluded that under most of photospheric conditions thermal properties are not seriously influenced by the magnetic field. However, when the gas pressure is low, the magnetic effect becomes very effective even if its field strength is very small (c.f., A,B,C and P in Figure 1).

References

Burgers, J : 1969, Flow Equations for Composite Gases, Academic Press.

Devoto, R.S. : 1967, Phys. Fluid **10**, 354

Devoto, R.S. and Li, C.P. : 1968, J. Plasma Phys. **2**, 17

Marshall, W. : 1957, AERE Reports T/R 2247, 2352, 2419, Harwell.

Oster, L. : 1968, Solar Phys. **3**, 543.

Rosseland, S, Jensen, E., and Tandberg-Hassen, E. : 1958, IAU Symp. **6**, 150.

Schwartz, C. : 1961, Phys. Rev. **124**, 1468.

Shkarofsky, I.P. : 1960, Can. J. Phys. **39**, 1619.

Spitzer, L. and Härm, R. : 1953, Phys. Rev. **89**, 977.

Ulmschneider, P. : 1970, Astron. Astrophys. **4**, 144.

Vanderslice, J.T., Weissman, S., Mason, E.A., and Fallon, R.J. : 1962, Phys. Fluids **5**, 155.

Vardya, M.S. : 1961, Astrophys. J. **133**, 107.

Wyller, A.A. : 1963, Astrophys. Norv. **8**, 53.

Yun, H.S. and Wyller, A.A. : 1972, Solar Phys. **27**, 44.