An Application of the Markov Process to the Management of Hospital Admissions

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1. Introduction

One of the principal uncertainties in hospital operations is the length of hospital stay for patients in bed and those scheduled to enter. This variable is an important factor in operating a scheduling system for elective patients and in planning man power and facility utilization.

Although the hospital schedules many patients for care and can control admissions to a considerable extent, there are sufficient emergency or unscheduled arrivals each day so that the admission process may be shown to be governed by chance factors. The underlying chance phenomena in admissions introduce large amounts of variability into a hospital system. The most important problems in the admission process, therefore, is to determine how many beds to set aside for emergency admissions and how many reservations to make with a reasonable degree of assurance that bed will be available when the patients arrive. From an administrative point of view, reservation procedures are necessary in order to draw deferable patients into the system in an orderly manner. In the first place, the author will introduce and summarize, for the build-up of main analysis, the important knowledge of determination of the optimum level of beds for emergency admissions that many renowned researchers have already done reasonably well. In the next place

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we will analyze the length of hospital stay for patients in terms of the absorbing state Markov chain process and try to relate its results to the scheduling system for elective patients.

2. The Optimum Level of Beds for Emergency Admissions

The first problem corresponds to the practical situation in which a physician, or an admitting officer, sets aside a percentage of beds for unscheduled admissions and accepts reservations for the remaining beds. Obviously a high level of beds saved for emergencies that never arrived will result in low average bed occupancy, assuming enough patients are kept available on a call list. On the other hand, a low level of beds set aside for emergencies will result in a high average occupancy level but it will increase the risk of an overflow admission of unscheduled arrivals.

We will introduce several suggested propositions instead of the further discussion of this problem.

(1) The number of admissions per day could be described by the Poisson distribution and the length of stay by the equally well known negative exponential distribution.

(2) The Poisson distribution could be used as an estimate of average occupancy and the addition of one or two standard deviations could establish an acceptable risk of bed shortage.

(3) The average daily number of patients occupying beds is a function of the admission rate and the average length of stay.

$$C=\lambda\cdot t$$

C: the average daily rate of bed accupancy

λ: the average number of daily admissions (the mean of the Poission distribution)

t: the average length of stay in days (the mean of the negative exponential distribution)

(4) Bed occupancy will rarely exceed or be less than the average daily rate of bed occupancy, plus or minus approximately four times the square root of it.

(5) The optimum level of beds to set aside for emergency admissions can be determined by computer simulation.

3. Absorbing State Markov Chain Analysis

A. Hypothetical Model

For the convenience of analysis, a process consists of ten states, two of which are absorbing, and the others non-absorbing.

Transition states

E: arrivals of patients

H: hospitalization of patients

(1): stay in hospital until the end of the first week

(2): stay in hospital until the end of the second week

(3): stay in hospital until the end of the third week

(4): stay in hospital until the end of the fourth week

(5): stay in hospital until the end of the fifth week

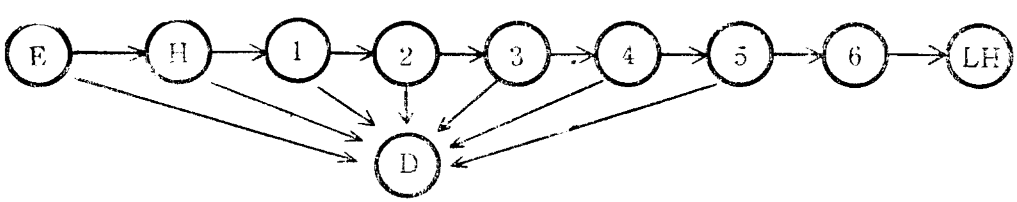
(6): stay in hospital until the end of the sixth week

Absorbing states

D: Discharges during every week of stay

LH: stay in hospital for over six weeks

Flow Diagram



Transition Probability Matrix in Canonical Form

	LH	D	E	H	1	2	3	4	5	6
LH	1	0	0	0	0	0	0	0	0	0
D	0	1	0	0	0	0	0	0	0	0
E	0	.9254	0	.0746	0	0	0	0	0	0
H	0	.3103	0	0	.6897	0	0	0	0	0
1	0	.5048	0	0	0	.4952	0	0	0	0
2	0	.4904	0	0	0	0	.5096	0	0	0
3	0	.5755	0	0	0	0	0	.4245	0	0
4	0	.5333	0	0	0	0	0	0	.4667	0
5	0	.0191	0	0	0	0	0	0	0	.3809
6	0	0	0	0	0	0	0	0	0	0

C. Mathematical Interpretation

The NC matrix provides very useful information for the analysis.

(1) On the average the patients admitted would stay in hospital for 2.32678 periods. (*i.e.*, 16.28746 days)

(2) On the average the patients with one week of stay would continue to be in hospital for 1.92370 more periods. (*i.e.*, 13.4659 days)

(3) On the average the patients with two weeks of stay would continue to be in hospital for 1.86553 more periods. (*i.e.*, 13.05871 days)

(4) On the average the patients with three weeks of stay would continue to be in hospital for 1.69087 more periods. (*i.e.*, 11.83609 days)

(5) On the average the patients with four weeks of stay would continue to be in hospital for 1.64447 periods. (*i.e.*, 11.51129 days)

(6) On the average the patients with five weeks of stay would continue to be in hospital for 1.38090 periods. (*i.e.*, 9.6663 days)

According to the NR matrix only 13 out of 1000 patients stay in hospital for more than 42 days. It refers to the fact that six weeks are sufficient as a planning horizon.

Length of Stay Unit: days

LOS	Frequency	Remarks			LOS	Frequency	Remarks		
1	27				30	2			
2	23				31	4			
3	20				32	5			
4	23				33	3			
5	30				34	4			
6	34				35	2	24	588	45
7	31	189		609	36	1			
8	39				37	0			
9	27				38	1			
10	30				39	4			
11	36				40	3			
12	23				41	3			
13	27				42	1	13	601	21
14	20	212	401	420	47	1			
15	18				50	2			
16	19				55	2			
17	12				59	1			
18	19				61	1			
19	13				63	1	8	609	8
20	15								
21	16	102	503	208					
22	10								
23	9								
24	13								
25	13								
26	5								
27	9								
28	2	61	504	106					
29	4								

* The above data come from the actual records of the hospital in the United States.

B. Mathematical Results

The Fundamental (N) Matrix

	E	H	1	2	3	4	5	6
E	1.0000	.7460	.05145	.02548	.01298	.00551	.00257	.00098
H	0	1.0000	.68970	.34154	.17405	.07383	.03448	.01313
1	0	0	1.00000	.49520	.25235	.10712	.04999	.01904
2	0	0	0	1.00000	.50960	.21633	.10096	.03846
3	0	0	0	0	1.00000	.42450	.19811	.07546
4	0	0	0	0	0	1.00000	.46670	.17777
5	0	0	0	0	0	0	1.00000	.38090
6	0	0	0	0	0	0	0	1.00000

	NC		NR	
			LH	D
E	1.17357	E	.00098	.99902
H	2.32678	H	.01313	.98687
1	1.92370	1	.01904	.98096
2	1.86553	2	.03846	.96154
3	1.69087	3	.07546	.92454
4	1.64447	4	.17777	.82223
5	1.38090	5	.38090	.61910
6	1.00000	6	1.00000	0

4. A Scheduling System for Elective Patients

From both the transition and NC matrices we can estimate roughly weekly discharge rates of patients admitted at a particular week for at least six weeks and thus establish six-week master plan for admissions of elective patients. The master plan is not complete for the actual scheduling system.

Nevertheless, NC matrix supplies the more detailed knowledge for an admitting officer to be capable of predicting the more accurate estimate of the next week discharges in advance.

A pilot study of length of stay carried out by the Berkely research team recommends a scheduling system for elective patients based on physicians' estimates. The main ideas of it can be explained as follows:

(1) The first estimate can be provided by the physician when he requests admission for a patient.

(2) The estimate can be revised after the patient is in hospital according to the appropriate schedule. The dates for revised estimates can be selected, after consultation with physicians, as being near the end of what is often a period of intensive treatment or testing during the patients' first few days of hospitalization. When the admitting officers plan the scheduling system for elective patients, they should consider physicians' etimates of the patient's length of stay as well as the statistical information given by the Markov chain analysis.

5. Conclusions

However, the mechanism for producing revised estimate is the principal means of varying the resulting precisions of estimate. Therefore, a scheduling system including physician's revision should be checked by a computer simulation to evaluate possible gains to admissions scheduling accruing from the use of these estimates.

The ability to accurately predict bed occupancy has long been an objective of hospital management. If the one were able to anticipate bed accupancy, then the one could more accurately plan for bed needs, schedule personnel, allocate service and supply.

We may conclude that even though the Markov chain analysis would not lead to ready-made answers for the scheduling system of elective patients, it can provide the more detailed and systematic knowledge for the solutions on the weekly base as well as the foundations for long run planning in relative sense.

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