NOTE

A Note on the Three-dimensional Correction Factor for the Virtual Inertia Coefficient of Ships in Vertical Vibrations

by

Keuck Chun Kim*

1.

Regarding to the calculation method for the added mass of the ship in vibrations, the traditional strip theory method which combines two-dimensional added mass with appropriate three-dimensional correction factors is still of great use for the ship vibration analysis, even though strong efforts are recently set toward a direct three-dimensional method, for instance, the work by P.N. Misra [1][1].

As to three-dimensional correction factors for vertical vibrations, adoption of deta derived from mathematical models such as ellipsoids-Lewis [2], Taylor [3] and Kruppa [4], and finitely-long elliptic cylinders-Kumai [5] prevails now. And then, it may be said that data derived from the former are reliable to fine ships and that data from the latter to full-form ships.

The researches on three-dimensional correction factors with finitely-long elliptic cylinders are much to the credit of T. Kumai, who has conducted a series of researches not only for vertical vibrations but for torsional vibrations [6].

Recently, for vertical vibrations of finitely-long elliptic cylinders C.Y. Kim [7] presented some results recalculated numerically in precision using Kumai's original problem formulation for the case of beam-draft ratios over 2.00. The numerical difference between them was of $O(10^{-3})$ and it seemed to count for little in practical application.

In case of beam-draft ratios less than 2.00, however, it is found that the difference between numerical results from the formula given in [5] and those recalculated in more precision by the author is of $O(10^{-2})$ for lower modes and of $O(10^{-1})$ for higher modes. Hence, considering that they are needed in case to deal correction factors as distributing ones in strip theory methods, in this note, the author supplements them with brief reviewals.

2.

According to [7], for beam-draft ratios over 2.00 the formula for three-dimensional correction factors of finitely-long elliptic cylinders, Jv, is as follows[2]:

$$J_V = \frac{-16}{\pi^2} \sum_{m}^{\infty} \sum_{r=1}^{\infty} \left(\frac{m}{m^2-s^2}\right)^2 (A_2^{(2r-1)})^2 \frac{Gek_{2r-1}(\xi_0, -q_m)}{Gek'_{2r-1}(\xi_0, -q_m)};$$

$$0 \leqq \eta \leqq \pi \quad \text{i.e. } B/T > 2.00 \qquad (1)$$

where

$$Gek'_{2r-1}(\xi_0, -q_m) = \left[\frac{d}{d\xi} Gek_{2r-1}(\xi, -q_m)\right]_{\xi=\xi_0}$$

$$q_m = \left(\frac{m\pi}{4\cosh\xi_0}\right)^2 \left(\frac{B}{L}\right)^2$$

$$m = \begin{cases} 2, 4, 6 \cdots\cdots \text{for } s=3, 5, 7 \cdots\cdots \text{nodes} \\ 1, 3, 5 \cdots\cdots \text{for } s=2, 4, 6 \cdots\cdots \text{nodes} \end{cases}$$

T, B and L; draft, breadth on waterline and length of the half-submerged elliptic cylinder.

In the case of beam-draft ratios less than 2.00, the formula for Jv can be obtained from the same problem formulation as [5] with a similar method as employed in [7] except the concern of the boundary condition at the free surface which should be replaced by

Received; May 21, 1975

* Member, College of Engineering, Seoul National University.

1) Numbers in [ ] indicate the References listed at the end of the paper.

2) Elliptic cylindrical co-ordinate system (ξ, η, z) which is related to cartesian co-ordinate system (x, y, z) by the transformation $x = h\cosh\xi\cos\eta$, $y = h\sin\xi\sin\eta$ and $z = z$ are employed. And all notations relating to Mathieu functions are after N. W. McLachlan's [8].

$$\phi=0, \quad at\ \eta=\frac{\pi}{2}\ \text{and}\ \frac{3\pi}{2} \tag{2}$$

Therefrom the formula turns out to be

$$J_V=\frac{-16}{\pi^2}\sum_m^\infty\sum_{r=1}^\infty\left(\frac{m}{m^2-s^2}\right)^2(B_1^{(2r-1)})^2\frac{Fek_{2r-1}(\xi_0,-q_m)}{Fek'_{2r-1}(\xi_0,-q_m)};$$

$$\frac{\pi}{2}\leq\eta\leq\frac{3\pi}{2}\ \text{i. e.}\ \frac{B}{T}<2.00 \tag{3}$$

where

$$q_m=\left(\frac{m\pi}{4\sinh\xi_0}\right)^2\left(\frac{B}{L}\right)^2$$

and other notations are same as Eq. (1).

Table 1. *Jv* derived from finitely-long elliptic cylinders.

B/T	L/B \ nodes	2	3	4	5	6	7
1.0	4.0	0.442	0.375	0.321	0.278	0.245	0.219
	5.0	0.500	0.433	0.376	0.330	0.293	0.263
	6.0	0.547	0.482	0.424	0.376	0.336	0.306
	7.0	0.587	0.524	0.466	0.416	0.375	0.341
	8.0	0.621	0.560	0.503	0.453	0.410	0.374
	9.0	0.650	0.592	0.536	0.486	0.442	0.405
	10.0	0.675	0.620	0.565	0.515	0.472	0.434
2.0	4.0	0.526	0.458	0.399	0.351	0.311	0.279
	5.0	0.585	0.520	0.460	0.409	0.367	0.332
	6.0	0.632	0.571	0.512	0.460	0.416	0.379
	7.0	0.670	0.612	0.556	0.504	0.460	0.421
	8.0	0.701	0.648	0.593	0.543	0.498	0.459
	9.0	0.728	0.678	0.626	0.577	0.352	0.493
	10.0	0.750	0.703	0.654	0.607	0.563	0.524
3.0	4.0	0.557	0.490	0.429	0.378	0.337	0.303
	5.0	0.616	0.553	0.493	0.441	0.396	0.359
	6.0	0.663	0.604	0.546	0.493	0.448	0.408
	7.0	0.698	0.644	0.589	0.537	0.492	0.452
	8.0	0.730	0.680	0.628	0.578	0.533	0.493
4.0	4.0	0.575	0.508	0.448	0.397	0.353	0.317
	5.0	0.634	0.571	0.510	0.457	0.411	0.373
	6.0	0.679	0.621	0.564	0.510	0.465	0.424
	7.0	0.715	0.663	0.608	0.557	0.510	0.470
	8.0	0.745	0.697	0.645	0.596	0.551	0.510

Note : For $\frac{L}{B}$ over 8 or 10 linear extrapolation may be admitted (refer to Fig. 1).

With the formula (3) numerical calculation was done for $\frac{B}{T}=1.00$ with the aid of the mathematic table [9][3] and the results are given in Table 1 together with Kumai's for $\frac{B}{T}=2.00$(circular cylinder) [5] and C.Y.Kim's for $\frac{B}{T}=3.0$ and 4.0 [7].

3.

As an example, in case of $\frac{L}{B}=8$ with $\frac{B}{T}=1.00$ and 3.00, *Jv* values obtained from Eqs. (3) and (1) and from the formulae given in [5] are resulted as follows;

No. of nodes		2	3	4	5	6	7
$\frac{B}{T}=1.00,\ \frac{L}{B}=8$							
	Eq. (3)	0.621	0.560	0.503	0.453	0.410	0.374
	[5]	0.543	0.467	0.394	0.332	0.280	0.237
$\frac{B}{T}=3.00,\ \frac{L}{B}=8$							
	Eq. (1)	0.730	0.680	0.628	0.578	0.533	0.493
	[5]	0.727	0.676	0.623	0.573	0.528	0.487

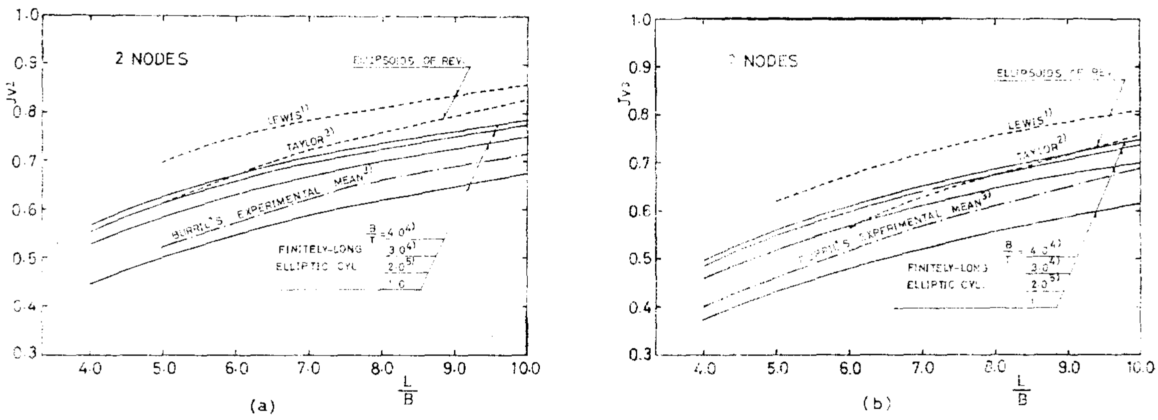
It seems that such numerical differences are mainly due to that an assumption $q_m < 1.0$ was made in [5] for simplification in obtaining the formulae for numerical calculations.

An another interest is to compare numerical results obtained from the formulae (1) and (3) with those from ellipsoids of revolution and with some experimental data. For this, Fig. 1 is prepared.

In Fig. 1. it is worthy of note that the experimental mean curves presented by Burril et al.[10] run nearly through the middle band of curves for $\frac{B}{T}=$ 1.00 and 2.00 of elliptic cylinders. The mean curves of Burril et al. are those obtained from their model experiments with prismatic bars of four different kinds of section contour; triangular section, ellipse-like section, midship-like section and circular section beam-draft ratios of which are 1.1548, 1.60, 1.60 and 2.00, respectively.

In concluding this article, the author would like to mention that, even though adoption of *Jv* values obtained from the mathematical model such as finitely-long elliptic cylinders are recommendable for full-from ships, more experimental studies with ship-like models are needed.

[3] In the table, values of $B_{2n-1}^{(2r-1)}(q_m)$ are given for $(2r-1)=1, 3, 5, \cdots\cdots 15$ and q_m from 0 to 25.



1) Data from [2], 2) Data from [3], 3) Data from [10]; section contours of models are triangle ($B/T=1.1548$), ellipse-like($B/T=1.60$), midship-like ($=1.60$) and circle ($B/T=2.00$), 4) Data from [7], 5) Data from [5].

Fig. 1. Comparision of Jv values from various sources.

References

1. P. N. Misra, "Added Mass in the Vibrational Analysis of Ships and Elastic Structures by Three-dimensional Approaches", *Report No. 74-71-S*, Det Norske Veritas, Jan. 1975.
2. F. M. Lewis, "The Inertia of the Water Surrounding a Vibrating Ship", *Trans. of SNAME*, Vol. 37, 1929.
3. J. Lockweed Taylor, "Some Hydrodynamic Inertia Coefficients", *Phil. Mag.*, Vol. 9, 1930.
4. C. Kruppa, "Beitrag zum Problem der hydrodynamischen Trägheitsgrössen bei elastischen Shiffsschwingungen", *Schiffstechnik*, Bd. 9, 1962.
5. T. Kumai, "On the Three-dimensional Correction Factor for the Virtual Inertia Coefficient in the Vertical Vibration of Ships I, *Jour. of SNA, Japan*, Vol. 112, 1962.
6. T. Kumai, "On the Three-dimensional Correction Factor for the Virtual Inertia Coefficient on the Torsional Vibration of Ships", *Jour. of SNA, Japan*, Vol. 108, 1960.
7. C.Y. Kim, "On the Three-dimensional Correction Factor for the Added Mass in the Vertical Vibration of the Ship", *Jour. of SNA, Korea*, Vol. 11, No. 2, 1974.
8. N. W. McLachlan, *Theory and Application of Mathieu Functions*, Dover Publication Inc., New York, 1964.
9. The National Applied Mathematics Lab., National Bureau of Standard (U. S. A.), *Tables relating to Mathieu Functions*, Columbia Univ. Press, New York, 1951.
10. L. C. Burril, W. Robson and R. L. Townsin, "Ship Vibration: Entrained Water Experiments", *Trans. of RINA*, Vol. 104, 1962.