韓國數學教育學會誌〈數學教育〉
1974. 6. Vol. XII No. 2

Journal of the Korea Society of Mathematical Education
June 1974. Vol. XII No. 2

ON THE STRUCTURE OF ω-BARRELLED SPACES

by

Dong hwa Kim

Korea University, Seoul, Korea

1. Introduction

Let E be a real or complex locally convex Hausdorff topological vector (abbreviated to locally convex) space and let E′ denote its dual.

A locally convex space E is said to be barrelled if every closed, balanced, convex, absorbing subset of E is a neighborhood of O or equivalently, if every $\sigma(E,E')$ bounded subset of E′ is equicontinuous.

A locally convex space E is said to be ω-barrelled if every countable $\sigma(E,E')$ −bounded subset of E′ is equicontinuous. In this paper, we study the structure of ω-barrelled space.

2. Main Theorem

Proposition 1. Every barrelled locally convex space is ω-barrelled.

Proof: It is immediate from the definition since every $\sigma(E,E')$ −bounded set in a barrelled space E is equicontinuous.

Corollary: Every Fréchet space (in particular Banach space) is ω-barrelled.

Proposition 2. ω-barrelled topology is stronger than the weak topology $\sigma(E,E')$.

Proof: Since the weak topology $\sigma(E,E')$ is the S-topology, where S is all subsets of E′ consisting of finite elements. The neighborhood of O in E for the weak topology is a neighborhood of O in ω-barrelled space E.

Theorem: A locally convex space E is ω-barrelled if and only if each barrel B which is the countable intersection of convex, circled and closed neighborhood of O in $(E,\sigma(E,E'))$ is itself a neighborhood of O in E.

Proof: Suppose that E is ω-barrelled. Let $B=\bigcap_{n=1}^{\infty}U_n$ be a barrel such that each U_n is a convex, circled, and closed neighborhood of O in E for the topology $\sigma(E,E')$. We can assume that $U_n=A_n^{\circ}$, where A_n is finite subset of E′.

Then

$$B^{\circ}=\left(\bigcap_{n=1}^{\infty}U_n^{\circ\circ}\right)^{\circ}=\left(\bigcup_{n=1}^{\infty}U_n^{\circ}\right)^{\circ\circ}\supset\bigcup_{n=1}^{\infty}U_n^{\circ}=\bigcup_{n=1}^{\infty}A_n^{\circ\circ}\supset\bigcup_{n=1}^{\infty}A_n$$

Since B is a barrel and hence absorbing, B° is $\sigma(E,E')$—bounded and so is $\bigcup_{n=1}^{\infty} A_n$.

Since E is ω-barrelled, it follows that $\bigcup_{n=1}^{\infty} A_n$ is equicontinuous.

Therefore,

$$\left(\bigcup_{n=1}^{\infty} A_n\right)^\circ = \bigcap_{n=1}^{\infty} A_n^\circ = \bigcap_{n=1}^{\infty} U_n = B$$

is a neighborhood of O in E.

For the converse, suppose the condition is satisfied. Let A be a countable $\sigma(E,E')$—bounded subset of E′.

If

$$A = \bigcup_{n=1}^{\infty} \{f_n\},$$

then it follows

$$B = A^\circ = \left(\bigcup_{n=1}^{\infty} \{f_n\}\right)^\circ = \bigcap_{n=1}^{\infty} \{f_n\}^\circ$$

is a barrel in E. And $\{f_n\}^\circ$ is a neighborhood of O. Hence B, being a barrel which is the countable intersection of convex, circled and closed neighborhood of O in E′ for the topology $\sigma(E,E')$.

$B^\circ = A^{\circ\circ}$ is equicontinuous.

But then $A \subset A^{\circ\circ}$ implies that A is equicontinuous. This proves that E is ω-barrelled.

References

1. Taylor, A.E; Introduction to Functional Analysis, Wiley, New York (1958)
2. John Horváth; Topological Vector Spaces and Distributions Vol.1. Addison-Wesley Pub. Co. (1966)
3. Husain, T; Two new classes of locally convex spaces. Math. Ann. 166, (1966)
4. S.Saxon and M.Levin; A note on the inheritance of properties of locally convex spaces by subspaces of countable codimension, Proc. Amer. Math. Soc. 29 (1971)

P.25에서 계속

留意해야 한다.

前述한바 바람직한 人間의 形成이라는 立場에서 算數科의 位置와 그 重要性을 共感하고 熱意를 다하여 教育目標達成에 努力함으로써 算數教育의 成果를 期待해야 할 것이다.

参 考 文 獻

○ 文教部 國民學校 教育課程

○ 具光祖, 李正實 算數教育 古今文化社

○ 具光祖外 初等數學教育의 現代化에 따른 教育內容의 改善 및 教師教育에 關한 研究 本誌 通卷 24號

○ 學校教育全書 數學教育 日本全國教育圖書株式會社

○ 川口廷外 算數教育現代化全書 日本 金子書房

(서울 教育大學)