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A NOTE ON A PAPER OF F. BAGEMIHL

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The object of this note is to extend a theorem of F. Bagemihl [1]. Let $C_r(f, p)$ denote the radial cluster set of a function $f(z)$ at the point p on the unit circle. Bagemihl has shown the following

THEOREM. *Let $f(z)$ and $g(z)$ be holomorphic functions in the open unit disc D. Suppose that A is a subarc of the unit circle, K is a subset of A of second category, and M is a metrically dense subset of A. If*

$$\infty \notin C_r(f, p) \cup C_r(g, p) \quad (p \in K)$$

and if

$$\infty \notin C_r(f, p) \supset C_r(g, p) \quad (p \in M),$$

then $f(z) \equiv g(z)$.

Bagemihl has shown it under a strong assumption of boundedness of the radial cluster sets. In fact he has presented two nonidentical holomorphic functions $f(z)$ and $g(z)$ in D such that for every point p on the unit circle we have $C_r(f, p) = C_r(g, p) = S$, where S is the Riemann sphere.

We present a simple extension of Bagemihl's result.

THEOREM. *Let $f(z)$ and $g(z)$ be meromorphic functions in the open unit disc D. Suppose that A is a subarc of the unit circle, K is a subset of A of second category, and M is a metrically dense subset of A, k is a complex number (possibly ∞). If*

$$k \notin C_r(f, p) \cup C_r(g, p) \quad (p \in K)$$

and if

$$k \notin C_r(f, p) \supset C_r(g, p) \quad (p \in M),$$

then $f(z) \equiv g(z)$.

Proof. Since $C_r(f, p)$ and $C_r(g, p)$ are closed subsets of the Riemann sphere S, we have

$$d(C_r(f, p), k) = d_1 > 0, \text{ and } d(C_r(g, p), k) = d_2 > 0,$$

where d denotes the chordal metric on the Riemann sphere.

Choose a point a ($\neq \infty$) so that

$$d(a, k) < min(d_1, d_2).$$

Define

$$F(z) = 1/(f(z) - a) \quad (z \in D),$$

and

$$G(z) = 1/(g(z) - a) \quad (z \in D).$$

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Then $F(z)$ and $G(z)$ are holomorphic functions in D. It follows readily from Bagemihl's theorem that $F(z)\equiv G(z)$, whence $f(z)\equiv g(z)$.

Reference

[1] F. Bagemihl, *A radial cluster set uniqueness theorem for holomorphic functions.* Monat. Math. **75**(1971), 289–290.

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