On The Added Mass and Damping of Chine Sections in Heaving Oscillation

—Comparisons with Equivalent Lewis Section—

by

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背骨型斷面柱狀體의 上下動搖에 있어서의 附加質量과 減衰力에 關하여

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요 약

Chine형 선체단면 주상체의 자유표면에서의 상하운동에 수반되는 부가질량과 감쇠력을 Ursell-Tasai 방법에 의하여 계산하였다.

Chine형에서 구하여진 결과는 같은 선체단면적계수와 반폭흘수비를 갖는 등가 Lewis형의 결과와 비교하였고 선체단면의 형상이 부가질량 및 감쇠력에 미치는 영향을 고찰하였다.

Chine형과 등가 Lewis형 선체단면 주상체에 파고가 일정한 횡파가 입사할 때의 상하운동을 고찰하였다. 이상과 같은 계산 및 고찰을 통하여 다음 결과를 얻었다.

1) 자유표면이 부가질량에 미치는 영향은 chine형이 등가 Lewis 형단면보다 큰 값을 갖는다.
2) 선체단면적계수와 반폭흘수비가 같은 경우에 감쇠력의 크기는 Lewis form, single chine, double chine 순서이다.
3) 선체단면계수와 반폭흘수 비는 상하운동에 가장 큰 영향을 미치는 것으로 생각되며, 선체단면의 기하학적인 변형이 운동에 미치는 영향은 중요한 요소가 되질 못한다.
4) 감쇠력계수와 선체단면계수와의 관계는 간단한 관계를 유지한다.

Nomenclature

A: amplitude ratio

B: beam

C_0: added mass coefficient $(\omega \to \infty)$

$C_1(\xi)$: added mass coefficient$=\dfrac{\text{added mass}}{1/2\,\rho\pi(B/2)^2}=K_4\cdot C_0$

E_H: exciting force due to incoming wave

F: hydrodynamic force

g: gravitational acceleration

h: amplitude of forced heaving

H_0: half beam to draft ratio

H_H: heave response

K: wave number

K_4: free surface coefficient of added mass

M: scale factor

m_H: added mass due to heave oscillation

N: damping force

p: hydrodynamic pressure

S: sectional area of cylinder

T: draft

T_e: kinetic energy

V: heaving velocity

η: amplitude of progressive wave

$\mu(\xi)$: damping coefficient$=\dfrac{\text{damping}}{\omega\cdot\frac{1}{2}\rho g\pi(B/2)^2}$

ξ_0: $K\cdot\dfrac{B}{2}=\dfrac{\omega^2}{g}\cdot\dfrac{B}{2}$

ρ: mass density

σ: sectional area coefficient

ϕ: velocity potential

Ψ: stream function

ω: circular frequency

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Abstract

The added mass and the damping for two-dimensional cylinder of a curvilinear section with chines, either single or double, in heave oscillations on the free surface are calculated by the Ursell-Tasai method [1,2]. The results are compared with that of the equivalent Lewis section which has the same sectional area coefficient and the half beam to draft ratio, and the effects of the shape of the sections on the added mass and damping are investigated.

The heave response of a cylinder of a chine section and that of the equivalent Lewis section in regular beam sea are calculated, and the differences of heave motions between those cylinders are investigated.

1. Introduction

The added mass and the damping of a cylinder which has a chine section may be important with respect to motions of fishing boats and high speed boats. Prof. C.W. Prohaska[3] and Prof. K.C. Kim[4] have studied the added masses of those ship sections using the Bieberbach transformation, when the wave number is infinite. In this paper, the free surface effects of these ship sections in heave oscillation are investigated by using the Bieberbach transformation

$$\frac{Z}{M}=\zeta+\sum_{n=1}^{\infty}(a_{2n-1}\cdot\zeta^{-(2n-1)}) \qquad (1)$$

and the Ursell-Tasai method[1,2].

The wave excitation forces in regular beam sea are also calculated by using the Haskind-Newman relation [5]. The heave responses in regular beam sea are then calculated.

The results are compared with those of the equivalent Lewis section which has the same sectional area coefficient and the half beam to draft ratio, and the effects of the change of the section on heave motion are investigated.

It seems to the author that hydrodynamic forces with respect to heave are mainly characterized by the sectional area coefficient and the half beam to draft ratio, and the other geometrical characteristics have secondary effects on the heave oscillation.

2. Formulation of the Problem

2.1 Definitions and Fundamental Equations

Let an infinite long cylinder be on the free surface of the water, and oscillate vertically about initial position. Coordinate systems are shown in Fig. 1.

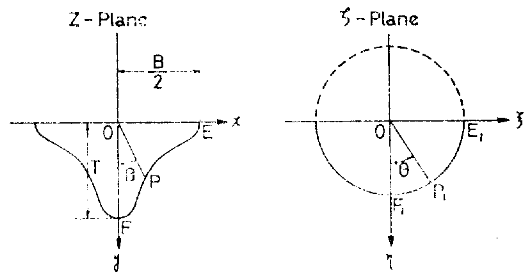

Fig. 1. Coordinate Systems; Mapping of Contour in to a Circle

Effects of viscosity and surface tension may be neglected. If the motion is started from rest, the fluid motion is irrotational and a velocity potential $\phi(x,y,t)$ exists which satisfy the Laplace equation:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0 \qquad \text{in the water.} \qquad (2)$$

When the amplitude of the oscillation is small, the linear assumptions may be justified. Neglecting the second order terms, the free surface condition is expressed as

$$\frac{\partial^2\phi}{\partial t^2}-g\frac{\partial\phi}{\partial y}=0 \quad \text{at } |x|>\frac{B}{2}, \quad y=0, \qquad (3)$$

where g is the gravitational acceleration.

The heave oscillation is assumed to be stationary one, and is given by

$$y_h=h\cos(\omega t+\delta) \qquad (4)$$

where y_h, h, ω and δ are the displacement, the amplitude the circular frequency and the phase of the forced heaving, respectively.

The continuity condition on the surface of the cylinder then, becomes as

$$\frac{\partial\phi}{\partial n}=-h\omega\sin(\omega t+\delta)\frac{\partial y}{\partial n}=V\frac{\partial y}{\partial n}\quad \text{on } C \qquad (5)$$

where C refers to the mean position of the cylinder surface. Let the depth of the water be infinite. The following condition must then be satisfied

$$\frac{\partial\phi}{\partial y}\longrightarrow 0 \qquad \text{when } y\longrightarrow\infty \qquad (6)$$

Let the cosine and sine parts of the velocity potential be ϕ_c and ϕ_s, that is

$$\phi=\phi_c(x,y)\cos\omega t+\phi_s(x,y)\sin\omega t \qquad (7)$$

Substituting (7) into, (2), (3), (5) and (6), the following eqs. are obtained,

$$\frac{\partial^2\phi_c}{\partial x^2}+\frac{\partial^2\phi_c}{\partial y^2}=0,\quad \frac{\partial^2\phi_s}{\partial x^2}+\frac{\partial^2\phi_s}{\partial y^2}=0 \quad \text{in the water} \qquad (8)$$

$$K\phi_c+\frac{\partial\phi_c}{\partial y}=0,\quad K\phi_s+\frac{\partial\phi_s}{\partial y}=0 \quad \text{at } |x|>\frac{B}{2},\ y=0 \qquad (9)$$

$$\frac{\partial\phi_c}{\partial n}\cos\omega t+\frac{\partial\phi_s}{\partial n}\sin\omega t=V\frac{\partial y}{\partial n}\quad \text{on } C \qquad (10)$$

$$\frac{\partial\phi_c}{\partial y}\longrightarrow 0,\quad \frac{\partial\phi_s}{\partial y}\longrightarrow 0 \quad \text{when } y\longrightarrow\infty \qquad (11)$$

In addition to those conditions, the velocity potential ϕ_c and ϕ_s must satisfy the appropriate radiation condition of the outgoing progressive waves at $|x|=\infty$ [1].

Let $\Psi_c(x,y)$ and $\Psi_s(x,y)$ be the cosine and the sine parts of the stream function $\Psi(x,y,t)$ corresponding to the velocity potential $\phi(x,y,t)$, that is

$$\Psi(x,y,t)=\Psi_c(x,y,t)\cos\omega t+\Psi_s(x,y,t)\sin\omega t \qquad (12)$$

The relations between ϕ_c and ϕ_s, and Ψ_c and Ψ_s are given as

$$\frac{\partial\phi_c}{\partial x}=\frac{\partial\Psi_c}{\partial y},\quad \frac{\partial\phi_s}{\partial x}=\frac{\partial\Psi_s}{\partial y},\quad \frac{\partial\phi_c}{\partial y}=-\frac{\partial\Psi_c}{\partial x},\quad \frac{\partial\phi_s}{\partial y}=-\frac{\partial\Psi_s}{\partial x} \quad \text{in the water.} \qquad (13)$$

From (10) and (13), the boundary values of Ψ_c and Ψ_s are prescribed as follow:

$$\Psi_c\cos\omega t+\Psi_s\sin\omega t=-Vx \qquad \text{on } C. \qquad (14)$$

2.2 Mathematical Representation of the Section Contour [4]

As a special case of the Bieberbach transformation (1), the following two-parameter family is considered:

$$\frac{Z}{M}=\zeta+a_{2m-1}\zeta^{-(2m-1)}+a_{2n-1}\zeta^{-(2n-1)} \qquad (15)$$

where $z=x+iy$ and $\zeta=ie^{\alpha}e^{-i\theta}$ refer to the physical plane and the mapped plane, respectively (see Fig.1). If the appropriate conditions are satisfied, the region occupied by the water and its image to x-axis in z-plane can be transformed onto the outer region of the unit circle in ζ-plane, and the x-and y-axes in z-plane corresponds to $\theta=\pm\frac{\pi}{2}$, $\theta=0$ in ζ-plane, respectively. The following conditions may, then, follow:

n and m=positive integers $(m<n)$
M=positive number (16)
a_{2m-1} and a_{2n-1}=real numbers.

Furthermore, to obtain a useful section contour, the following conditions must be satisfied, that is

$$\begin{aligned}&\mathrm{Re}\left[z\left(\frac{\pi}{2}\right)\right]\geq\mathrm{Re}\,[z(\theta)]\geq 0\\ &\mathrm{Im}\,[z(0)]\geq\mathrm{Im}[z(\theta)]\geq 0\end{aligned}\qquad 0\leq\theta\leq\frac{\pi}{2} \qquad (17)$$

From the transformation (15), the section contour C is given in the term of θ as

$$\begin{aligned}\frac{x}{M}&=\sin\theta+(-1)^{m-1}a_{2m-1}\sin(2m-1)\theta+(-1)^{n-1}a_{2n-1}\sin(2n-1)\theta\\ \frac{y}{M}&=\cos\theta+(-1)^{m}a_{2m-1}\cos(2m-1)\theta+(-1)^{n}a_{2n-1}\cos(2n-1)\theta\end{aligned} \qquad (18)$$

Let B, T, $2S$, H_0 and σ be the beam, the draft, the sectional area, the half beam to draft ratio and the sectional area coefficient of the cylinder section. These can be written in the terms of M, a_{2m-1} and a_{2n-1} as follows:

$$\frac{B}{2}=M(1+a_{2m-1}+a_{2n-1}) \qquad (19-1)$$

$$T=M[1+(-1)^m a_{2m-1}+(-1)^n a_{2n-1}] \qquad (19-2)$$

$$S=\frac{1}{2}\oint xdy=\frac{\pi}{2}\left(\frac{B}{2}\right)^2\frac{1-(2m-1)a^2_{2m-1}-(2n-1)a^2_{2n-1}}{(1+a_{2m-1}+a_{2n-1})^2} \qquad (19-3)$$

$$Ho=\frac{B/2}{T}=\frac{1+a_{2m-1}+a_{2n-1}}{1+(-1)^m a_{2m-1}+(-1)^n a_{2n-1}} \qquad (19-4)$$

$$\sigma=\frac{S}{BT}=\frac{\pi}{4}[1-(2m-1)a^2_{2m-1}-(2n-1)a^2_{2n-1}]/\{1+(-1)^m a_{2m-1}+(-1)^n a_{2n-1}\}[1+a_{2m+1}+a_{2n-1}] \qquad (19-5)$$

In general, a_{2m-1} and a_{2n-1} are determined from (19-4) and (19-5).

In this paper, (a_1, a_7) and (a_1, a_{11}) families are considered. (a_1, a_7) and (a_1, a_{11}) families corresponds to the single and double chine sections, respectively. The conditions of the mapping(17) are given in its explicit form as

$$0\le a_{2n-1}\le\frac{1}{2n-1}\qquad a_1=0$$
$$0\le a_{2n-1}<\frac{1-|a_1|}{2n-1}\qquad a_1\neq 0\qquad \text{for } n=4,6. \tag{20}$$

From this condition, σ and H_0 can not be taken arbitrary. For each H_0, σ must satisfy the inequality:

$$\sigma_{\min}(H_0)\le\sigma\le\sigma_{\max}(H_0) \tag{21}$$

$\sigma_{\min}(H_0)$ and $\sigma_{\max}(H_0)$ are given as[4]

$$\sigma_{\min}(H_0)=\frac{\pi}{4}H_0\left[1-\left(\frac{H_0-1}{H_0+1}\right)^2\{1+(a_{2n-1})^2{}_{\max}\}-(2n-1)(a_{2n-1})^2{}_{\max}\right]\Big/\left[1+\left(\frac{H_0-1}{H_0+1}\right)\{1+(a_{2n-1})_{\max}\}+(a_{2n-1})_{\max}\right]^2\qquad\text{for } n=4,6, \tag{22}$$

$$\sigma_{\max}(H_0)=\frac{\pi}{4}\qquad \text{for } n=4,6, \tag{23}$$

where $(a_{2n-1})_{\max}$ means to take the maximum of a_{2n-1} determined by the inequality (20). In this paper, H_0 is taken to be unity, and it, then, follows:

$$\sigma_{\min}(1)=\begin{cases}0.5154 & \text{for } n=4,\\ 0.6000 & \text{for } n=6.\end{cases} \tag{24}$$

3. Numerical Procedures

By the transformation (18), the free surface $y=0$ in z-plane corresponds to $\theta=\pm\frac{\pi}{2}$ in ζ-plane, and the free surface condition becomes as

$$\xi_0\left\{\frac{1-(2m-1)a_{2m-1}-(2n-1)a_{2n-1}}{1+a_{2m-1}+a_{2n-1}}\right\}\phi\mp\frac{\partial\phi}{\partial\theta}=0\qquad \text{at } \theta=\pm\frac{\pi}{2} \tag{25}$$

where $\quad\xi_0=K\,\frac{B}{2}$ (26)

The singular velocity potentials which satisfy the Laplace Eq. and the free surface condition are obtained as

$$\phi_{2l}=\Big[e^{-2l\alpha}\cos 2l\theta+\frac{\xi_0}{1+a_{2m-1}+a_{2n-1}}\times\left\{\frac{e^{-(2l-1)\alpha}}{2l-1}\cos(2l-1)\theta+\frac{(-1)^{m-1}(2m-1)a_{2m-1}}{2l+2m-1}\times e^{-(2l+2m-1)\alpha}\cos(2l+2m-1)\theta+\frac{(-1)^{n-1}(2n-1)a_{2n-1}}{2l+2m-1}\times e^{-(2l+2n-1)\alpha}\cos(2l+2n-1)\theta\right\}\Big]\begin{matrix}\cos\omega t\\ \sin\omega t\end{matrix} \tag{27}$$

$l=1,\ 2,\ 3,\ \cdots\cdots$

The stream function ψ_{2l} corresponding to the velocity potential ϕ_{2l} are given as

$$\psi_{2l}=\Big[e^{-2l\alpha}\sin 2l\theta+\frac{\xi_0}{1+a_{2m-1}+a_{2n-1}}\times\left\{\frac{e^{-(2l-1)\alpha}}{2l-1}\sin(2l-1)\theta+\frac{(-1)^{m-1}(2m-1)a_{2m-1}}{2l+2m-1}\times e^{-(2l+2m-1)\alpha}\sin(2l+2m-1)\theta+\frac{(-1)^{n-1}(2n-1)a_{2n-1}}{2l+2n-1}\times e^{-(2l+2n-1)\alpha}\sin(2l+2n-1)\theta\right\}\Big]\begin{matrix}\cos\omega t\\ \sin\omega t\end{matrix}$$

$$l=1,\ 2,\ 3,\ \cdots\cdots \tag{28}$$

To satisfy the radiation condition of the outgoing progressive wave, the appropriate wave making singularity must be added to the sets of the velocity potentials ϕ_{2l}. According to the Ursell-Tasai method [1,2], a source singularity at the origin of z-plane is taken as the wave making singularity. The stream function $\psi(x,y,t)$ is, then, expressed as

$$\psi(x,y,t)=\frac{g\eta}{\pi\omega}\Big[\Psi_c(K,\ x(\alpha,\theta),\ y(\alpha,\theta))\cos\omega t+\Psi_s(K,x(\alpha,\theta),\ y(\alpha,\theta))\sin\omega t+\cos\omega t\sum_{l=1}^{\infty}p_{2l}(\xi_0)\psi_{2l}(\xi_0,\alpha,\theta)+\sin\omega t\sum_{l=1}^{\infty}q_{2l}(\xi_0)\psi_{2l}(\xi_0,\alpha,\theta)\Big] \tag{29}$$

$\Psi_c(K,x,y)$ and $\Psi_s(K,x,y)$ are given as

$$\Psi_c(K,x,y)=\pi e^{-ky}\sin Kx$$
$$\Psi_s(K,x,y)=\int_0^\infty\frac{e^{-Kx}}{K^2+k^2}\{k\sin ky+K\cos ky\}\,dk-\pi e^{-Ky}\cos Kx \tag{30}$$

where η is the amplitude of the diverging wave, and $p_{2l}(\xi_0)$ and $q_{2l}(\xi_0)$ are the integral constants. Substituting (29) into (14), the boundary conditions on C is obtained as

$$\frac{\pi\omega}{g\eta}[-V(\xi_0,t)x(0,\theta)]=\Psi_c(K,x(0,\theta),y(0,\theta))\cos\omega t+\Psi_s(K,x(0,\theta),y(0,\theta))\sin\omega t+\cos\omega t\sum_{l=1}^{\infty}p_{2l}(\xi_0)\psi_{2l}(\xi_0,\ 0,\ \theta)+\sin\omega t\sum_{l=1}^{\infty}q_{2l}(\xi_0)\psi_{2l}(\xi_0,0,\theta) \tag{31}$$

$$\text{at } 0\le\frac{\pi}{2}$$

Put $\theta=\frac{\pi}{2}$, then

$$\frac{\pi\omega}{g\eta}\left[-V(\xi_0,t)x\left(0,\ \frac{\pi}{2}\right)\right]$$
$$=\Psi_c\left(K,x\left(0,\frac{\pi}{2}\right),\ y\left(0,\frac{\pi}{2}\right)\right)\cos\omega t$$
$$+\Psi_s\left(K,x\left(0,\frac{\pi}{2}\right),\ y\left(0,\frac{\pi}{2}\right)\right)\sin\omega t$$
$$+\cos\omega t\sum_{l=1}^{\infty}p_{2l}(\xi_0)\psi_{2l}\left(\xi_0,\ 0,\ \frac{\pi}{2}\right)$$
$$+\sin\omega t\sum_{l=1}^{\infty}q_{2l}(\xi_0)\psi_{2l}\left(\xi_0,\ 0,\frac{\pi}{2}\right). \qquad (32)$$

Eliminating $V(\xi_0,t)$ from (31) and (32), the following equations are obtained:

$$\Psi_{co}(\xi_0,\ a_{2m-1},\ a_{2n-1},\ \theta)$$
$$-[\sin\theta-(-1)^m a_{2m-1}\sin(2m-1)\theta$$
$$-(-1)^n a_{2n-1}\sin(2n-1)\theta]/[1+a_{2m-1}+a_{2n-1}]$$
$$\times\Psi_{co}\left(\xi_0,a_{2m-1},a_{2n-1},\ \frac{\pi}{2}\right)$$
$$=\sum_{l=1}^{\infty}p_{2l}(\xi_0)f_{2l}(\xi_0,a_{2m-1},a_{2n-1},\theta) \qquad (33\text{-}1)$$

$$\Psi_{so}(\xi_0,\ a_{2m-1},\ a_{2n-1},\theta)$$
$$-[\sin\theta-(-1)^m a_{2m-1}\sin(2m-1)\theta$$
$$-(-1)^n a_{2n-1}\sin(2n-1)\theta]/[1+a_{2m-1}+a_{2n-1}]$$
$$\times\Psi_{so}\left(\xi_0,a_{2m-1},\ a_{2n-1},\ \frac{\pi}{2}\right)$$
$$=\sum_{l=1}^{\infty}q_{2l}(\xi_0)f_{2l}(\xi_0,a_{2m-1},a_{2n-1},\theta) \qquad (33\text{-}2)$$

where

$$\Psi_{co}(\xi_0,a_{2m-1},a_{2n-1},\theta)=\Psi_c(K,x(0,\theta),y(0,\theta))$$
$$\Psi_{so}(\xi_0,a_{2m-1},a_{2n-1},\theta)=\Psi_s(K,x(0,\theta),y(0,\theta))$$

$$f_{2l}(\xi_0,\ a_{2m-1},\ a_{2n-1},\theta)=$$
$$-\left[\psi_{2l}(\xi_0,\ 0,\ \theta)-\frac{x(0,\ \theta)}{x\left(0,\frac{\pi}{2}\right)}\psi_{2l}\left(\xi_0,\ 0,\ \frac{\pi}{2}\right)\right]$$
$$=-\left[\sin 2l\theta+\frac{\xi_0}{1+a_{2m-1}+a_{2n-1}}\left\{\frac{\sin(2l-1)\theta}{2l-1}\right.\right.$$
$$+\frac{(-1)^{m-1}(2m-1)a_{2m-1}}{2l+2m-1}\sin(2l+2m-1)\theta$$
$$\left.+\frac{(-1)^{n-1}(2n-1)a_{2n-1}}{2l+2n-1}\sin(2l+2n-1)\theta\right\}$$
$$-\frac{(-1)^{l-1}\xi_0}{(1+a_{2m-1}+a_{2n-1})^2}\left\{\frac{1}{2l-1}\right.$$
$$\left.-\frac{(2m-1)a_{2m-1}}{2l+2m+1}-\frac{(2n-1)a_{2n-1}}{2l+2n-1}\right\}$$
$$\times\left\{\sin\theta-(-1)^m a_{2m-1}\sin(2m-1)\theta\right.$$
$$\left.\left.-(-1)^n a_{2n-1}\sin(2n-1)\theta\right\}\right] \qquad (34)$$

The unknown $p_{2l}(\xi_0)$ and $q_{2l}(\xi_0)$ are determined by solving the equation (33-1) and (33-2), respectively.

Let $\bar{A}$ be the amplitude ratio:

$$\bar{A}=\frac{\text{amplitude of progressive wave}}{\text{amplitude of forced heaving}}=\left|\frac{\eta}{h}\right|. \qquad (35)$$

From (4) and (32), $\bar{A}$ is given as

$$\bar{A}=\frac{\pi\xi_0}{\sqrt{A_0^2+B_0^2}} \qquad (36)$$

where

$$A_0=\Psi_{co}\left(\xi_0,\ a_{2m-1},\ a_{2n-1},\ \frac{\pi}{2}\right)$$
$$+\sum_{l=1}^{\infty}p_{2l}(\xi_0)(-1)^{l+1}\frac{\xi_0}{1+a_{2m-1}+a_{2n-1}}$$
$$\times\left\{\frac{1}{2l-1}-\frac{(2m-1)\ a_{2m-1}}{2l+2m-1}-\frac{(2n-1)\ a_{2n-1}}{2l+2n-1}\right\} \qquad (37\text{-}1)$$

$$B_0=\Psi_{so}\left(\xi_0,\ a_{2m-1},\ a_{2n-1},\ \frac{\pi}{2}\right)$$
$$+\sum_{l=1}^{\infty}q_{2l}(\xi_0)\ (-1)^{l+1}\frac{\xi_0}{1+a_{2m-1}+a_{2n-1}}$$
$$\times\left\{\frac{1}{2l-1}-\frac{(2m-1)a_{2m-1}}{2l+2m-1}-\frac{(2n-1)\ a_{2n-1}}{2l+2n-1}\right\} \qquad (37\text{-}2)$$

The velocity potential $\phi\ (x,y,t)$ corresponding to the stream function $\Psi(x,y,t)$ can be easily derive from (29), that is

$$\phi(x,y,t)=\frac{g\eta}{\pi\omega}\left[\Phi_c(K,x(\alpha,\theta),y(\alpha,\theta))\cos\omega t\right.$$
$$+\Phi_s(K,\ x(\alpha,\theta),\ y(\alpha,\theta))\ \sin\omega t$$
$$+\cos\omega t\sum_{l=1}^{\infty}p_{2l}(\xi_0)\phi_{2l}(\xi_0,\ \alpha,\ \theta)$$
$$+\sin\omega t\sum_{l=1}^{\infty}q_{2l}(\xi_0)\phi_{2l}(\xi_0,\alpha,\theta), \qquad (38)$$

where

$$\Phi_c(K,x,y)=\pi e^{-Ky}\cos Kx$$
$$\Psi_s(K,x,y)=-\int_0^{\infty}\frac{e^{-kx}}{K^2+k^2}\left\{k\cos ky-K\sin ky\right\}dk$$
$$+\pi e^{-Ky}\sin Kx \qquad (39)$$

From the expression of the velocity potential ϕ, the hydrodynamic pressure $p=-\rho\frac{\partial\phi}{\partial t}$ can be derived. Hence, the hydrodynamic force acting on the cylinder in the direction of y-axis can be expressed as follows:

$$F=\left(\frac{g\eta}{\pi}\right)\rho B(M_0\cos\omega t-N_0\sin\omega t). \qquad (40)$$

$M_0(\xi_0,a_{2m-1},a_{2n-1})$ and $N_0\ (\xi_0,a_{2m-1},\ a_{2n-1})$ are given as

$$M_0=\int_0^{2\pi}\Phi_{s0}(\xi_0,\ a_{2m-1},\ a_{2n-1},\ \theta)$$
$$\times[\{\cos\theta+(-1)^{m-1}a_{2m-1}\cos(2m-1)\theta$$
$$+(-1)^{n-1}a_{2n-1}\cos(2n+1)\theta\}$$
$$/(1+a_{2m-1}+a_{2n-1})]\times d\theta$$
$$+\frac{1}{1+a_{2m-1}+a_{2n-1}}\left[\sum_{l=1}^{\infty}(-1)^{l-1}q_{2l}(\xi_0)\right.$$
$$\times\left\{\frac{1}{4l^2-1}+\frac{(2m-1)^2a_{2m-1}}{4l^2-(2m-1)^2}-\frac{(2n-1)^2a_{2n-1}}{4l^2-(2n-1)^2}\right\}$$
$$+\frac{\pi}{4}\ \frac{\xi_0}{1+a_{2m-1}+a_{2n-1}}\left\{q_2\right.$$
$$+(-1)^{m-1}a_{2m-1}q_{2m}+(-1)^{n-1}a_{2n-1}q_{2n}$$
$$\left.\left.+(-1)^{n-m}q_{2(n-m)}(2n-1)a_{2m-1}a_{2n-1}\right\}\right] \tag{41-1}$$

$$N_0=\int_0^{2\pi}\Phi_{c0}(\xi_0,\ a_{2m-1},\ a_{2n-1},\theta)$$
$$\times[\{\cos\theta+(-1)^{n-1}a_{2m-1}\cos(2m-1)\theta$$
$$+(-1)^{n-1}a_{2n-1}\cos(2n-1)\theta\}$$
$$/(1+a_{2m-1}+a_{2n-1})]\times d\theta$$
$$+\frac{\xi_0}{1+a_{2m-1}+a_{2n-1}}\left[\sum_{l=1}^{\infty}(-1)^{l-1}p_{2l}(\xi_0)\right.$$
$$\times\left\{\frac{1}{4l^2-1}+\frac{(2m-1)^2a_{2m-1}}{4l^2-(2m-1)^2}-\frac{(2n-1)^2a_{2n-1}}{4l^2-(2n-1)^2}\right\}$$
$$+\frac{\pi}{4}\ \frac{\xi_0}{1+a_{2m-1}+a_{2n-1}}\left\{p_2\right.$$
$$+(-1)^{m-1}a_{2m-1}p_{2m}+(-1)^{n-1}a_{2n-1}p_{2n}$$
$$\left.\left.+(-1)^{n-m}p_{2(n-m)}(2m-1)\ a_{2m-1}a_{2n-1}\right\}\right] \tag{41-2}$$

where

$$\Phi_{c0}(\xi_0,a_{2m-1},a_{2n-1},\theta)=\Phi_c(K,x(0,\theta),y(0,\theta))$$
$$\Phi_{s0}(\xi_0,a_{2m-1},a_{2n-1},\theta)=\Phi_s(K,x(0,\theta),y(0,\theta)) \tag{42}$$

From (4), (32), (37-1) and (37-2), the velocity $\frac{dy_h}{dt}$ and the acceleration $\frac{d^2y_h}{dt^2}$ of the forced heaving can be written as

$$\frac{dy_h}{dt}=\left(\frac{2g\eta}{\pi B\omega}\right)\left\{-A_0(\xi_0)\cos\omega t-B_0(\xi_0)\ \sin\omega t\right\} \tag{43}$$

$$\frac{d^2y_h}{dt^2}=\left(\frac{2g\eta}{\pi B}\right)\left\{A_0(\xi_0)\sin\omega t-B_0(\xi_0)\ \cos\omega t\right\} \tag{44}$$

From (40), (43) and (44), the hydrodynamic force can be expressed in its component form as

$$F=-2\rho\left(\frac{B}{2}\right)^2\left(\frac{M_0B_0+N_0A_0}{A_0^2+B_0^2}\right)\frac{d^2y_h}{dt^2}$$
$$-2\rho\omega\left(\frac{B}{2}\right)^2\left(\frac{M_0A_0-N_0B_0}{A_0^2+B_0^2}\right)\frac{dy_h}{dt} \tag{45}$$

Hence, it follows that

$$\text{added mass}=2\rho\left(\frac{B}{2}\right)^2\left(\frac{M_0B_0+N_0A_0}{A_0^2+B_0^2}\right) \tag{46}$$

$$\text{damping}=2\rho\omega\left(\frac{B}{2}\right)^2\left(\frac{M_0A_0-N_0B_0}{A_0^2+B_0^2}\right) \tag{47}$$

Let the free surface effect K_4 of the added mass be defined as

$$K_4=\frac{\text{added mass}}{\frac{1}{2}\rho\pi\left(\frac{B}{2}\right)^2C_0} \tag{48}$$

$$\text{where}\ \ C_0=\frac{\text{added mass in case of }\omega\to\infty}{\frac{1}{2}\rho\pi\left(\frac{B}{2}\right)^2} \tag{49}$$

K_4 is, then, obtained for $(a_1,\ a_{2n-1})$ family as

$$K_4=\frac{4}{\pi}\left(\frac{M_0B_0+N_0A_0}{A_0^2+B_0^2}\right)\frac{(1+a_1+a_{2n-1})^2}{(1+a_1)^2+(2n-1)a^2_{2n-1}}$$
$$\text{for } n=4,6. \tag{50}$$

From the consideration of the energy transmission by the diverging waves, the followingr elation must hold [1, 2]:

$$\frac{\rho g^2\eta^2}{\pi^2\omega}(M_0A_0-N_0B_0)=\frac{1}{2}\ \frac{\rho g^2\eta^2}{\omega} \tag{51}$$

The left hand side of (51) refers to the work done by the cylinder, and the right hand side refers to the average energy flux by the diverging waves.

4. Results of Calculations

Numerical calculations were conducted for the section contours shown in Fig. 2. In the present work, the half beam to draft ratio H_0 is taken unity, and the effect of the sectional area coefficient σ is calculated.

The exponential integrals in (30) and (39) are calculated by using formulae given in Appendix 1. The infinite series in (29) and (38) were truncated at $l=6$. The least square method was used to solve the linear equation for the integral constants $p_{2l}(\xi_0)$ and $q_{2l}(\xi_0)$, $(l=1, 2, \cdots\cdots, 6)$, and the component of the matrix were integrated numerically by using the Simpson's rule. The numerical calculations were checked by the equation(see Eq. (51)):

$$\text{Error}=\frac{\pi^2}{2}-(M_0A_0-N_0B_0) \tag{52}$$

An example of the calculation is given in Table 1.

In Fig. 3-1~3-4 and in Fig. 4-1~4-2, the free surface effect K_4, the amplitude ratio $\bar{A}$ and the damping force of the chine sections are compared with those of the equivalent Lewis section which has the same σ and H_0 with the chine sections. It seems to

Table 1. Accuracy of the numerical calculation single chine

$\sigma=0.75$ $H_0=1.00$ $l=6$

ξ_0	0.2	0.4	0.8	1.2
$\bar{A}$	0.296	0.497	0.752	0.888
K_4	1.033	0.730	0.606	0.624
Error(10^{-4})	1	4	10	25

ξ_0	1.6	2.0	2.4	3.2
$\bar{A}$	0.960	0.995	1.009	1.007
K_4	0.667	0.711	0.748	0.805
Error(10^{-4})	63	128	186	114

ξ_0	$l=10$		$l=12$
	5.0	3.2	3.2
$\bar{A}$	0.956	1.0017	1.0010
K_4	0.876	0.8047	0.8047
Error(10^{-4})	441	41	13

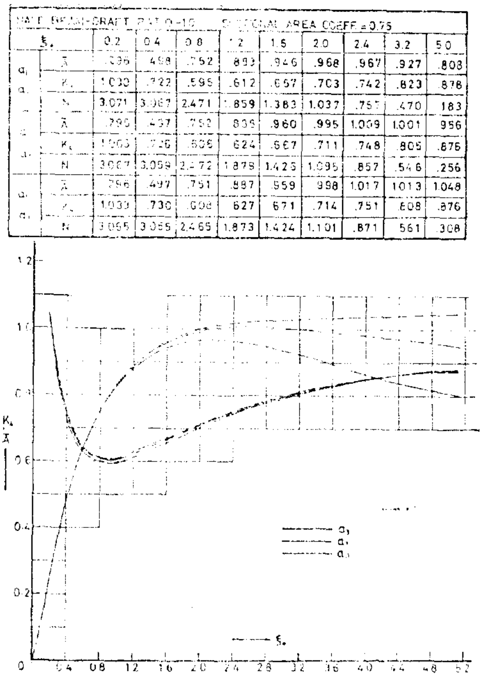

Fig. 3-1. K_4, $\bar{A}$ *vs.* ξ_0; $H_0=1.0$, $\sigma=0.75$

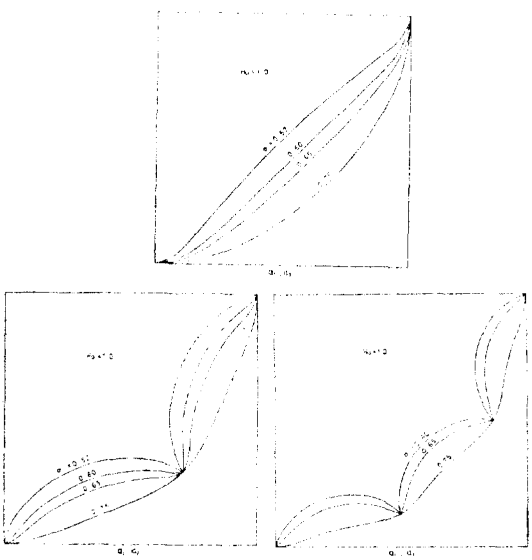

Fig. 2. Section Contours

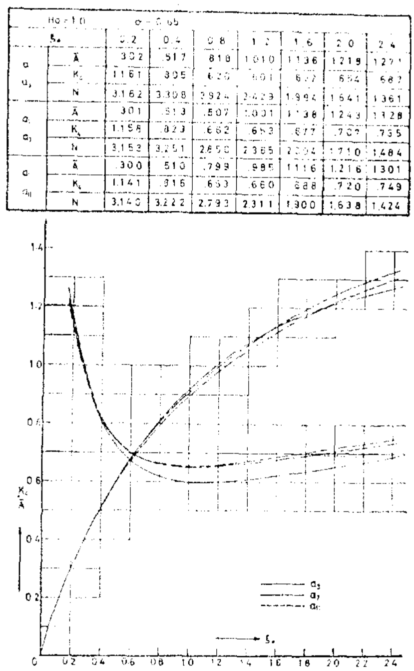

Fig. 3-2. K_4, $\bar{A}$ *vs.* ξ_0; $H_0=1.0$, $\sigma=0.65$

the author that there exists any significant differenses.

In Fig. 5-1~5-3, the effects of σ on the added mass coefficient C_1 and the damping coefficient μ are shown. These quantities are in a rather simple relation with σ.

$H_0=1.0$	$\sigma=0.60$							
ξ_0		0.2	.4	.8	1.2	1.6	2.0	2.4
a_1 a_3	$\bar{A}$	.304	.525	.845	1.064	1.218	1.330	1.413
	K_4	1.233	.856	.647	.611	.620	.644	.670
	N	3.226	3.408	3.122	2.690	2.296	1.959	1.680
a_1 a_7	$\bar{A}$	.302	.518	.837	1.040	1.201	1.330	1.441
	K_4	1.216	.871	.698	.680	.697	.721	.745
	N	3.184	3.320	2.990	2.577	2.230	1.958	1.747
a_1 a_{11}	$\bar{A}$	.301	.515	.815	1.017	1.166	1.286	1.389
	K_4	1.187	.857	.696	.685	.706	.734	.760
	N	3.167	3.279	2.907	2.465	2.103	1.830	1.624

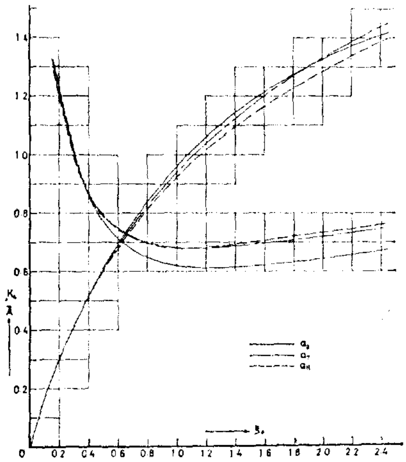

Fig. 3-3. K_4, $\bar{A}$ vs ξ_0; $H_0=1.0$, $\sigma=0.60$

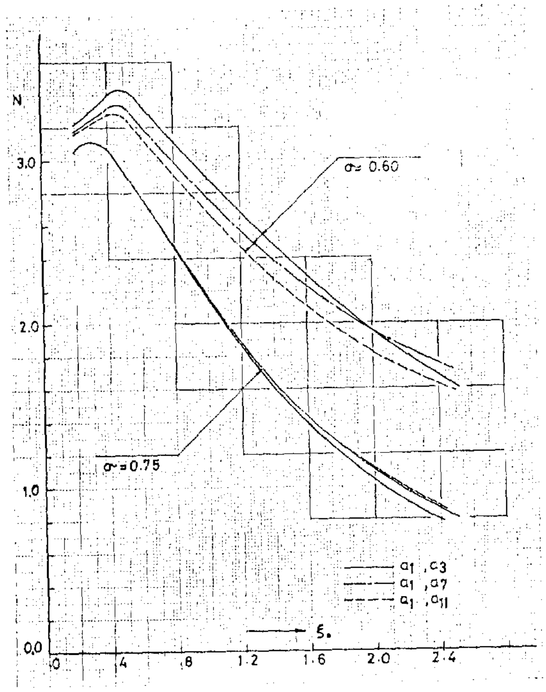

Fig. 4-1. N vs ξ_0; $H_0=1$, $\sigma=0.60$ and $\sigma=0.75$

$H_0=1.0$	$\sigma=0.52$							
ξ_0		0.2	0.4	0.8	1.2	1.6	2.0	2.4
a_1 a_3	$\bar{A}$	.307	.535	.881	1.136	1.333	1.489	1.617
	K_4	1.351	.948	.708	.648	.639	.649	.665
	N	3.288	3.545	3.398	3.075	2.748	2.455	2.200
a_1 a_7	$\bar{A}$	.304	.525	.851	1.089	1.277	1.436	1.577
	K_4	1.300	.949	.762	.732	.739	.756	.774
	N	3.224	3.411	3.169	2.822	2.521	2.282	2.093

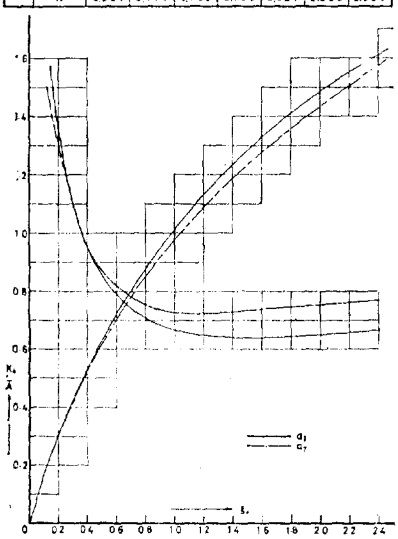

Fig. 3-4. K_4, $\bar{A}$ vs ξ_0; $H_0=1.0$, $\sigma=0.52$

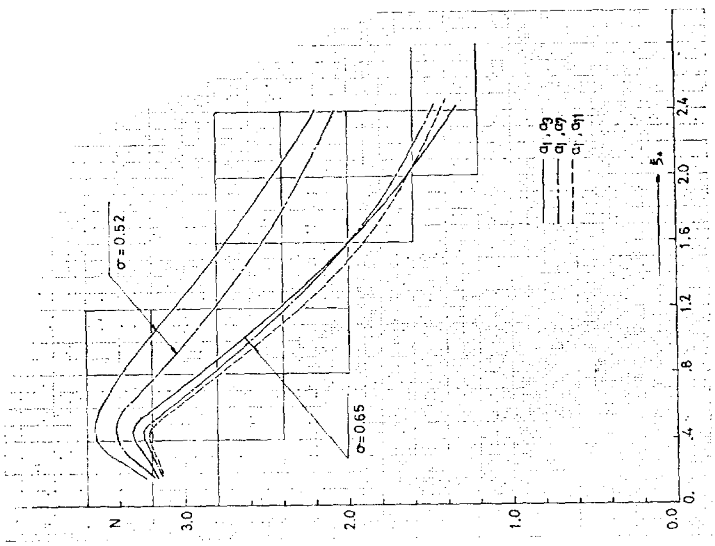

Fig. 4-2. N vs ξ_0; $H_0=1$, $\sigma=0.52$ and $\sigma=0.65$

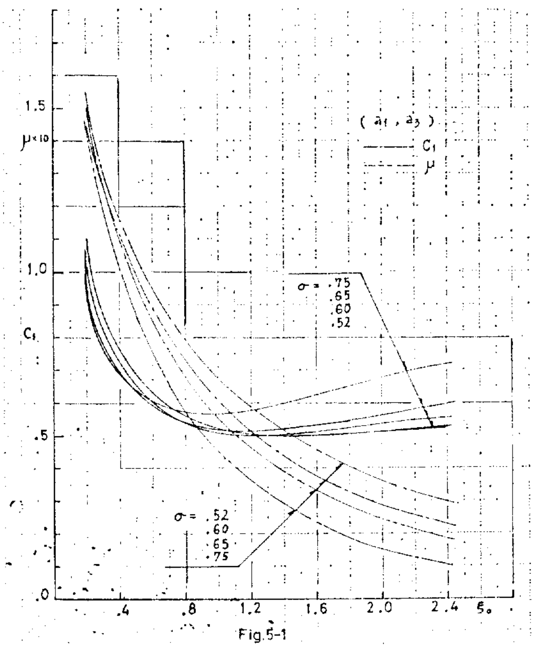

Fig. 5-1. C_1, μ *vs.* ξ_0; Lewis Form

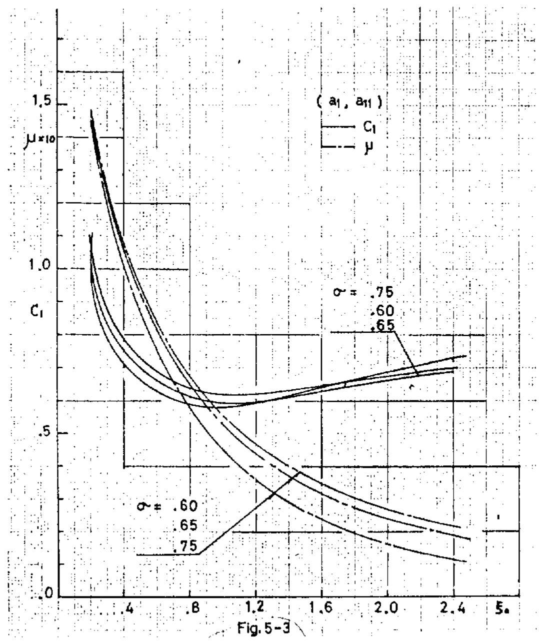

Fig. 5-3. C_1, μ *vs.* ξ_0; Double Chine Form

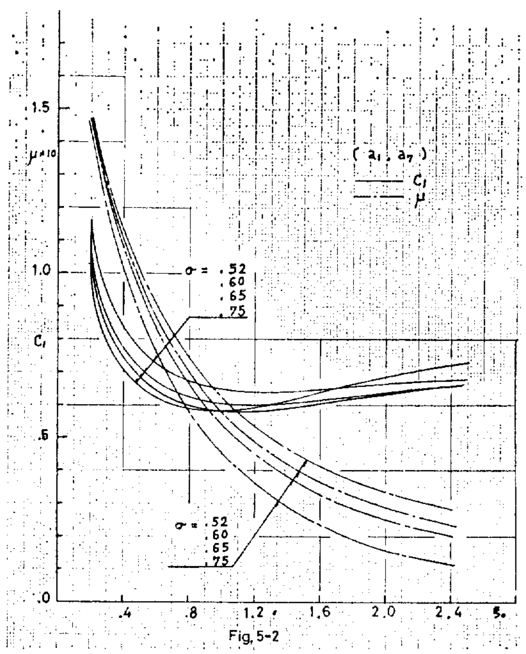

Fig. 5-2. C_1, μ *vs.* ξ_0; Single Chine Form

5. Heave Response in Regular Beam Sea

In section 4, the effects of σ on K_4, $\bar{A}$, C_1 and μ are shown numerically. In this section, the heave response in regular beam sea is discussed.

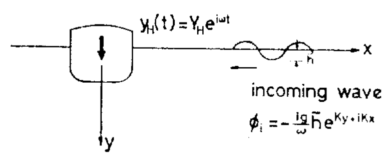

Fig. 6

Let

$$y_H(t) = Y_H e^{i\omega t} \tag{53}$$

be the displacement of the ship in y-direction. The equation of the vertical motion can be written as

$$m\ddot{y} + \rho g B y_H = -m_H \ddot{y}_H - N_H \dot{y}_H + E_H e^{i\omega t}, \tag{54}$$

where $m = \rho S = \rho\sigma BT$

B = beam,

and

m_H = added mass due to heave oscillation

$$= K_4 \cdot C_0 \cdot \frac{1}{2}\rho\pi\left(\frac{B}{2}\right)^2$$

$$N_H = \text{damping} = \frac{\rho g}{\omega K} |A_H|^2 \tag{55}$$

E_H = exciting force due to the incoming wave

$$= \frac{i\omega^2\rho}{K^2} \cdot \bar{A}_H \cdot \bar{h} \tag{56}$$

Substituting (53) and (55) into (54), the heave response $H_H(\xi_0)$:

$$H_H(\xi_0)=\frac{Y_H}{\bar{h}} \tag{57}$$

is derived as

$$H_H(\xi_0)=\frac{\frac{i\omega^2\rho}{K^2}\bar{A}_H}{[-(m+m_H)\omega^2+N_H i\omega+\rho gB]}$$

$$=\frac{i\bar{A}_H}{-\left[\sigma BT+K_4C_0\frac{1}{2}\pi\left(\frac{B}{2}\right)^2\right]K^2+i|\bar{A}_H|^2+KB} \tag{58}$$

Hence, $H_H(\xi_0)$ is given as

$$|H_H(\xi_0)|=|\bar{A}_H|\,/\left[\left\{KB-\sigma(BK)\ (TK)-K_4C_0\frac{1}{2}\pi\left(K\frac{B}{2}\right)^2\right\}^2+\bar{A}_H{}^4\right]^{1/2} \tag{59}$$

In Fig. 7, numerical results are given for a single chine section, double chine section and the equivalent Lewis form section which have the same sectional area coefficient $\sigma=0.60$. The difference between these are not significant in spite of the large differences of their section contours. To the author. It seems to be an interesting phenomena.

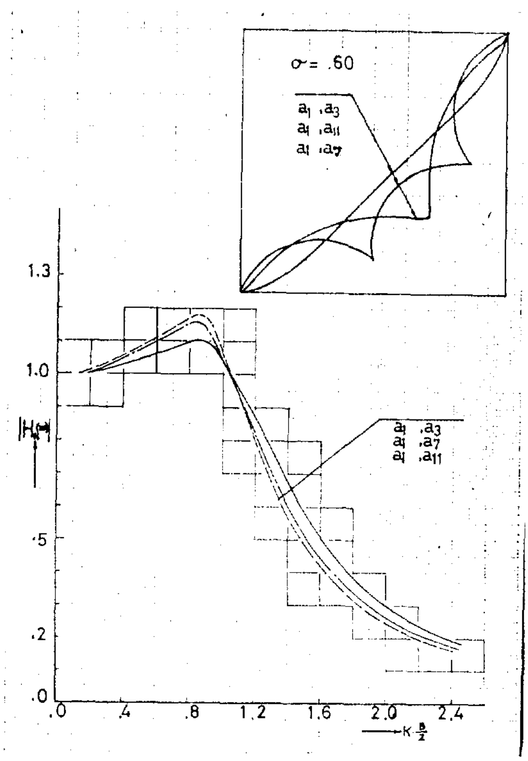

Fig. 7. Heave Response in Regular Beam Sea; $H_0=1$, $\sigma=0.60$

6. Conclusions

The following conclusions are deduced as a result of the present work.

(1) The free surface effect of the added mass of the chine section, either single or double, has a larger value than that of the equivalent Lewis section.

(2) If the sectional area coefficient and the half beam to draft ratio are the same, the damping coefficient becomes larger in the order of the equivalent Lewis form, the single chine and the double chine in the range of low circular frequency.

(3) The sectional area coefficient and the half beam to draft ratio seem to have the primary effects on the heaving motion, and the effects due to the other geometrical characteristics seem to be secondary.

(4) Added mass coefficient and damping coefficient of heaving oscillation seem to be in a rather simple relation with the sectional area coefficient.

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Appendix I

$$\Psi_0=\frac{g\eta}{\pi\omega}[\Psi_c(K,x,y)\cos\omega t+\Psi_s(K,x,y)\sin\omega t] \quad \text{(A-1-1)}$$

$$\Psi_c=\pi\cdot e^{-Ky}\cdot\sin Kx$$

$$\Psi_s=\int_0^\infty\frac{e^{-\kappa x}}{K^2+\kappa^2}\{\kappa\sin\kappa y+K\cos\kappa y\}d\kappa-\pi e^{-Ky}\cos Kx$$

$$\phi_0=\frac{g\eta}{\pi\omega}[\phi_c(K,x,y)\cos\omega t+\phi_s(K,x,y)\sin\omega t] \quad \text{(A-1-2)}$$

$$\phi_c=\pi\cdot e^{-Ky}\cdot\cos Kx$$

$$\phi_s=-\int_0^\infty\frac{e^{-\kappa x}}{K^2+\kappa^2}\{\kappa\cos\kappa y-K\sin\kappa y\}d\kappa+\pi e^{-Ky}\sin Kx$$

Evaluation of the exponential integral

$$\int_0^\infty\frac{\cos Bx+x\sin Bx}{1+x^2}\cdot e^{-Ax}dx$$

$$=\frac{1}{2}\int\frac{e^{-(A-iB)x}}{1+x^2}dx+\frac{1}{2}\int\frac{e^{-(A+iB)x}}{1+x^2}dx$$

$$+\frac{1}{2i}\int\frac{x\cdot e^{-(A-iB)x}}{1+x^2}dx-\frac{1}{2i}\int\frac{xe^{-(A+iB)x}}{1+x^2}dx$$

$$\begin{cases}Z_1=A-iB\\ Z_2=A+iB\end{cases}$$

where

$$\int_0^\infty\frac{e^{-Z_1x}}{1+x^2}dx=\sin(Z_1)C_i(Z_1)-\cos(Z_1)S_i(Z_1)+\cos(Z_1)\frac{\pi}{2}$$

$$\int_0^\infty\frac{e^{-Z_2x}}{1+x^2}dx=\sin(Z_2)C_i(Z_2)-\cos(Z_2)S_i(Z_2)+\cos(Z_2)\frac{\pi}{2}$$

$$\int_0^\infty\frac{xe^{-Z_1x}}{1+x^2}dx=-\cos(Z_1)C_i(Z_1)-\sin(Z_1)S_i(Z_1)+\sin(Z_1)\frac{\pi}{2}$$

$$\begin{cases}C_i(Z)=\gamma+\ln Z+\int_0^z\frac{\cos t-1}{t}dt\\ S_i(Z)=\int_0^z\frac{\sin t}{t}dt\end{cases}$$

$$=e^{-B}\Big[-\frac{e^{-iA}}{Z_i}\cdot C_i(Z_1)-\frac{e^{-iA}}{2}S_i(Z_1)+\frac{e^{-iA}}{2}\cdot\frac{\pi}{2}+\frac{e^{iA}}{2i}\cdot C_i(Z_2)-\frac{e^{iA}}{2}S_i(Z_2)+\frac{e^{iA}}{2}\cdot\frac{\pi}{2}\Big]$$

where

$$\frac{e^{iA}-e^{-iA}}{2i}C_i(Z_i)-\frac{e^{iA}\{C_i(Z_1)-C_i(Z_2)\}}{2i}$$

$$=\sin A\Big\{\gamma+\frac{1}{2}\log(A^2+B^2)+\sum_{n=1}^\infty(-1)^n\frac{(A^2+B^2)^n\cdot\cos 2n\theta}{2n(2n)!}\Big\}$$

$$+\cos A\Big\{\tan^{-1}\frac{B}{A}+\sum_{n=1}^\infty\frac{(-1)^n(A^2+B^2)^n\sin 2n\theta}{2n(2n)!}\Big\}$$

$$-\frac{e^{iA}+e^{-iA}}{2}S_i(Z_1)+\frac{e^{iA}}{2}\{S_i(Z_1)-S_i(Z_2)\}$$

$$=-\cos A\sum_{n=0}^\infty\frac{(-1)^n(A^2+B^2)^{\frac{2n+1}{2}}\cos(2m+1)\theta}{(2n+1)(2n+1)!}$$

$$+\sin A\sum_{n=0}^\infty\frac{(-1)^n(A^2+B^2)^{\frac{2n+1}{2}}\sin(2n+1)\theta}{(2n+1)(2n+1)!}$$

Hence

$$\int_0^\infty\frac{\cos Bx+x\sin Bx}{1+x^2}\cdot e^{-Ax}dx$$

$$=e^{-B}\Big[\sin A\Big\{\gamma+\log(A^2+B^2)^{\frac{1}{2}}+\sum_{n=1}^\infty\frac{(-1)^n(A^2+B^2)^n\cos 2n\theta}{2n(2n)!}+\sum_{n=0}^\infty\frac{(-1)^n(A^2+B^2)^{\frac{2n+1}{2}}\sin(2n+1)\theta}{(2n+1)(2n+1)!}\Big\}$$

$$+\cos A\Big\{\frac{\pi}{2}+\tan^{-1}\frac{B}{A}+\sum_{n=1}^\infty\frac{(-1)^n(A^2+B^2)^n\sin 2n\theta}{2n(2n)!}+\sum_{n=0}^\infty\frac{(-1)^n(A^2+B^2)^{\frac{2n+1}{2}}\cos(2n+1)\theta}{(2n+1)(2n+1)!}\Big\}\Big] \quad \text{(A-1-3)}$$

Appendix II

The Haskind-Newman Relation [5]

Let ϕ_i be the velocity potential of the incoming wave:

$$\phi_i=-\frac{ig}{\omega}\bar{h}e^{Ky+iKx} \quad \text{(A-2-1)}$$

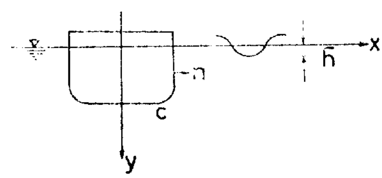

Fig. 8

where h: amplitude of incoming wave

Let ϕ_d be the diffraction velocity potential.

Then, the velocity potential ϕ of the diffraction problem due to ϕ_i

$$\phi=\phi_i+\phi_d \quad \text{(A-2-2)}$$

$$\frac{\partial\phi}{\partial n}=\frac{\partial}{\partial n}(\phi_i+\phi_d) \quad \text{on } C \quad \text{(A-2-3)}$$

The velocity potential ϕ_H of the forced heaving of the unit velocity amplitude is defined as

$$\frac{\partial\phi_H}{\partial n}=\frac{\partial y}{\partial n} \quad \text{on } C \quad \text{(A-2-4)}$$

The heave component E_H of the wave excitation force due to the in coming wave is

$$E_H=\int_c\left(-p\frac{\partial y}{\partial n}\right)ds \qquad \text{(A-2-5)}$$

where $pe^{i\omega t}$=hydrodynamic pressure

$$E_H=i\omega\rho\int_c\left[(\phi_i+\phi_d)\frac{\partial y}{\partial n}\right]ds$$

$$e_H=\frac{E_H}{i\omega\rho}=\int_c\left[\phi_i\frac{\partial y}{\partial n}+\phi_d\frac{\partial y}{\partial n}\right]ds$$

$$=\int_c\left[\phi_i\frac{\partial\phi_H}{\partial n}+\phi_d\frac{\partial\phi_H}{\partial n}\right]ds$$

by Green's theorem

$$=\int_c\left[\phi_i\frac{\partial\phi_H}{\partial n}+\phi_H\frac{\partial\phi_d}{\partial n}\right]ds$$

$$=\int_c\left[\phi_i\frac{\partial\phi_H}{\partial n}-\phi_H\frac{\partial\phi_i}{\partial n}\right]ds$$

by Green's theorem

$$=-\left\{\int_{\substack{0\\x=R+}}^{-\infty}\left[\phi_i\left(\frac{\partial\phi_H}{\partial x}\right)-\phi_H\left(-\frac{\partial\phi_i}{\partial x}\right)\right](-dy)\right.$$

$$\left.+\int_{\substack{-\infty\\x=R-}}^{0}\left[\phi_i\left(\frac{\partial\phi_H}{\partial x}\right)-\phi_H\left(\frac{\partial\phi_i}{\partial x}\right)\right](dy)\right\}$$

$$=-\left\{\int_{\substack{-\infty\\x=R+}}^{0}\left[-\phi_i\frac{\partial\phi_H}{\partial x}+\phi_H\frac{\partial\phi_i}{\partial x}\right]dy\right.$$

$$\left.+\int_{\substack{-\infty\\x=R-}}^{0}\left[\phi_i\frac{\partial\phi_H}{\partial x}-\phi_H\frac{\partial\phi_i}{\partial x}\right]dy\right\}$$

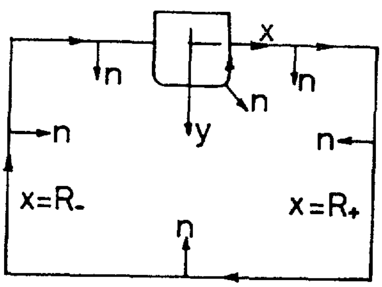

Fig. 9

The assymptotic form of ϕ_H is given as

$$\phi_H\sim\begin{cases}-\dfrac{ig}{\omega}\left(\dfrac{\bar{A}_H}{i\omega}\right)e^{Ky-iKx} & x\to\infty\\ -\dfrac{ig}{\omega}\left(\dfrac{\bar{A}_H}{i\omega}\right)e^{Ky+iKx} & x\to\infty\end{cases}$$

where $\bar{A}$: wave amplitude ratio including the phase difference.

Then, at $x=R_+$

$$-\phi_i\frac{\partial\phi_H}{\partial x}+\phi_H\frac{\partial\phi_i}{\partial x}$$

$$\sim-\left[-\frac{ig}{\omega}\bar{h}e^{Ky+iKx}\right]\left[-\frac{gK}{\omega}\left(\frac{\bar{A}_H}{i\omega}\right)e^{Ky-iKx}\right]$$

$$+\left[-\frac{ig}{\omega}\left(\frac{\bar{A}_H}{iw}\right)e^{Ky-iKx}\right]\cdot\left[\frac{gK}{\omega}\bar{h}e^{Ky+iKx}\right]$$

$$=-\frac{2\omega}{K}\bar{A}_H\cdot\bar{h}e^{2Ky}$$

at $x=R_-$

$$\phi_i\frac{\partial\phi_H}{\partial x}-\phi_H\frac{\partial\phi_i}{\partial x}$$

$$\sim\left[-\frac{ig}{\omega}\bar{h}e^{Ky+iKx}\right]\cdot\left[\frac{gK}{\omega}\left(\frac{\bar{A}_H}{i\omega}\right)e^{Ky+iKx}\right]$$

$$-\left[-\frac{ig}{\omega}\left(\frac{\bar{A}_H}{i\omega}\right)e^{Ky+iKx}\right]\left[\frac{gK}{\omega}\bar{h}e^{Ky+iKx}\right]=0$$

Hence, the following relation between the wave excitation force due to the incoming wave and the amplitude ratio for heave oscillation is derived:

$$\frac{iE_H}{\omega\rho}=\lim_{\substack{R_+\to\infty\\R_-\to\infty}}\left\{\int_{\substack{-\infty\\x=R+}}^{0}\left[-\phi_i\frac{\partial\phi_H}{\partial x}+\phi_H\frac{\partial\phi_i}{\partial x}\right]dy\right.$$

$$\left.+\int_{\substack{-\infty\\x=R-}}^{0}\left[\phi_i\frac{\partial\phi_H}{\partial x}-\phi_H\frac{\partial\phi_i}{\partial x}\right]dy\right\}$$

$$=-\frac{2\omega}{K}\bar{A}_H\bar{h}\int_{-\infty}^{0}e^{2\kappa y}\,dy=-\frac{\omega}{K^2}\bar{A}_H\bar{h}$$

$$E_H=\frac{i\omega^2\rho}{K^2}\bar{A}_H\bar{h}$$

(The Haskind-Newman Relation) (A-2-6)