無限固狀電子프라즈마內에서의 電波傳播特性

논 문
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Characteristics of Wave Propagation in an Unbounded Solid State Electron Plasma

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Abstract

This paper deals with wave propagations in solid state electron plasmas from the view point of treating the plasma as a conducting fluid, and especially consideration is extended to the effect of diffusion on the permittivities and dispersion relations. The analysis is based on the conception of the self-consistent field approximation. It is shown for the cases of the specific physical configurations that the positions of the null elements in the permittivity tensors are not affected by the diffusion terms, and the diffusion effect appears only in the case of the space-charge wave. It is also shown that the magnitude of the real part of wave vector is in proportion to the 3/2nds power of the field in some regions.

I. INTRODUCTION

On analyzing the characteristics of wave propagations in solid state electron plasmas, the permittivities play a critical role as in dielectrics, since the permittivities are especially closely related with lattice vibrations, dispersion relations and wave propagation velocities in the media [1], [2], and calculating the permittivity for an electron gas in solid state materials subjected under the external excitations there are several methods known as random phase approximation, independent-pair approximation, self-consistent field approximation, or time-dependent Hartree-Fock approximation [3], [4].

In this paper discussion is based on the conception of the self-consistent field approximation, and the concept of plasma equivalent permittivities allows the collective interactions between the charged particles and lattices to be described in terms of a familiar electrical property using a hydrodynamical model for the plasma, to be rendered less difficult to analogize with that for the solid state nonplasma materials, and to be characterized the plasma as a dielectric medium.

It is of particular interest in the field of electro-optics to investigate the properties of the permittivities of materials to be utilized, and it is furthermore one of the principal targets in the current research to seek the plasmas which possess a proper permittivity suitable for the intended specific applications as fully discussed by Crawford [5].

It is a great convenience in the consideration of plasma waves to treat the plasma as a dielectric medium with its circumference. The presence of electrons in the plasma gives rise

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to a convection current, and this current must be accounted for in the derivation of an equivalent permittivity. When a magnetostatic field is present, the plasma is anisotropic and the plasma permittivity is represented by a tensor. Anisotropy in a plasma medium is due to the fact that electrons orbit magneticfield lines in one direction only and the plasma properties enter the wave equation only through the tensor dielectric constant [6] and one tends to investigate the tensor from a viewpoint of mathematical symmetry transformations, because the latter can be used to simplify and unify the results of the problems concerned with the former [7].

In solids, because of the thermal vibrations of the lattice, scattering is a very important part of any problem. Aside from phonon scattering of the carriers, there are Rutherford scattering and scattering effect from neutral impurities as well as faults. For dense semiconductor electron-hole plasmas electron-hole scattering also must taken into consideration. For solids, the value of the scattering frequency ranges about 10^{10} to 10^{13} sec^{-1} as the temperature increases from liquid Helium to the room temperature. And it is usually necessary that to observe plasma-wave phenomena in solids the relaxation time of the charge carriers due to the above scatterings must be made sufficiently large enough so that the weve phenomena may not be damped out [4], [8]. This requires the application of high purity samples at low temperatures, and it is assumed in this paper that the time is a constant [4], because the devices which utilize the characteristics of the plasmas operate in the constant environment under the ordinary conditions and the constituents of the devices remain unchanged. These assumptions simplify the problems to a great deal.

With both finite temperature and scattering in the solid there are diffusion effects whenever the carriers are bunched. Since most of the interesting interactions involve such bunching [4], [8], diffusion can play a critical role in the permittivity tensors, the dispersion relations, etc. According to the classical binary collision theory for a Lorentzian gas, the transverse diffusion is proportional to the applied magnetic field for ordinary magnetostatic fields, while for very large magnetostatic fields the direct diffusion is proportional to the inverse square of the field [9], [10], and the diffusion will give its influence on the tensors, the dispersion relations and the wave propagation through the media, and that kind of phenomena occurs in solid state electron magnetoplasmas [4].

It is of fundamental importance, therefore, to investigate the expressions of the permittivity tensors, the dispersion relations and consequently the wave propagation through the solid state electron magnetoplasmas when the diffusion effect is taken into consideration, and these problems are discussed in this paper to obtain the general features of the mutual interrelations between the plasmas and waves.

The propagation of waves in a plasma can be described phenomenologically be solving Maxwell's equations with the Boltzmann equation together, and in the analysis we will concentrate only on the response of the electron gas assuming that the motion of the ionized donors in semiconductors can be neglected. It is aslo assumed herein that the medium is an infinite one, monochromatic plane waves which have the form of exp j $(\omega t - \bar{k}\cdot\bar{r})$ propagate through in the medium and general features of wave do not depend on the details of the band structure [4], [11]-[16], and rationalized MKS units are used throughout the paper.

II. THEORETICAL CONSIDERATIONS

Maxwell's equations are

$$\nabla\times\bar{E}=-\frac{\partial\bar{B}}{\partial t} \quad (1)$$

$$\nabla\times\bar{H}=\bar{J}+\frac{\partial\bar{D}}{\partial t} \quad (2)$$

$$\nabla\cdot\bar{B}=0 \quad (3)$$

$$\nabla\cdot\bar{D}=\rho \quad (4)$$

and in the microscopical description the source terms $\bar{J}$ and ρ are related to the dynamical motion of electrons as

$$\rho_{(\bar{r},t)}=\sum_i -e\delta[\bar{r}-\bar{r}_{i(t)}] \quad (5)$$

$$\bar{J}_{(\bar{r},t)}=\sum_i -e\bar{v}_i\delta[\bar{r}-\bar{r}_{i(t)}] \tag{6}$$

The macroscopic equations of the hydrodynamic model are obtained from the microscopic Boltzmann equation [APP.A]

$$Df=\left(\frac{\partial f}{\partial t}\right)_c \tag{7}$$

by taking moments of the velocity distribution. The first moment of the Boltzmann equation (7) using the Lorentz equation leads to the equation of motion [APP. A]:

$$\frac{d\bar{v}}{dt}=\eta(\bar{E}+\bar{v}\times\bar{B})-\nu\bar{v}-\frac{1}{mn}\nabla\cdot\underline{P} \quad , \tag{8}$$

where η is the charge to the mass ratio of the carrier and ν the effective collision frequency leading to momentum change and n and $\underline{P}$ are the particle density and intrinsic pressure tensor, respectively, and the latter is derived from the random-walk theory and expressed, assuming the pressure is isotropic because the plasma is considered as a conducting fluid, as

$$\underline{P}=\frac{m^*n\langle v_T^2\rangle}{3}\underline{1} \quad , \tag{9}$$

where $\langle v_T^2\rangle=\frac{3k_BT}{m^*}$ is the average kinetic velocity of the particles of effective temperature T and k_B Boltzmann's constant, and obtaining Eq. (8) the small effect of the gravitational gradient in making the vertical direction unique is neglected [6], [17]—[20].

Because, in this problem, Maxwell's equations and the Boltzmann equation have constant coefficients and construct a linear system, the problem may be solved by taking Fourier transforms in t and r, and the following basic equations can be obtained:

$$j\bar{k}\times\bar{E}=j\omega\mu_0\bar{H} \tag{10}$$

$$-j\bar{k}\times\bar{H}=\bar{J}+j\omega\epsilon\bar{E} \tag{11}$$

$$j\bar{k}\cdot\bar{B}=0 \tag{12}$$

$$-j\bar{k}\cdot\bar{D}=\rho \tag{13}$$

$$j\Omega\bar{v}=\eta\bar{E}+\bar{v}\times\bar{w}_c+\eta\bar{v}_0\times\bar{B}+j\frac{v_T^2\rho}{3\rho_0}\bar{k} \tag{14}$$

$$\bar{J}=\rho_0\bar{v}+\rho\bar{v}_0 \tag{15}$$

and

$$\bar{k}\cdot\bar{J}=\omega\rho \quad , \tag{16}$$

where $\bar{w}_c=\eta\bar{B}_0$ and Eq. (16) follows from Eqs. (11) and (13). Notice that we dropped $\langle\ \rangle$ symbol in Eq. (14) for convenience. In deriving Eqs. (10) to (16), the following operators are used [21]:

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\bar{v}_0\cdot\nabla \tag{17}$$

$$j\Omega=\frac{d}{dt}+\nu \tag{18}$$

And in notation the subscript 0 refers to the $d.\ c.$ value and the higher order terms are neglected because of its small magnitude.

The general equation for the wave propagation through the plasma including the diffusion effect can be obtained from Eqs. (10) to (16) [APP. B] as

$$\bar{E}+\frac{k^2}{\omega^2\epsilon\mu_0}[\hat{k}(\hat{k}\cdot\bar{E})-\bar{E}]-\frac{1}{\omega\epsilon Q}\Big\{\frac{\rho_0}{\omega}[W\eta\bar{R} +k\eta\bar{S}(\bar{v}_0\cdot\bar{E})+\frac{k^2v_T^2\omega_p^2\epsilon}{3\rho_0Q}\bar{S}(\hat{k}\cdot[\bar{R}+[\frac{k}{W}\bar{T} (\bar{v}_0\cdot\bar{E})])/[1-\frac{k^2v_T^2}{3WQ}(\hat{k}\cdot\bar{T})])+\bar{v}_0k\omega_p^2\epsilon (\hat{k}\cdot[\bar{R}+\frac{k}{W}\bar{T}(\bar{v}_0\cdot\bar{E})])/[1-\frac{k^2v_T^2}{3WQ} (\hat{k}\cdot\bar{T})]\Big\}=0 \quad , \tag{19}$$

where

$$Q=\Omega(\Omega^2-\omega_c^2)$$
$$\bar{R}=\Omega^2\bar{E}-\bar{w}_c(\bar{w}_c\cdot\bar{E})+j\Omega(\bar{w}_c\times\bar{E})$$
$$\bar{S}=\Omega^2\hat{k}-\bar{w}_c(\bar{w}_c\cdot\hat{k})+j\Omega(\bar{w}_c\times\hat{k})$$
$$\bar{T}=\Omega^2\hat{k}-\bar{w}_c(\bar{w}_c\cdot\hat{k})$$

The Helmholtz equation can be written, from the assumption of a perturbation in the form of $\exp j(\omega t-\bar{k}\cdot\bar{r})$, as

$$\bar{k}\times\bar{k}\times\bar{E}+\frac{\omega^2}{c^2}\underline{\epsilon}\cdot\bar{E}=0 \quad , \tag{20}$$

where $\underline{\epsilon}$ and c are the tensor relative permittivity and the velocity of light, respectively. From Eq. (20) and its associated equation for $\bar{J}$, the following expression will be obtained:

$$\underline{\epsilon}\cdot\bar{E}=\epsilon_l\bar{E}+\frac{1}{j\omega\epsilon_0}(\rho_0\bar{v}+\rho\bar{v}_0) \quad , \tag{21}$$

and using Eq. (11) with its related expression for ρ, the above expression becomes

$$\underline{\epsilon}\cdot\bar{E}=\epsilon_l\bar{E}-\frac{1}{\omega\epsilon_0Q}\Big\{\frac{\rho_0}{\omega}[W\eta\bar{R}+k\eta\bar{S}(\bar{v}_0\cdot\bar{E}) +\frac{k^2v_T^2\omega_p^2\epsilon_0}{3\rho_0Q}\bar{S}(\hat{k}\cdot[\bar{R}+\frac{k}{W}\bar{T}(\bar{v}_0\cdot\bar{E})])/ [1-\frac{k^2v_T^2}{3WQ}(\hat{k}\cdot\bar{T})]]+\bar{v}_0k\omega_p^2\epsilon_0(\hat{k}\cdot[\bar{R} +\frac{k}{W}\bar{T}(\bar{v}_0\cdot\bar{E})])/[1-\frac{k^2v_T^2}{3WQ}(\hat{k}\cdot\bar{T})]\Big\}, \tag{22}$$

where ϵ_l is the relative scalar permittivity of the lattice. Expression (22) is the general expression by which the permittivity tensor can be defined for the case where the diffusion effect is taken into consideration.

Ⅲ. DISCUSSION

A. Permittivity Tensors

To begin with the permittivity tensors in which the diffusion effects are taken into consideration will be derived for the extreme cases in its physical configuration.

For the case where the plasma is in the state of nonstreaming (in this case, $\frac{d}{dt}=\frac{\partial}{\partial t}$), and the wave vector and the magnetostatic field are in the z and y directions, respectively, the following permittivity tensor is obtained from Expression (22):

$$\underline{\epsilon}=\begin{pmatrix} \epsilon_l-BC & 0 & -jBD \\ 0 & \epsilon_l-BF & 0 \\ jBD & 0 & \epsilon_l-BE \end{pmatrix}, \tag{23}$$

where

$$B=\frac{\rho_0}{\omega\epsilon\Omega(\Omega^2-\omega_c^2)}$$
$$C=\eta\Omega^2+\frac{k^2\epsilon v_T^2\omega_p^2\omega_c^2\Omega^2}{\rho_0[3\Omega(\Omega^2-\omega_c^2)\omega-k^2v_T^2\Omega^2]}$$
$$D=\eta\omega_c\Omega+\frac{k^2\epsilon v_T^2\omega_p^2\omega_c\Omega^3}{\rho_0[3\Omega(\Omega^2-\omega_c^2)\omega-k^2v_T^2\Omega^2]}$$
$$E=\eta\Omega^2+\frac{k^2\epsilon v_T^2\omega_p^2\Omega^4}{\rho_0[3\Omega(\Omega^2-\omega_c^2)\omega-k^2v_T^2\Omega^2]}$$
$$F=\eta(\Omega^2-\omega_c^2),$$

and for the case where the wave vector and the magnetostatic field are in the z and x directons, respectively, the permittivity tensor can be expressed, following the same procedure as applied to Expression (22) to derive Expression (23), as

$$\underline{\epsilon}=\begin{pmatrix} \epsilon_l-BF & 0 & 0 \\ 0 & \epsilon_l-BC & jBD \\ 0 & -jBD & \epsilon_l-BE \end{pmatrix}, \tag{24}$$

and for the case where both of the wave vector and the magnetostatic field are in the z direction, the following permittivity tensor can be obtained:

$$\underline{\epsilon}=\begin{pmatrix} \epsilon_l-BC' & jBD' & 0 \\ -jBD' & \epsilon_l-BC' & 0 \\ 0 & 0 & \epsilon_l-BF' \end{pmatrix}, \tag{25}$$

where

$$C'=\eta\Omega^2$$
$$D'=\eta\omega_c\Omega$$
$$F'=\eta(\Omega^2-\omega_c^2)+\frac{k^2\epsilon v_T^2\omega_p^2(\Omega^2-\omega_c^2)}{\rho_0[3\Omega\omega-k^2v_T^2]}.$$

Next, for the case where the plasma is in the state of streaming and its direction of the uniform flow is in the z direction with the wave vector and the magnetostatic field in the z and y directions, respectively, the following permittivity tensor is obtained from Expression (22):

$$\underline{\epsilon}=\begin{pmatrix} \epsilon_l-BC'' & 0 & -jBD'' \\ 0 & \epsilon_l-BF'' & 0 \\ jBD_0'' & 0 & \epsilon_l-BE'' \end{pmatrix}, \tag{26}$$

where

$$C''=\frac{\eta}{\omega}\Omega^2(\omega-kv_0)+\frac{k^2\epsilon v_T^2\omega_p^2\omega_c^2\Omega^2(\omega-kv_0)}{\omega\rho_0[3\Omega(\Omega^2-\omega_c^2)(\omega-kv_0)-k^2v_T^2\Omega^2]}$$
$$D''=\eta\omega_c\Omega+\frac{k^2\epsilon v_T^2\omega_p^2\omega_c\Omega^3}{\rho_0[3\Omega(\Omega^2-\omega_c^2)(\omega-kv_0)-k^2v_T^2\Omega^2]}$$
$$D_0''=\frac{\eta}{\omega}\omega_c\Omega(\omega-kv_0)+\frac{3k\epsilon\omega_p^2\omega_c\Omega^2(\Omega^2-\omega_c^2)(\omega-kv_0)}{\rho_0[3\Omega(\Omega^2-\omega_c^2)(\omega-kv_0)-k^2v_T^2\Omega^2]}\left(\frac{kv_T^2\Omega^2}{3\Omega(\Omega^2-\omega_c^2)\omega}+v_0\right)$$
$$E''=\eta\omega^2+\frac{3k\epsilon\omega_p^2\Omega^3(\Omega^2-\omega_c^2)\omega}{\rho_0[3\Omega(\Omega^2-\omega_c^2)(\omega-kv_0)-k^2v_T^2\Omega^2]}\left(\frac{kv_T^2\Omega^2}{3\Omega(\Omega^2-\omega_c^2)\omega}+v_0\right)$$
$$F''=\frac{\eta}{\omega}(\Omega^2-\omega_c^2)(\omega-kv_0),$$

and for the case where the wave vector and the field are in the z and x directions, respectively, the tensor becomes as

$$\epsilon_l=\begin{pmatrix} \epsilon_l-BF'' & 0 & 0 \\ 0 & \epsilon_l-BC'' & jBD'' \\ 0 & -jBD_0'' & \epsilon_l-BE'' \end{pmatrix}, \tag{27}$$

and for the case where both of the vector and the field are in the z direction, the following result is obtained:

$$\epsilon_l=\begin{pmatrix} \epsilon_l-BC''' & jBD''' & 0 \\ -jBD''' & \epsilon_l-BC''' & 0 \\ 0 & 0 & \epsilon_l-BF''' \end{pmatrix} \tag{28}$$

where

$$C'''=\frac{\eta}{\omega}\Omega^2(\omega-kv_0)$$
$$D'''=\frac{\eta}{\omega}\omega_c\Omega(\omega-kv_0)$$
$$F'''=\eta(\Omega^2-\omega_c^2)+(kv_T^2+3\Omega v_0\omega)\frac{k\epsilon\omega_p^2(\Omega^2-\omega_c^2)}{\rho_0[3\Omega(\omega-kv_0)-k^2v_T^2]}$$

Observing Expressions (23) to (28) it is seen that each expression constitutes a antisymmetric permittivity tensor ($\epsilon_{ij}=-\epsilon_{ji}$: $i\neq j$) except those of (26) and (27), and each matrix, which is for the extreme case in its physical configuration, has four null elements and comparing these matrices with the ones obtained from Equations (B—3) and (B—4) [APP.B], in which the diffusion and/or the streaming terms are neglected at the first place, they are similar in the form of a matrix to each other.

Therefore, it can be seen that there is no influence upon the matrix form, which expresses the permittivity tensor, due to the diffusion and/or the streaming terms, and as a result of the above argument the 2nd terms of C, C″, C‴, D, D″, D_0'', D‴, E, E″, F′, F″, F‴ and the 3rd term of D_0'' can be regarded as perturbation terms and the degree of its perturbation on the permittivities depends upon the relative magnitudes of variables and parameters, and also it might be expected that these terms have influence on the type of waves, which propagate through in an solid atate electron plasma, since the terms are, in general, complex quantities, and on instabilities [22].

Expressions (23) and (24) are equivalent to each other in their physical meanings as it can be recognized by.comparing their corresponding eigenvalues, but Expression (25) is quite different from those two expressions in its physical reality, even though it is equivalent to the other two expressions in its mathematical form if one disregards differences between the corresponding terms in the matrices and pays only attention to the number of null elements and its positions in the matrices, and this is due to the fact that Expression (25) is for the case of the longitudinally applied field. The same arguments hold good with the relations between Expressions (26), (27) and (28), too.

The permittivity tensor has connection with the conductivity tensor as

$$\underline{\underline{\epsilon}}=\epsilon_l\underline{\underline{1}}+\frac{\underline{\underline{\sigma}}}{jw\epsilon_0} \qquad , \qquad (29)$$

where $\underline{\underline{\sigma}}$ is the conductivity tensor, and the tensor (25) can be splitted as

$$\underline{\underline{\epsilon}}=\epsilon_l\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}-B\begin{pmatrix}C'&0&0\\0&C'&0\\0&0&F'\end{pmatrix}+jBD'\begin{pmatrix}0&1&0\\-1&0&0\\0&0&0\end{pmatrix} \qquad (30)$$

In the absence of magnetic fields the transport equation in the relaxation time approximation for a longitudinal phonon in a free-electron gas is

$$\frac{\partial f}{\partial t}+\bar{v}\cdot\nabla f+\bar{F}\cdot\nabla_k f=-\frac{f-f_0}{\tau} \qquad , \qquad (31)$$

where the symbol v is substituted for u for convenience. From Eq. (31) with the definition of the electric current density $\bar{J}$, $\bar{J}$ is given by

$$\bar{J}=\underline{\underline{\sigma}}\cdot\left(\bar{E}+\frac{m\bar{u}}{e\tau}\right)+\text{nec}_s\bar{R} \qquad (32)$$

with $\bar{u}$ for the local lattice velocity, n the perturbed electron density, c_s the sound velocity and $\bar{R}$ the diffusion vector, and the conductivity tensor is

$$\sigma_{\mu\nu}=-\frac{e^2\tau}{4\pi^3}\int d^3k\frac{v_\mu v_\nu}{1-i\omega\tau+i\bar{q}\cdot\bar{v}\tau}\cdot\frac{\partial f_0}{\partial\varepsilon} \qquad (33)$$

with $\bar{q}$ for the wavevector of the phonon. Solving Eq. (33) with the appropriate equilibrium distribution function, i.e., in this case, Fermi distribution fumction, for $\bar{q}$ parallel to the z axis, it will be obtained:

$$\sigma_{xx}=\sigma_{yy}\neq 0 \qquad (34)$$
$$\sigma_{zz}\neq 0$$

and the off-diagonal terms are zero [3]. Therefore, it can be inferred at least that, on the analogy of the results of the foregoing argument, the 3rd term on the right-hand side of Expression (30), that is, the off-diagonal terms in the permittivity tensor, denotes the anisotropic properties of the plasma arising from a result of the existence of the applied magnetostatic field, and this fact is more fully emphasized when Expressions (26), (27) and (28) are observed relating with those of the nonstreaming cases. The motion of charged particles is in the form of a helix with cycloiding around the magnetostatic field lines when an electron plasma is submerged in the magnetostatic field, and the sense of the rotation of the particle is determined by the Lorentz force equation. This is the origin of D_0'' term which differs from D'' and this fact exhibits a striking contrast with the nonstreaming cases.

In Expression (30) the diffusion term is included only in the 2nd term of the right-hand side of the expression and, in this sense, the permittivity tensor (25) is quite different from those of (23) and (24) in its physical characteristics, and in the latters the diffusion terms are in both of the

2nd and 3rd terms as perturbation. Therefore, when the directions of the wave vector and the magnetostatic field are same, the effect of diffusion appears only in ϵ_{zz} element and such a mutual relation holds for in the streaming cases, i.e., among Expressions (26), (27) and (28).

The above situation shows that the diffusion effects occur only in the longitudinal direction in the plasma when the field and the wave vector are in the same direction whether the plasma is in the state of streaming or not.

The Galilean transformation [23] is applicable for nonrelativistic translations and the problems dealt herein are indeed in this case, since, e.g., the expression C can be obtained from the C'' by simply replacing $(\omega - v_0 k)$ by ω. Therefore, in some plasma problems, the characteristics of streaming states can be deduced from that of nonstreaming states.

Substituting the tensor (30) into the constitutive relation $\bar{D} = \underline{\epsilon} \cdot \bar{E}$ [14] it is obtained:

$$\bar{D} = \epsilon_l \bar{E} - (BC' (\hat{x}\hat{x}\cdot + \hat{y}\hat{y}\cdot) + BF'\hat{z}\hat{z}\cdot)\bar{E} - jBD'\hat{z} \times \bar{E} \tag{35}$$

and Expression (36) is an alternative statement of the constitutive relation for the tensor (30), and the states and the degrees of perturbations are understandable in another representation from the said expression.

B. Dispersion Relations

Next, using Equation (19), the dispersion relations in which the diffusion effects are included can be derived. Because of the existence of the intimate relation between a permittivity and a dispersion equation and from the results of the foregoing symmetry considerations on the permittivity tensors, consideration is only required for the cases where the magnetostatic fields are in the x and z directions, respectively, when the wave vector is in the z direction.

For the case where the plasma is in the state of streaming, and its direction of the uniform flow is in the z direction with the wave vector and the magnetostatic field in the z and x directions, respectively, the following dispersion equation is obtained from Equation (19):

$$\begin{vmatrix} 1+A-BF'' & 0 & 0 \\ 0 & 1+A-BC'' & jBD'' \\ 0 & -jBD_0'' & 1-BE'' \end{vmatrix} = 0, \tag{36}$$

where

$$A = -\frac{k^2}{\omega^2 \epsilon \mu_0},$$

and for the case where both of the vector and the field are in the z direction, from the same equation, the following result is obtained:

$$\begin{vmatrix} 1+A-BC''' & jBD''' & 0 \\ -jBD''' & 1+A-BC''' & 0 \\ 0 & 0 & 1-BF''' \end{vmatrix} = 0. \tag{37}$$

The dispersion equations (36) and (37) indicate that there exist three kinds of waves which can propagate through in the plasma as far as the specific physical models chosen are concerned, and these are linearly and circularly polarized and longitudinal electrokinetic plane waves and these waves are the normal modes of propagation in the medium. It is obvious from the equations to see that there is no influences on the characteristics of the propagation of the linearly and the circularly polarized plane waves due to the diffusion effect of the medium. For the case where $\omega - kv_0 = 0$ the dispersion relations for both the linearly and circularly polarized plane waves become the one for transverse plane waves in free space except for its difference in permittivities and this fact shows partially the validity of the expressions obtained, and the validity will be more fully emphasized by the numerical results of the computer calculations comparing these results with the available publications or by the analytical methods.

The solution of the dispersion equation may, in general, yield growing, evanescent and decaying waves. To distingish between these waves which can be existed in an system the instabilities of the system must be investigated. For identifying an absolute instability, δ function of both time and space is taken as the source function to the system. Then the responce of the system can be written as

$$R_{(t,z)} = \frac{1}{(2\pi)^2} \int_{C_L} \int_{C_F} G_{(\omega,k)} e^{j(\omega t - kz)} dk d\omega, \tag{38}$$

where C_L is the Laplace contour and C_F the Fourier one, and the inversion of the Green's function $G_{(\omega,k)}$ becomes $D_{(\omega,k)}^{-1}$. For identifying a convective instability a source function of $\delta(z)e^{j\omega_r t}$ is taken and, in this case, the response of the system becomes as

$$R_{(t,z)}=\frac{1}{(2\pi)^2}\int_{C_L}\int_{C_F}G_{(\omega,k)}\frac{e^{j(\omega t-kz)}}{j(\omega-\omega_r)}dkd\omega \quad , \tag{39}$$

and the relation $D_{(\omega,k)}^{-1}=G_{(\omega,k)}$ is still held. Therefore, as a result, the distinction between the waves can be achieved by the investigation of the singularities of $D_{(\omega,k)}^{-1}$ since the singularities and the instabilities are closely related with each other, and the types of the instabilities can be determined by the positions of the singularities in the contour planes, and as a consequence of the foregoing results the types of waves can be distinguished each other. For example, one of the zeros of the dispersion relation for the case of the linearly polarized plane wave is expressed as

$$k=G+H-J/3 \quad , \tag{40}$$

where

$$G=(K+(L+K)^{1/2})^{1/2};H=(K-(L+K)^{1/2})^{1/2}$$
$$J=-(\omega-j10\omega_c)/v_0$$
$$K=[9\mu_0 v_0^2\{\omega^3\epsilon-\omega\rho_0\eta-j10\omega_c(\omega^2\epsilon-\rho_0\eta)\}$$
$$-27\mu_0 v_0^2\omega(\omega^2\epsilon-\rho_0\eta-j10\omega\omega_c\epsilon)+2(\omega^3$$
$$-300\omega\omega_c^2-j10\omega_c\{3\omega^2-100\omega_c^2\})]/54v_0^3$$
$$L=[3\mu_0 v_0^2(\rho_0\eta-\omega^2\epsilon)-\omega^2+100\omega_c^2+j20\omega\omega_c]/9v_0^2$$

C. Numerical Results

If a state of any medium is represented by the function

$$\Psi_{(\bar r,t)}=\Psi_0\exp j(\omega t-\bar k_r\cdot\bar r) \quad , \tag{41}$$

then the surfaces of constant state are defined by

$$\omega t-\bar k_r\cdot\bar r=K \quad , \tag{42}$$

where K is a constant, and the phase velocity with which these surfaces are propagated is defined by Eq. (42). For the case where the directions of the wave vector and displacement are same, the phase velocity can be expressed as

$$v_p=\frac{\omega}{k_r} \quad . \tag{43}$$

Next, if a state of any medium is represented by the function

$$\Psi_{(\bar r,t)}=\Psi_{0(k_r)}\exp j(\omega t-\bar k_r\cdot\bar r)dk_r \quad , \tag{44}$$

then, if the range of values for k is small and centered about some specific value kr_0

$$kr_0-\delta kr\lessgtr kr\lessgtr kr_0+\delta kr \quad , \tag{45}$$

Eq. (44)may be replaced by

$$\Psi_{(\bar r,t)}=\int_{kr_0-\delta kr}^{kr_0+\delta kr}\Psi_0(k_r)\exp j(\omega t-\bar k_r\cdot\bar r)dk_r \tag{46}$$

with

$$\omega(kr)=\omega(kr_0)+\left(\frac{\partial\omega}{\partial kr}\right)_{kr_0}\delta kr+\frac{1}{2}\left(\frac{\partial^2\omega}{\partial k_r^2}\right)_{kr_0}(\delta k_r)^2+\cdots\cdots \quad , \tag{47}$$

If the expressions for k_r and ω are subsuituted into Eq. (46), then, for the case where the directions of the wave vector and displacement are the same, the wave packet can be represented by

$$\Psi_{(z,t)}=\exp j[\omega_{(k_{r0})}t-k_{r0}z]\cdot\varphi_{(z,t)} \quad , \tag{48}$$

where

$$\varphi_{(z,t)}=\int_{kr_0-\delta k_r}^{kr_0+\delta k_r}\Psi_{0(k_r)}\exp j(-\delta k_r)\left[z-\left(\frac{\partial\omega}{\partial k_r}\right)_{kr_0}t-\frac{1}{2}\left(\frac{\partial^2\omega}{\partial k_r^2}\right)_{kr_0}\delta k_r t-\cdots\right]dk_r \tag{49}$$

If the total range of wave numbers is small, so that for all δk_r

$$\frac{1}{2}\left(\frac{\partial^2\omega}{\partial k_r^2}\right)_{kr_0}\delta k_r^2 t\ll 1 \quad ,$$

then the planes of constant packet amplitude are defined by

$$z-\left(\frac{\partial\omega}{\partial k_r}\right)_{kr_0}t=K \quad , \tag{50}$$

where K is a constant, and the group velocity can be expressed as

$$v_g=\left(\frac{\partial\omega}{\partial k_r}\right)_{kr_0} \quad . \tag{51}$$

Since the concept of the group velocity is wholly preeise only when the wave packet is composed of elementary waves lying within an infinitely narrow region of the spectrum [24], it seems that the result of the concept is quite suitable in conformity with using in the discussion. If the dispersion of a medium is normal, a wave packet may travel a great distance without appreciable diffusion, and since the energy is presumed to be localized in the region occupied by the field it is obvious that the velocity of energy propagation must be approximately equal to the group velocity. If, on the contrary, the

TABLE I
A PORTION OF DATA USED (1)

v_0	ω_c	ω	k_{lin}	k_{cir}	$k_{lon}v_{T=0}$	$k_{lon}v_{T=10^5}$
0.0E+00	0.0E+00	0.1E+09	0.1348E+05+ j 0.0000E+00	0.1348E+05+ j 0.0000E+00	No Roots Can be Found.	0.1231E+04+ j 0.4679E−02
0.0E+00	0.0E+00	0.1E+11	0.1348E+05+ j 0.0000E+00	0.1348E+05+ j 0.0000E+00	〃	0.1231E+05+ j 0.0000E+00
0.0E+00	0.0E+00	0.1E+13	0.1897E+05+ j 0.0000E+00	0.1897E+05+ j 0.0000E+00	〃	0.1417E+08+ j −0.0000E+00
0.0E+00	0.1E+10	0.1E+09	0.9578E+03+ j 0.9483E+03	0.1001E+04+ j 0.8968E+03	〃	0.8747E+04+ j −0.8660E+04
0.0E+00	0.1E+10	0.1E+11	0.1047E+05+ j 0.4337E+04	0.1031E+05+ j 0.3986E+04	〃	0.1352E+06+ j −0.5602E+05
0.0E+00	0.1E+10	0.1E+13	0.1897E+05+ j 0.4789E+02	0.1896E+05+ j 0.4781E+02	〃	0.1417E+08+ j −0.8244E+05
0.0E+00	0.1E+11	0.1E+09	0.3015E+03+ j 0.3012E+03	0.3154E+03+ j 0.2851E+03	〃	0.2754E+05+ j −0.2751E+05
0.0E+00	0.1E+11	0.1E+11	0.3154E+04+ j 0.2851E+04	0.3266E+04+ j 0.2675E+04	〃	0.2893E+06+ j −0.2619E+06
0.0E+00	0.1E+1 1	0.1E+13	0.1892E+05+ j 0.4752E+03	0.1888E+05+ j 0.4671E+03	〃	0.1418E+08+ j −0.8237E+06
0.0E+00	0.1E+14	0.1E+06	0.3014E+00+ j 0.3014E+C0	0.3152E+C0+ j 0.2853E+C0	〃	0.2752E+05+ j −0.2752E+05
0.0E+00	0.1E+14	0.1E+08	0.3015E+01+ j 0.3012E+01	0.3154E+01+ j 0.2851E+01	〃	0.2752E+06+ j −0.2753E+06
0.0E+00	0.1E+14	0.1E+10	0.3165E+02+ j 0.2870E+02	0.3310E+02+ j 0.2717E+02	〃	0.2690E+07+ j −0.2823E+07
0.0E+00	0.1E+14	0.1E+12	0.1336E+04+ j 0.6798E+02	0.1343E+04+ j 0.6696E+02	〃	−0.3654E+08+ j 0.4025E+08
0.0E+00	0.1E+14	0.1E+14	0.1334E+06+ j 0.6740E+02	0.1334E+06+ j 0.6545E+02	〃	−0.4069E+09+ j 0.3686E+09
0.1E+05	0.0E+00	0.1E+09	0.3339E+04+ j 0.1508E+07	0.3333E+04+ j 0.1508E+07	0.3849E+30+ j 0.1209E+25	0.4836E+22+ j −0.1047E+28
0.1E+05	0.0E+00	0.1E+11	0.3849E+09+ j 0.1448E+04	0.3849E+09+ j 0.1448E+04	0.3849E+30+ j 0.1209+25	0.1496E+22+ j 0.1047E+28
0.1E+05	0.0E+00	0.1E+13	0.3849E+12+ j 0.0000E+00	0.3849E+12+ j 0.0000E+00	0.3849E+30+ j 0.1209E+25	0.1496E+22+ j −1047E+28
0.1E+05	0.1E+10	0.1E+09	0.2680E+09+ j 0.2758E+09	0.2259E+09+ j 0.3154E+09	0.3849E+30+ j −0.1209E+25	0.4836E+22+ j −0.1047E+28
0.1E+05	0.1E+10	0.1E+11	0.2482E+09+ j −0.5985E+09	0.3127E+09+ j −0.6244E+09	0.3849E+30+ j −0.1209E+25	0.1496E+22+ j −0.1047E+28
0.1E+05	0.1E+10	0.1E+13	0.3849E+12+ j −0.5774E+10	0.3855E+12+ j −0.5777E+10	0.3849E+30+ j −0.1209E+25	0.1496E+22+ j −0.1047E+28
0.1E+05	0.1E+11	0.1E+09	0.8594E+10+ j 0.8616E+10	0.7270E+10+ j 0.9875E+10	0.3849E+30+ j −0.1209E+25	0.4836E+22+ j −0.1047E+28

v_e	ω_c	ω	k_{iin}	k_{iir}	$k_{ion\,v_T=0}$	$k_{ion\,v_T=10^5}$
0.1E+05	0.1E+11	0.1E+11	0.7283E10+ j0.9863E+10	0.5892E+10+ j0.1106E+11	0.3849E+30+ j −0.1209E+25	0.1496E+22+ j −0.1047E+28
0.1E+05	0.1E+11	0.1E+13	0.3835E+12+ j −0.5776E+11	0.3893E+12+ j −0.5805E+11	0.3849E+30+ j −0.1209E+25	0.1496E+22+ j −1047E+28
0.1E+05	0.1E+14	0.1E+06	0.2722E+15+ j0.2722E+15	0.2303E+15+ j0.3120E+15	0.3849E+30+ j −0.1209E+25	−0.6055E+18+ j0.1046E+28
0.1E+05	0.1E+14	0.1E+08	0.2722E+15+ j0.2722E+15	0.2303E+15+ j0.3120E+15	0.3849E+30+ j −0.6667E+10	0.1185E+23+ j −0.1047E+28
0.1E+05	0.1E+14	0.1E+10	0.2722E+15+ j0.2722E+15	0.2303E+15+ j0.3120E+15	0.3849E+30+ j −0.6667E+10	0.5057E+22+ j −0.1047E+28
0.1E+05	0.1E+14	0.1E+12	0.2718E+15+ j0.2726E+15	0.2299E+15+ j0.3124E+15	0.3849E+30+ j −0.6667E+10	−0.3340E+22+ j0.1047E+28
0.1E+05	0.1E+14	0.1E+14	0.2303E+15+ j0.3120E+15	0.1863E+15+ j0.3499E+05	0.3849E+30+ j0.1209E+25	0.4836E+22+ j −0.1047E+28
0.1E+06	0.0E+00	0.1E+09	0.3347E+03+ j0.4920E+06	0.3333E+03+ j0.4920E+06	0.1217E+29+ j0.0000E+00	0.2236E+29+ j0.0000E+00
0.1E+06	0.0E+00	0.1E+11	0.1117E+08+ j0.0000E+00	0.1117E+08+ j0.0000E+00	0.1217E+29+ j0.0000E+00	0.2236E+29+ j0.0000E+00
0.1E+06	0.0E+00	0.1E+13	0.1217E+11+ j0.4634E+05	0.1217E+11+ j0.4634E+05	0.1217E+29+ j0.3778E+23	0.2236E+29+ j0.0000E+00
0.1E+06	0.1E+10	0.1E+09	0.8110E+07+ j0.8360E+07	0.6770E+07+ j0.9634E+07	0.1217E+29+ j0.6667E+05	0.2236E+29+ j −0.8333E+05
0.1E+06	0.1E+10	0.1E+11	0.8247E+07+ j −0.1946E+08	0.1023E+08+ j −0.2029E+08	0.1217E+29+ j −0.6667E+05	0.2236E+29+ j −0.8333E+05
0.1E+06	0.1E+10	0.1E+13	0.1217E+11+ j −0.1826E+09	0.1219E+11+ j −0.1827E+09	0.1217E+29+ j −0.3778E+23	0.2236E+29+ j −0.8333E+05
0.1E+06	0.1E+11	0.1E+09	0.2716+09+ j 0.2721E+09	0.2298E+09+ j0.3119E+09	0.1217E+29+ j −0.6667E+06	0.2236E+29+ j −0.8333E+06
0.1E+06	0.1E+11	0.1E+11	0.2302E+09+ j0.3116E+09	0.1863E+09+ j0.3495E+09	0.1217E+29+ j −0.6667E+06	0.2236E+29+ j −0.8333E+060
0.1E+06	0.1E+11	0.1E+13	0.1213E+11+ j0.1827E+10	0.1231E+11+ j −0.1836E+10	0.1217E+29+ j −0.3778E+23	0.2236+29+ j −0.8333E+06
0.1E+06	0.1E+14	0.1E+06	0.8607E+13+ j0.8606E+13	0.7283E+13+ j0.9866E+13	0.1217E29+ j −0.3778E+23	0.2236E+29+ j −0.7556E+23
0.1E+06	0.1E+14	0.1E+08	0.8607E+13+ j0.8606E+13	0.7283E+13+ j0.9866E+13	0.1217E+29+ j −0.3778E+23	0.2236E+29+ j −0.8333E+09
0.1E+06	0.1E+14	0.1E+10	0.8607E+13+ j0.8606E+13	0.7283E+13+ j0.9866E+13	0.1217E+29+ j −0.6667E+09	0.2236E+29+ j −0.8333E+09
0.1E+06	0.1E+14	0.1E+12	0.8594E+13+ j0.8619E+13	0.7269E+13+ j0.9878E+13	0.1217E+29+ j0.3778E+23	0.2236E+29+ j −0.8333E+09
0.1E+06	0.1E+14	0.1E+14	0.7283E+13+ j0.9866E+13	0.5892E+13+ j0.1106E+14	0.1217E+29+ j −0.3778E+23	0.2236E+29+ j −0.8333E+09

dispersion is anomalous, the identity of group velocity and energy velocity in the system cannot be usually existed. Therefore, in the case of normal dispersion, the velocity of energy propagation is equivalent to the group velocity. The above mentioned Expressions (43) and (51) play an important role in discussing and analyzing the numerical results obtained graphically.

Some of the computer (results used in the discussion for the waves mentioned previously are given in Tables I (In the Table, k_{lin}, k_{cir} and k_{lon} stand for the wave vectors of linearly and circularly polarized and longitudinal electrokinetic plane waves.) and II for the case of $T\approx 77°K$ and of the typical range of the other parameters of the n-type extrinsic semiconductors The data for the circularly polarized plane waves are those for the left-handed circularly polarized plane waves $[\bar{E}_+=E_+(\hat{x}-j\hat{y})]$. It is very clear to see from Table I that for both the linearly and the circularly polarized plane waves the tendency of the variation of the wave vector vs. the angular frequency and applied field is quite similar in both cases except for the magnitude of each corresponding wave vector. (Notice some of the data are not shown in Tables I and II because of their great quantities.) These similarlities in the variation stem from the fact that the both waves are transverse ones. In the case of the circularly polarized plane waves $\frac{d\omega}{dk_r}$ is infinite in the region of $\omega=10^5$ to 5.10^{10} and show some deviation from an infinity $\omega=10^{11}$ up to 10^{14} for $v_o=0$ and $\omega_c=0$ as far as the data show. Therefore, it shows that the dispersion is an anomalous one in the region of $\omega=10^5$ to 5.10^{10} and it becomes a normal dispersion above this region, that is, a transition occurs. At $v_o=0$ and $\omega_c=10^{12}$ the dispersion digram for k_r are nearly parabolic and the deviation from the parabolic variation is due to collisions and

T A B L E II

A PORTION OF DATA USED (2)

ω_C	ω	$k_{v_T=1.7\cdot 10^4}$	$k_{v_T=1.75\cdot 10^4}$
0.1E+02	0.1E+06	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33
0.1E+02	0.3E+06	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33
0.1E+02	0.1E+07	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33
0.1E+02	0.3E+07	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33
0.1E+02	0.1E+08	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+02	0.3E+08	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+02	0.1E+09	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+02	0.1E+09	0.5482E+32+ j −0.9424E−01	--0.1600E+22+ j 0.1280E+33
0.1E+02	0.1E+10	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33
0.1E+02	0.3E+10	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33
0.1E+02	0.1E+11	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+02	0.3E+11	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+02	0.1E+12	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+02	0.3E+12	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+02	0.1E+13	0.5482E+32+ j −0.1547E+27	--0.1600E+22+ j 0.1280E+33
0.1E+02	0.3E+13	0.5482E+32+ j −0.9424E−01	0.4377E+27+ j 0.1280E+33
0.1E+02	0.1E+14	0.5482E+32+ j −0.9424E−01	0.4377E+27+ j 0.1280E+33
0.1E+02	0.3E+14	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33
0.1E+02	0.1E+15	0.5482E+32+ j −0.1547E+27	--0.1600E+22+ j 0.1280E+33
0.1E+13	0.1E+06	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33
0.1E+13	0.3E+06	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33

ω_C	ω	$k_{v_T=1.7\cdot10^4}$	$k_{v_T=1.75\cdot10^4}$
0.1E+13	0.1E+07	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33
0.1E+13	0.3E+07	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33
0.1E+13	0.1E+08	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+13	0.3E+08	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+13	0.1E+09	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+13	0.3E+09	0.5482E+32+ j −0.9424E+10	−0.1600E+27+ j 0.1280E+33
0.1E+13	0.1E+10	0.5482E+32+ j −0.1547E+27	−0.1600E+27+ j 0.1280E+33
0.1E+13	0.3E+10	0.5482E+32+ j −0.1547E+27	−0.1600E+27+ j 0.1280E+33
0.1E+13	0.1E+11	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+13	0.3E+11	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+13	0.1E+12	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+13	0.3E+12	0.5482E+32+ j −0.1547E+27	0.4377E+27+ j 0.1280E+33
0.1E+13	0.1E+13	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33
0.1E+13	0.3E+13	0.5482E+32+ j −0.9424E+10	0.4377E+27+ j 0.1280E+33
0.1E+13	0.1E+14	0.5482E+32+ j −0.9424E+10	0.4377E+27+ j 0.1280E+33
0.1E+13	0.3E+14	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33
0.1E+13	0.1E+15	0.5482E+32+ j −0.1547E+27	−0.1600E+22+ j 0.1280E+33

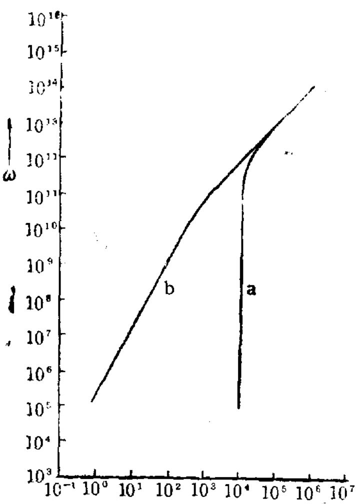

Fig. 1. Dispersion diagrams for left-handed circularly polarized plane waves: (a) $v_o=\omega_c=0$, (b) $v_o=0$ and $\omega_C=10^{12}$

applied fields, and the phase velocity becomes very large for high frequencies in comparing with those for the case of low frequencies, and, in this case, the dispersion is a normal one, and the direction of power flow is same as that of wave propagation, i.e., the waves are positive-energy-carring waves (Fig. 1). In the streaming cases for the circularly polarized plane waves the magnitude of k_r is in proportion to the 3/2nds power of the field intensity in the range of $\omega_c=10^9$ to 10^{14} for $v_o=10^4$ and 10^5, and $\omega=10^8$ and 10^{12}, respectively, (Fig. 2), and the dispersion is, in the same manner with those for the nonstreaming case, an anomalous one up to $\omega=10^{11}$ and as further the frequency increases the dispersion becomes a normal one.

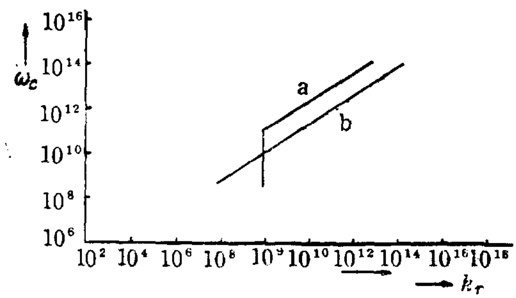

Fig. 2. Magnetic field vs. wave vector relations for left-handed circularly polarized plane waves: (a) $V_o=10^4$ and $\omega=10^8$, (b) $V_o=10^5$ and $\omega=10^{12}$

In the case of the longitudinal electrokinetic plane waves there exist nonpropagating electrostatic waves as it can be seen from Table I for $v_o=0$ and $v_T=0$ ($v_T=0$ is assumed to comparing with the case of $v_T\neq0$), and all the waves are characterized in their dispersive properties by the anomalous dispersion when $v_T=0$ and $v_o\neq 0$, since $\frac{d\omega}{dk_r}$ is infinite everywhere. But it is obvious that the waves are normally dispersive, for example, for $v_T=10^5$, $v_o=0$ and $\omega_C=0$, since, in this example, the dispersion relation is expressed graphically by the straight line. For $v_T=10^5$, $v_o=0$ and $\omega_C=10^{12}$ the relation is expressed as a parabolic curve in the figure and the waves are anomalously dispersive in lower frequency regions and normally dispersive in the other frequency regions (Fig.3). For $v_T=10^5$, $v_o\neq 0$ and $\omega_C\neq 0$ no variation of k_r vs. the frequency is

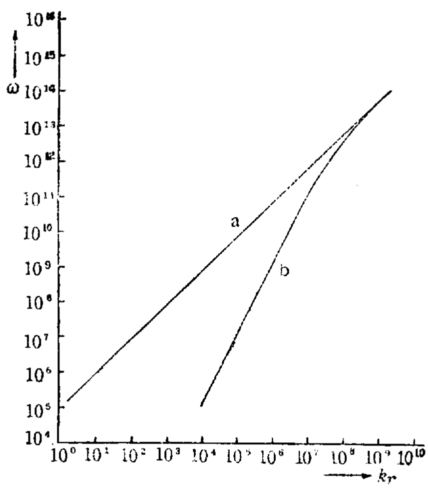

Fig. 3. Dispersion diagram for the longitudinally polarized plane waves: (a) $v_o=\omega_c=0$, (b) $v_o=0$ and $\omega_c=10^{12}$

observed, and furthermore no variation of k_r can be observed when the applied field intensities are varied, and this result seems to be a physically reasonable, since the result is for the longitudinal electrokinetic plane waves. It is very interesting to observe the transition region where the characteristics of the variation of the wave vector are quite different before and behind the region. For streaming cases with $v_o=10^4$ in the dispersion relation, for example, k_r is a constant for $1.5\ 10^4\leq v_T\leq 1.7\cdot 10^4$, whereas k_i is a constant for $1.95\cdot 10^4\geq v_T\geq 1.75\cdot 10^4$ (Table Ⅱ), and the numerical results suggest that the transitions do not occur in the same manner as the above whenever the parameters are varied. As the parameters indicate it seems that the origin of the existence of the transition is due to the thermal velocity of the charged particle and its environmental circumstances, i.e., lattice vibrations. In the above example the phase velocity of the wave are proportional to the angular frequency in the region of $1.5\cdot 10^4\leq v_T\leq 1.7\cdot 10^4$

For the nonstreaming cases the magnitude of the imaginary part of the wave vector of the circularly polarized plane wave shows no uniformity in its variation as the field and angular frequency vary. On the other hand, however, for the case of streaming ($v_0=10^4$ and 10^5) the variation cannot be observed in the range of $\omega\approx 10^5$ to 10^{11}, i.e., the magnitude of the imaginary part shows a constant value and a deviation from this value occurs at the higher angular frequencies. It can also be observed that the magnitude of the imaginary part increases in proportion as the applied magnetic field does throughout the whole range of v_0. For the case of the longitidinal electrokinetic plane waves, the magnitude of the imaginaly part varies randomly in the dispersion diagram at $v_T=0$ and $v_0=10^4$ and 10^5. At $v_T=10^5$ and $v_0=0$ the magnitude slowly increases as the field strength increases, but, on the contrary, the magnitude is a constant at $v_0=10^4$. This tendency of variation has no more soundness at $v_0=10^5$. The magnitude varies nonuniformly throughout the whole regions indicated in Table Ⅰ, but the magnitude is a constant in the range of $\omega=10^5\sim 10^6$ regardless of the variation of the field strength and increases proportionately with the field in the other portions of the frequency ranges.

IV. SUMMARY AND CONCLUSIONS

The result of the general equation of wave propagation through in the solid state electron plasma in which the diffusion effect are taken into consideration is applied to the specific physical configurations of the streaming and nonstreaming states of the plasmas, and the permittivity tensors and dispersion relations are obtained for the each case and compared analytically with each other to see how the expressions and relations are affected by the effect of diffusion, and for the specific values of the parameters and variables the dispersion relations are examined using the results of the computer calculations and the following conclusions are obtained:

(a) In both the permittivity tensors and dispersion relations in their matrix and deteterminant forms, the diffusion terms do not affect any influences on the positions of the null elements;

(b) In some particular cases the expressions of the magnitude of the off-diagonal terms are different from each other;

(c) In the dispersion relation, the diffusion effect appears only in the case of the space-charge wave so far;

(d) In the dispersion relations the normal and anomalous dispersions coexist in some cases;

(e) The magnitude of the real part of the wave vector is in proportion to the 3/2nds power of the magnetic field in a certain range of the field.

APPENDIX A
THE DERIVATION OF EQUATION (8)

Let $f(\bar{r},\bar{u},t)\,d\bar{r}\,d\bar{u}$ be the number of charged carriers having coordinates lying between $\bar{r}$ and $\bar{r}+d\bar{r}$ and velocities between $\bar{u}$ and $\bar{u}+d\bar{u}$ in phase space. We normalize $f(\bar{r},\bar{u},t)$ according to

$$\int f(\bar{r},\bar{u},t)\,d\bar{u}=n(\bar{r},t) \qquad \text{(A—1)}$$

where $n(\bar{r},t)$ is the number density of the carriers in the plasma at $\bar{r}$ and t. The equation for f can be written in the form

$$Df=\left(\frac{\partial f}{\partial t}\right)_c \quad , \qquad \text{(A—2)}$$

with

$$D\equiv\frac{\partial}{\partial t}+\bar{u}\cdot\frac{\partial}{\partial\bar{r}}+\frac{\bar{F}}{m}\cdot\frac{\partial}{\partial\bar{u}} \quad ,$$

where $\bar{F}$ is the external force and m the mass of the carrier.

On multiplying Eq. (A—2) by ϕ, which is any property associated with the carrier, and integrating throughout the velocity-space, we have

$$\int\phi\,Df\,d\bar{u}=n\Delta\langle\phi\rangle \quad , \qquad \text{(A—3)}$$

where

$$n\Delta\langle\phi\rangle\equiv\int\phi\left(\frac{\partial f}{\partial t}\right)_c d\bar{u}$$

and Eq. (A—3) can be rewritten after some manipulations as

$$\frac{\partial}{\partial t}(n\langle\phi\rangle)+\frac{\partial}{\partial\bar{r}}\cdot n\langle\phi\bar{u}\rangle-n\Big(<\frac{\partial\phi}{\partial t}> +<\bar{u}\cdot\frac{\partial\phi}{\partial\bar{r}}>+\bar{a}\cdot<\frac{\partial\phi}{\partial\bar{u}}>\Big)=\int\phi\left(\frac{\partial f}{\partial t}\right)_c d\bar{u} \quad , \qquad \text{(A—4)}$$

where

$$\int\phi f\,d\bar{u}\equiv n\langle\phi\rangle \quad ,$$

Let $\bar{v}$ be the mean velocity defined by

$$\bar{v}_{(\bar{r},t)}\equiv\frac{1}{n_{(\bar{r},t)}}\int\bar{u}\,f\,d\bar{u},$$

$\bar{V}$ the peculiar velocity

$$\bar{V}\equiv\bar{u}-\bar{v},$$

then, making the specific stosszahlansatz on the right-hand side of Eq. (A—4), the equation can be written as

$$\frac{d}{dt}(n\langle\phi\rangle)+n\langle\phi\rangle\frac{\partial}{\partial\bar{r}}\cdot\bar{v}+\frac{\partial}{\partial\bar{r}}\cdot n\langle\phi\bar{V}\rangle -n\Big\{<\frac{d\phi}{dt}>+>\cdot\frac{\partial\phi}{\partial\bar{r}}>+\Big(\bar{a}-\frac{d\bar{v}}{dt}\Big)\cdot\langle\frac{\partial\phi}{\partial\bar{V}}\rangle -<\frac{\partial\phi}{\partial\bar{V}}\bar{V}>:\frac{\partial}{\partial\bar{r}}\bar{v}\Big\}=\int\phi\Big\{-\frac{1}{\tau}(f-f_0)\Big\}d\bar{u}, \qquad \text{(A—5)}$$

where f_0 is the equilibrium distribution function in the absence of the external perturbation and τ the relaxation time.

In the case of $\bar{\phi}=m\bar{V}$, Eq. (A—5) becomes

$$\nabla\cdot\underline{P}-\rho\Big(\bar{a}-\frac{d\bar{v}}{dt}\Big)=-\frac{\rho}{\tau}\bar{v} \quad , \qquad \text{(A—6)}$$

where $\rho=mn$, and Eq. (8) follows immediatly Eq. (A—6).

APPENDIX B
THE DERIVATION OF EQUATION (19)

Substitution of Eq. (10) into the third term of the right-hand side of Eq. (14) leads, after some manipulations, to the following result:

$$j\Omega\bar{w}_c x\bar{v}=\eta\frac{W}{\omega}\bar{w}_c x\bar{E}+\bar{w}_c x(\bar{v}x\bar{w}_c) +\frac{\eta}{\omega}(\bar{v}_0\cdot\bar{E})(\bar{w}_c x\bar{k})+\frac{v_T^2}{3\rho_0}\bar{w}_c xj\bar{k}\rho \quad , \qquad \text{(B—1)}$$

where $W\equiv\omega-\bar{k}\cdot\bar{v}_0$. Similarly,

$$j\Omega\bar{w}\cdot\bar{v}=\eta\frac{W}{\omega}\bar{w}_c\cdot\bar{E}+\frac{\eta}{\omega}(\bar{v}_0\cdot\bar{E})(\bar{w}_c\cdot\bar{k}) +\frac{v_T^2}{3\rho_0}\bar{w}_c\cdot j\bar{k}\rho \quad . \qquad \text{(B—2)}$$

By recombining Eqs. (14), (B—1) and (B—2), the following result will be obtained;

$$j\Omega(\Omega^2-\omega_c^2)\bar{v}=\frac{W}{\omega}\eta\{\Omega^2\bar{E}+j\Omega(\bar{w}_c\mathbf{X}\bar{E})-(\bar{w}_c\cdot\bar{E})\bar{w}_c+\frac{\eta}{\omega}(\bar{v}_0\cdot\bar{E})\{\Omega^2\bar{k}-(\bar{w}_c\cdot\bar{k})\bar{w}_c+j\Omega(\bar{w}_c x\bar{k})\}+\frac{jv_T^2\rho}{\rho_0}\{\bar{k}\Omega^2-(\bar{w}_c\cdot\bar{k})\bar{w}_c+j\Omega(\bar{w}_c x\bar{k})\}\quad, \qquad (B-3)$$

From Eqs. (15) and (16)

$$\rho=\frac{1}{W}\bar{k}\cdot(\rho_0\bar{v})\quad, \qquad (B-4)$$

and Eq. (19) is obtained from Eqs. (10), (11), (15), (16), (B—3) and (B—4) .

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