A Model of Sunspots with a Magnetic Monopole-like Field Configuration

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Abstract

Observational implication for a possible presence of a magnetic monopole-like field in the visible layers of sunspots is examined by constructing a magnetostatic model of sunspots with a monopole-like field configuration.

The resulting monopole approximation for a magnetic structure of spots is found to be compatible with the observations within a certain limited range of optical depth, which happens to lie mostly in its visible range.

1. Introduction

It has been known for some time that sunspot magnetic fields in the visible layers appear to be emerging from a hypothetical magnetic monopole embedded deep in sunspot umbrae. Observational evidence for a possible presence of the monopole-like field has been often mentioned in the literature (e.g., Beckers and Schröter (1969)), but the true configuration is known to be far more complex. Some workers such as Mattig (1969), however, claim that the complexity of the observed field may be a consequence of possible magnetic inhomogeneities.

Recently, Mattig(1969) has shown that for a sunspot in horizontal magnetostatic equilibrium, the horizontal pressure difference between the normal photosphere and the spot must be equivalent to about the double of magnetostatic pressure, $B^2/(8\pi)$, in order to account for the observed Wilson depression. What is intriguing most regarding Mattig's result is the fact that an identical result can be obtained from a spot with a magnetic field configuration described by a hypothetical magnetic monopole, located deep in a sunspot umbra (See Equation (8) in Section 2). The present work is to investigate whether or not the observational implication for the possible presence of a magnetic monopole-like field in sunspots is physically compatible with observations. A theoretical test is made by constructing a magnetostatic sunspot model based on a monopole-like field configuration. The resulting physical properties are compared with the observations.

2. Equations

For a single sunspot in horizontal

magnetostatic equilibrium, the horizontal pressure gradient is described by

$$\frac{\partial P}{\partial r}=\frac{B_z}{4\pi}\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}\right) \tag{1}$$

where P is the gas pressure, and B_r and B_z represent the radial and vertical components of the magnetic field. The first term on the right-hand side of the equation arises from the curvature of the lines of force and the second term is due to the magnetostatic pressure.

Under the configuration generated by a hypothetical magnetic monopole(with pole strength, m) located at a depth, $z=Z_M$ deep in the umbra on its axis of symmetry, each component of the magnetic field is given by

$$B_z(r,z)=\frac{m}{(1+\alpha^2)^{3/2}(Z_M-z)^2}=D(\alpha)\xi^2(z) \tag{2}$$

$$B_r(r,z)=\frac{m\alpha}{(1+\alpha^2)^{3/2}(Z_M-z)^2}=\alpha D(\alpha)\xi^2(z) \tag{3}$$

with

$\alpha\equiv r\xi(z), \quad \xi(z)\equiv 1/(Z_M-z)$

and $D(\alpha)=m/(1+\alpha^2)^{3/2}$, (4)

where the z-axis is taken to be perpendicular to the solar surface and directed toward the center of the sun.

Making use of Equations (2), (3) and (4), one finds that

$$\frac{\partial B_r}{\partial z}=-\frac{\partial Bz}{\partial r} \tag{5}$$

since the partial derivatives are found to be

$$\frac{\partial B_r}{\partial z}=\frac{\partial}{\partial z}[\alpha D(\alpha)\xi^2]$$

$$=2\xi\frac{d\xi}{dz}\alpha D(\alpha)+\xi^2 r\frac{d\xi}{dz}\frac{d}{d\alpha}[\alpha D(\alpha)]$$

$$=3m\alpha\xi^3/(1+\alpha^2)^{5/2}$$

$$\frac{\partial B_z}{\partial r}=\frac{\partial}{\partial r}[D(\alpha)\xi^2]=\xi^3\frac{d}{d\alpha}[D(\alpha)]$$

$$=-3m\alpha\xi^3/(1+\alpha^2)^{5/2},$$

Consequently, as indicated by Equation (5), both the magnetostatic and the curvature pressure equally contribute to the horizontal pressure equilibrium of the spot.

With the aid of Equation(5), Equation (1) is further simplified as

$$\frac{\partial}{\partial r}\left[p(r,z)+2\frac{B^2{}_z(r,z)}{8\pi}\right]=0, \tag{6}$$

or

$$p(r,z)=p(0,z)+2\left[\frac{B_z{}^2(0,z)}{8\pi}-\frac{B_z{}^2(r,z)}{8\pi}\right] \tag{7}$$

when $r\to\infty$, Equation (7) becomes

$$\Delta p(z)=[p(\infty,z)-p(0,z)]=2\left[\frac{B_z{}^2(0,z)}{8\pi}\right] \tag{8}$$

since the field strength in the normal photosphere, $B_z(\infty,z)$ vanishes. Accordingly, the vertical field gradient is given by

$$\frac{dB_z(0,z)}{dz}=2\pi g\Delta\rho/B_z(0,z), \tag{9}$$

noting that $d\,\Delta P(z)/dz=g\,\Delta\rho$ (where $\Delta\rho=\rho_\odot-\rho^*$is the horizontal density difference between the normal photosphere and the umbra and g is the surface gravity of the sun).

3. Results and Discussions

Utilizing a typical umbral model for a large sunspot by Yun(1970), the field strength and its vertical gradient (on the umbral axis) are calculated as a function of depth, z by the use of Equations (8) and (9). The results are summarized in Table I. In the table, the first column refers to optical depth, $\tau^*_{5,000}$ at 5,000 Å. The zero point of the geometric scale in the second clumn is taken to be a depth in the normal photosphere where $T=T^\odot{}_{eff}$ (i.e., $T^\odot{}_{eff}=5,780°$K). The third and fourth columns represent the gas pressure, P and the temperature, T for the umbra and the last two columns, for the normal photosphere Yun(1971.) Finally, the computed field strength, $B_z(0,z)$ and its vertical gradient, $dB_z(0,z)/dz$ are presented in the fifth and sixth columns.

In the calculation the Wilson depression of the umbera is taken to be 600 km, adopting an estimate by Wittmann and Schröter (1969). The negative values for the computed field gradients below $\tau^*_{5000}>1$ are due to a decline of the horizontal pressure difference, ΔP after reaching a maximum near optical depth of unity.

As seen from Table I, the computed field gradients, in general, are found to be slightly larger as compared with the observations (e.g., 0.5 gauss/km to 2.0 gauss/km (Bray and Loughhead (1964)), regardless of the adopted value of the Wilson depreesion. The computed field strength is comparable with the observations for a typical sunspot with an umbral area $A_u=100$. However, below $\tau^*_{5000}>1$, the computed strength begins to decrease with depth, indicating that the true sunspot field starts to depart from the monopole-like field configuration below this point. Thus, it appears that the monopole-like configuration is not suitable representation for such a deeper region of the sunspot atmospheres. Nevertheless, as noted from Table I, the monopole-like field configuration yields compatible results with the observations (within the observational uncertainty) for a certain limited range of optical depth, which corresponds mostly to the visible layers. It may be concluded that, for the benefit of its simplicity, the monopole approximation can be utilized to represent the magnetic structure of sunspots, when our interest resides only in the visible layers of sunspots.

Table I. An umbral model of a large sunspot ($A_u=100$)

log τ^*_{5000}	z (km)	log P^*	log T^*	B_z (gauss)	dB_z/dz (gauss/km)	log P_0	log T_0
−2.0	440	4.391	3.56	2,974	3.6	5.862	4.13
−1.0	538	4.906	3.60	3,294	2.7	5.975	4.15
−0.4	604	5.230	3.61	3,430	1.3	6.044	4.16
0.0	650	5.449	3.62	3,450	0.6	6.085	4.17
0.4	696	5.663	3.65	3,364	−4.5	6.134	4.18
0.8	740	5.848	3.71	3,162	−12.1	6.175	4.19

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