Metacompact Subsets

Chung, In Jae

I. Introduction

In this paper term metacompact (countably metacompact) subsets means β-metacompact(β-countably metacompact) subsets. By a point finite filter with respect to a subset is meant a filter of the subset if there is an element in every point finite relative open covering with respect to the subset such that the element meets every members of the filter. Characterizations of β-metacompact subsets are given, using systems of relative closed subsets with respect to a subset of a space as well as using the notion of limit points of point finite filter of a subset.

If we define α-metacompact(α-countably metacompact) and σ-metacompact as well as in definition 1 and definition 2 in the following section, then every α-metacompact subset is σ-metacompact and every α-metacompact(α-countably metacompact) subset is also β-metacompact(β-countably metacompact) But we will prove the following theorems

(1) A closed subset of the interior of a β-metacompact(β-countably metacompact) subset of a space is α-metacompact(α-countably metacompact)

(2) A σ-metacompact, α-countably metacompact subset in a topological space is α-metacompact.

II. Some Definitions and Results.

Definition 1. A subset M of a topological space (X, T) is α-metacompact(σ-metacompact) if every open cover by members of T has an open point finite(σ-point finite) refinement by members of T. Definition 2. A subset M of a topological space (X, T) is α-countably metacompact if every couatably open cover by members of T has an open point finite refinement by members of T. Definition 3. A subset M of a topological space is β-metacompact(β-countably metacompact) if M is a metacompact(countably metacompact) subspace.

Definition 4. A system $\sigma=\{F\}$ of relative closed sets with respect to M is called tangent to point finite relative open coverings with respect to M or simply point finite tangent of M, if in each point finite relative open covering with respect to M there is an element V intersecting all $F\varepsilon\sigma$.

Theorem 1. In order that a subset M of a space be β-metacompact it is necessary and sufficient that each point finite tangent system of M has a nont void intersection.

Proof. Let M be not β-metacompact and let u be a relative open covering of M, which does not have a point finite relative open refinement with respect to M. If v be a point finite relative open covering of M, them there is a $V\varepsilon v$ which does not contained in any member of u. It is clear that $\{C_MU : U\varepsilon u\}$ is a point finite tangent of M and $\bigcap_{U\varepsilon u} C_MU=\phi$.

Conversely let $\sigma=\{F\}$ be a point finite tangent of M and let $\bigcap F=\phi$, $F\varepsilon\sigma$. Then $\{C_MF : F\varepsilon\sigma\}$ is a relative open covering of M. Let u be a point finite relative open refinement with respect to M of $\{C_MF : F\varepsilon\sigma\}$. There is a $V\varepsilon u$ such that $V\cap F \neq \phi$ for all $F\varepsilon\sigma$. On the other hand, since u is a refinement of a covering $\{C_MF : F\varepsilon\sigma\}$, there is a C_MF such that $V\subset C_MF$. This means that $V\cap F=\phi$. This is contradiction to $V\cap F\neq\phi$.

We write $u>v$ if u is a refinement of v. Take V in any point finite relative open covering with respect to subset M of a space. The system $\xi=\{V\}$

thus obtained is directed. This system ξ is called a point finite thread of M if for any two $U\varepsilon\xi\cap u$, $V\varepsilon\xi\cap v$, a $W\varepsilon\xi\cap w$ can be chosen in a point finite relative open covering w with respect to M with $w>u$, $w>v$ for any point finite relative open coverings u, v with respect to M. We shall say that the subset M of a space has property K_p if for every point finite tangent system $\sigma=\{F\}$ of M the sets $U\varepsilon u$ (having common points with all $F\varepsilon\sigma$) can be chosen in such a way as to form a point finite thread.

Lemma 1. If $\xi=\{V\}$ is a point finite thread of a subset M of a regular space and $x\varepsilon\cap(\bar{V}\cap M)$, then all of the neighborhoods N_x in M of the point $x\varepsilon M$ are among the V. In fact, $\cap V=\cap(\bar{V}\cap M)$. For the given N_x, we take smaller N_x' with $\overline{N_x'}\cap M\subset N_x$ and $u=\{N_x, M-(N_x'\cap M)\}$; then $V=N_x$.

It follows from this lemma that the intersection of all elements of a thread cannot contain more than one point.

Theorem 2. A subset M of a regular space is β-metacompact if and only if both of the following conditions are fulfilled.

(a) the subset M has the property K_p

(b) each point finite thread $\xi=\{V\}$ of M has a non-void intersection.

Proof. Let M be β-metacompact, and $\sigma=\{F\}$ a point finite tangent system of M. Then $\bigcap_{F\varepsilon\sigma} F$ contains a point x. In any point finite relative open covering with respect to M take $V\ni x$. The system $\xi=\{V\}$ thus obtained is a point finite thread of M. Let $V\varepsilon\xi\cap u$, $V'\varepsilon\xi\cap u'$ be given in point finite relative open covering u, u'. Let us choose neighborhoods N_x, N_x' of x so that $N_x\cap M\subset V\cap V'$, $N_x'\cap M\subset N_x$, and take the covering $u_1=\{N_x, M-(N_x'\cap M)\}$. Take any point finite relative open covering u'' with respect to M following u, u', u_1; then the set $V''\varepsilon\xi\cap u''$ containing x and contained in some element of u_1, must be contained in N_x; therefore $V''\cap M\subset N_x\cap M\subset V\cap V'$.

Sufficiency: Let $\sigma=\{F\}$ be a point finite tangent system of M and $\xi=\{V\}$ a dual thread with $x\varepsilon\cap V=\cap(\bar{V}\cap M)$. As all V, *i.e* N_x, intersect all $F\varepsilon\sigma$, we have $x\varepsilon\bigcap_{F\varepsilon\sigma} F$ and thus M is β-metacompact.

Difinition 5. A filter $\mathfrak{F}$ of a subset M of a space is called a point finite filter of M if there is an element V in every point finite relative open covering with respect to M such that $V\cap B\neq\phi$ for every $B\varepsilon\ \mathfrak{F}$

Theorem 3. A subset M of a regular space is β-metacompact if and only if it has the property K_p and each point finite filter of M has a limit point.

Proof. If M be β-metacompact, it has property K_p. Let $\mathfrak{F}$ be a point finite filter of M. $\{\bar{B}\cap M: B\varepsilon\mathfrak{F}\}$ is a point finite tangent system of M, so that it has common point x which is a limit point of $\mathfrak{F}$.

Let $\sigma=\{F\}$ be an arbitrary point finite tangent system of M, $\xi=\{V\}$ a point finite dual thread of M. In fact filter (ξ) generated by ξ is a point finite filter of M. By hypothesis, it has a limit point x which is the only limit point of all $\bar{V}\cap M$. Thus by above lemma, all neighborhoods of x are among the V, so that x belongs to all $F\varepsilon\sigma$ and $\bigcap_{F\varepsilon\sigma} F\neq\phi$.

Theorem 4. Let F be a closed subset of the interior, G, of a β-metacompact(β-countably meta compact) subset M of a topological space. Then F is α-metacompact(α-countably metacompact).

Proof. Let u be an open cover of F. The family consisting of $\{U\cap M : U\varepsilon u\}$ and the set M-F is an open cover of M using the relative topology for M and has a point finite open refinement with respect to M. Let v consist of members of this refinement contained in a member of $\{U\cap M : U\varepsilon u\}$. Let $w=\{V\cap G : V\varepsilon v\}$. For $V\varepsilon v$, $V=T\cap M$ where T is open in X. So $W=V\cap G$ is open in X. Hence w is a point finite open refinement of u and F is α-metacompact.

Corollary. A closed subset of a metacompact (countably metacompact) space is α-metacompact

(α-countably metacompact)

Lemma 2. If the covering $\{A_\alpha : \alpha\varepsilon \mathbf{A}\}$ of X has a point finite refinement $\{B_\beta : \beta\varepsilon \mathbf{B}\}$, then it also has a precise point finite refinement $\{C_\alpha : \alpha\varepsilon \mathbf{A}\}$. Furthermore, if each B_β is an open set, then each C_α can be chosen to be an open set also.

We ommit the proof of above lemma because it was proved in [2, p. 162].

Theorm 5. Let M be a σ-metacompact, α-countably metacompact in a topological space (X, T). Then M is α-metacompact.

Proof. Let u be an open cover of M. There is an open σ-point finite refinement of u, $v=\bigcup v_n$ where each v_n is point finite. Let $W_n=\bigcup\{V : V\varepsilon v_n\}$. Then $\{W_n\}$ is a countable open cover of M. The covering $\{W_n\}$ has a countable open point finite refinement $\{T_n\}$ by above lemma. Let $S_n=\{V\cap \dot{T}_n : V\varepsilon v_n\}$, $S=\bigcup S_n$. Any point x in M meets a finite number of $\{T_n\}$, T_{x_1}, , T_{x_n} and also meets a finite number of members of S_{x_2}, , S_{x_n}, ··· respectively. Hence x meets a finite number of members of S.

References

1. C.E. AULL, Paracompact subsets, General topology and its relation to modern analysis and algebra Ⅱ, pp. 45~51, Academic Press, New York and London (1967).
2. DUGUNDJI, JAMES, Topology, pp. 160~169, Allyn and Bacon, Boston (1966).
3. V. PONOMAREV, On paracompact spaces and related questions, General topology and its relation to modern analysis and algebra Ⅰ, pp. 302~306, Academic Press, New York and London (1962).

<요 약>

Metacompact인 부분 집합들을 5가지 형으로 분류하여 β-metacompact인 집합과 그 집합에 관하여 폐집합인 집합들과의 관계를 규명하였고, 위상공간의 임의의 부분집합의 Point finite filter를 정의하여 이 filter가 limit point를 갖는 조건과 이 집합이 β-metacompact인 조건과를 비교하였다.

다음에 5가지형의 metacompact들 사이의 관계를 조사하여 다음과 같은 관계를 얻었다.

(1) β-metacompact (β-countably metacompact)인 부분집합의 내점들의 폐 부분집합은 α-metacompact (α-countably metacompact)이다.

(2) σ-metacompact이고, α-countably metacompact인 부분집합은 α-metacompact이다.

(蔚山工大)

12 page에서 계속

位가 比較的 均等하게 分布되어 있는 것을 前提로 하지 않으면 안된다.

따라서 米穀豫想收穫高調査에 있어서 山間部와 平野部로 區分되어 있는 것과 같이 統計單位의 性質이 地域에 따라 다를 때에는 單純抽出法보다는 層別抽出法이나 集落抽出法을 利用하는 것이 要求된다는 것을 銘心하여야 할 것이다.

亂 數 表

이 表는 H. Burke Horton의 Random Decimal Digits의 一部를 轉載한 것이다.

02946	96520	81881	56247	17623	47441	27821
85697	62000	87957	07258	45054	58410	92081
26734	68426	52067	23123	73700	58730	06111
47829	32353	95941	72169	48374	03905	06865
76603	99339	40571	41186	04981	17531	97372
47526	26522	11045	83515	45639	02485	43905
70100	85732	19741	92951	98832	38188	24080
86819	50200	50889	06493	66638	03619	90906
41614	30074	23403	03656	77580	87772	86877
17930	26194	53836	53692	67125	98175	00912
24649	31845	25736	75231	83808	98997	71829
97899	34061	54308	59358	56462	85166	97301
76801	49594	81002	30397	52728	15101	72070
62567	08480	61873	63162	44873	35302	04511
49723	15275	09399	11211	67352	41526	23497
42658	70183	89417	57656	35370	14915	16560
65080	35569	79392	14937	06081	74957	87787
02906	38119	72407	71417	58478	99297	43519
75153	86376	63852	60557	21211	77299	74967
14192	49525	78844	13664	98964	64425	33536

(昌德女高 敎師)