케이블로 支持하는 空氣膨脹式 플라스틱 溫室의 構造上의 分析

Structural Analysis of An Experimental Cable-Supported Air-Inflated Green House

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적 요

공기 팽창식 이중 플라스틱 온실이 집약 농업 생산에 있어서 그 이용도가 증가되고 있다. 낮은 압력(0.25인치 H_2O)의 공기를 이용하여 이중 플라스틱 막층을 분리시키는 개념은 플라스틱 덮개를 고착시키는데 효과적인 방법을 가능케 하였다.

어떤 형태로던 온실에 있어서 구조체의 부재의 양을 줄인다든가 간소화 하는것은 온실내부에 그늘진 부분의 면적이 줄어든다는 것과 구조물의 초기 비용을 덜어준다는 점에서 이중의 효과를 얻게 된다.

최근에 개발된 시험적인 케이블 지지, 공기 팽창식 플라스틱 덮개와 여러가지 구조 요소의 하중, 응력, 변형 등의 해석을 다루었다.

첫째, 지붕을 이루고 있는 플라스틱 덮개의 처짐량을 계산하는 공식을 개발하였다. 지붕단면의 모양을 긴 원통의 단면으로 가정함으로써 단면의 기하학과 플라스틱 막응력을 추정할 수 있게 하였다.

둘째, 팽창된 이중 플라스틱막을 온실 내부에서 지지하기 위하여 사용된 케이블의 초기 인장과 팽창압력에 따른 최종 인장력 및 변형 처짐을 해석하여 근사 관계식을 개발하였다.

셋째, 케이블 대용으로 개발된 3/4인치 강파이프의 사용이 제시되었고 따라서 하중과 양단 조건에 따른 응력 및 변형 처짐을 계산하여 설계에 사용할 수 있는 공식들을 제공하였다.

넷째, 풍하중이 온실 시스템 전체에 미치는 영향을 재래적인 방법에 의하여 추정하였고 그 결과가 실제적으로 경험할 수 있는 결과와는 상당한 차이가 있음을 지적하였다.

플라스틱 온실 설계에 있어서 관건이 되고 있는 플라스틱 덮개, 지지 케이블 및 지지 강파이프등의 실제 하중 조건 밑에서의 변형의 일부를 측정함으로써 개발된 추정공식들이 설계 목적상 이용이 가능함을 보여주고 있다.

I. Introduction

The utilization of air inflated, double layer plastic green-houses in intensive agricultural production is increasing rapidly. A major reason for the widespread popularity of these structures is their relatively low cost. Recent improvements in their design have demonstrated a trend towards the simplification and minimization of structural members in the framework and towards simpler schemes for attaching the plastic cover. The concept of using air under low pressures(less than 0.25 inches of water) to separate the film layes has enabled the development of efficient schemes for attaching the plastic cover.

Any reduction or simplification of structural members in a greenhouse can have a two fold benefit in that shading in the house is diminished and the initial cost of the structure is reduced. Therefore, a careful analysis of the various structural components is essential if a minimal design is to be achieved. Fortunately, relatively small factors of safety are justified for these components as they are loaded by the plastic film covers which will fail before the framing if the design is adequate. Also, a failure of the cover or a partial failure of structural components does not

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pose as serious a safety hazard as in other types of buildings nor are the economic consequences as severe.

Recently the author has reviewed various types of air inflated and air supported greenhouses.(3) Also, the development of an experimental, modular, cable-supported greenhouse was reported. An analysis of the loads, stresses and deformations of the plastic cover and of the various structural components are discussed here.

In previous work the relationship between the deflection of rectangular sections of plastic films from the plane of support, the internal air pressure, the dimensions of the section and the film stress in the plastic has been presented.(4)

II. The Development

This analysis is based upon the assumption that deflections of the film from the plane of support are relatively small, which is the case in the type of structure shown in Fig. 1. When the length of the rectangular plastic film is at least three times the width the deflection of the plastic film midway between the sides of the rectangular section is given by : (4)

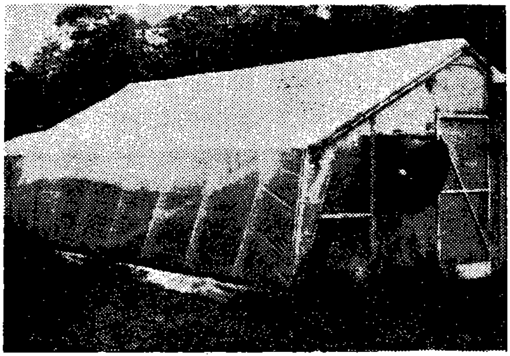


Fig 1. 17′×32′ Typical slant-leg rigid frame greenhouse

(1) $Z=\frac{pa^2}{2S}$

where ; z = deflection, inches
p = pressure between the films, psi
a = onehalf the short side of the rectangle, inches
S = stress in the film lb. per lineal inch

In the case of the cable supprt greenhouse shown in Fig. 2a, 2b, 2c. The outer roof sections are supported in a rectangular plane 12 ft. by 48 ft.

In this structure the maximum deflection of the upper film of 18 inches is significant when compared to the dimensions "a" which is 72 inches. For this situation a new analysis of the relationship between dimensions, deflection, pressure and

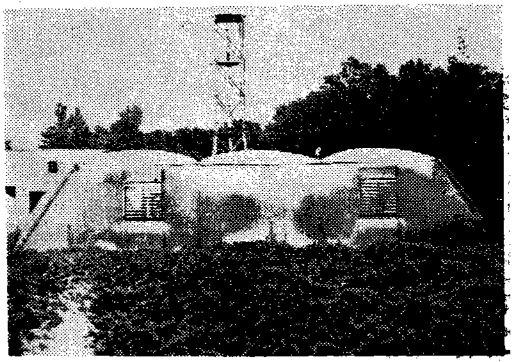
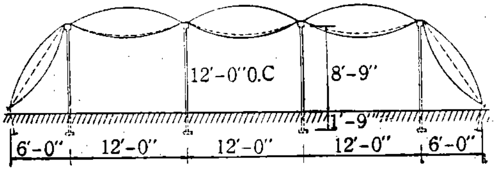


Fig 2a. 48′×48′ Experimental cable-supported air-inflated greenhouse.



Air inflated cable supported plastic film greenhouse

Fig 2b. Cross section of 48′×48′ experimental greenhouse.

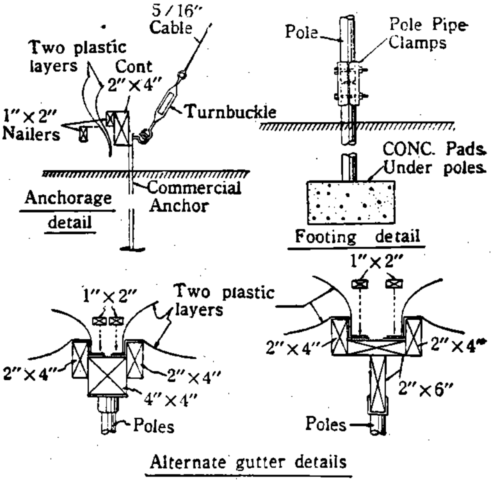


Fig 2c. Construction details of 48′×48′ experimental greenhouse.

stress in the film is needed. Assuming that the shape of a roof section can be approximated by a long section of a cylinder and that the ends are fixed, geometry of the section and the analysis of film stresses can be established. The geometry of the section and identification of the forces, stresses and strains is given in Fig. 2. The symbols not previously defined are :

t=Thickness of the plastic film, inches

R=Radius of curvature of roof section after inflation, inches

A=One half the arc length of the plastic film between gutter after inflation, inches

A'=One half the arc length of the plastic film between the gutter before inflation, inches

α=The angle subtended by the arc A, radians

σ=Tensile stress in the plastic film perpendicular to the gutters after inflation, psi

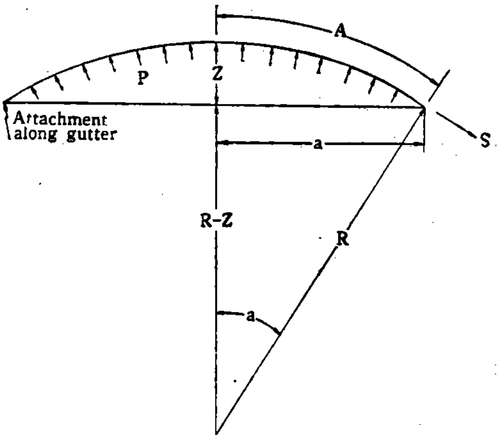
ε=Strain in the palastic film perpendicular to the gutters after inflation, inches/inch

σ_2=Tensile stress in the plastic film parallel to the gutters after inflation, psi

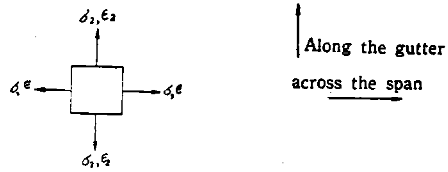
ε_2=Strain in the plastic film parallel to the gutters after inflation, inches/inch

E=Modules of elasticity of the plastic film, psi

μ=Poisson's ratio



Cross section of roof. gutter spacing=2a



element of plastic film

Fig. 3. Geometry and Stresses in a Roof Section of the Cable Support House

For the plastic film to be in equilibrium, the internal pressure must be balanced by the lineal stress in the plastic :

$$(2) \quad S=PR$$

and

$$(3) \quad S=\sigma t$$

Since there can be no strain parallel to the gutters :

$$(4) \quad \varepsilon_2=\frac{\sigma_2}{E}-\mu\frac{\sigma}{E}=0$$

therefore :

$$(5) \quad \sigma_2=\mu\sigma$$

From the geometry shown in Fig. 3.

$$(6) \quad R=\frac{Z^2+a^2}{2Z}$$

$$(7) \quad \alpha=\tan^{-1}\frac{a}{R-Z}=\sin^{-1}\frac{a}{R}$$

$$(8) \quad A=R\alpha$$

The increase in the arc length from A' to A is due to the lineal stress S. Therefore, from the principles of elasticity :

$$(9) \quad A=A'\left[1+\frac{\sigma}{E}(1-\mu^2)\right]$$

In order to have an expression relating the stress and pressure combine equations and to get :

$$(10) \quad \frac{ap}{\sigma t}=\sin\left[\frac{A'p}{t}\left(\frac{1}{\sigma}+\frac{1-\mu^2}{E}\right)\right]$$

Thus, if A' has been determined as well as E, μ, a and t equation(10) relates σ to p. For any value of p, σ can be solved for graphically or by trial and error. Once σ has been computed, R can be found from(2) and (3) :

$$(11) \quad R=\frac{\sigma t}{p}$$

and the deflection Z computed from :

$$(12) \quad Z=R-\sqrt{R^2-a^2}$$

For this structure the span of 12 feet (a=72 inches) was selected to fit nominal lumber sizes and widths of plastic tubing (28 ft. circumference). Next the vertical deflection of 18 inches was selected to give the desired shape. Having picked these two parameters, formula (6), (7) and (8) can be used to compute the radius of curvature R and the final arc length of the plastic film between the gutters. In this case : R=153 inches, A=74.96 inches (2A=149.92). Next the desired operating pressure p is selected, in this case 0.4 in HO (0.0144 psi). From formula (2) the lineal stress S in the plastic film can be calculated to be 2.2 lb/inch. Next the thickness of the film is slected, in this case 5 mil (0.005 inch). The film used was nominally 4 mil but by actual measurement it was found to be about 5 mil. From formula (3) the stress in the film, σ is 440 psi. Since this stress is well below the yield stress of polyethylene film (3,100 to 5,500 psi (3)), nominal 4 mil film is acceptable. Now formula (9) can be used to compute the arc length A′ before inflation. Assuming the elastic modulus E is 40,000 psi (determined by actual tests) and Poisson's ratio is 0.38 the computation gives $A'=74.26$ inches ($2A'=148.52$ inches). As the span, 2a, is 144 inches this means that a total of 4.52 inches of slack should be left in the top layer of plastic when the gutter connections are made fast, assuming that there is no slippage of this connection when the roof is inflated. Application of 0.4 inches of water pressure will stretch the film an additional inch resulting in a final rise, z, of 18 inches. Formula (1) predicts a stress of 415 psi.

Small changes in pressure will have a relatively small effect on R and Z but the effect on stress in the plastic will be almost proportional to the change in the pressure. For example, in the preceeding analysis the maximum anticipated operating pressure of 0.4 inches of water pressure was selected for design purposes. However, the fan which inflates the air space between the roof layers is usually throttled back so that the normal operating pressure is 0.2 inches of water. Formula (10) can be used to compute the stress which is 234 psi in this case. Next formula (11) can be used to calculate the radius of curvature R which is 162 inches. Finally formula (12) can be used to compute the deflection z which is 16.9 inches.

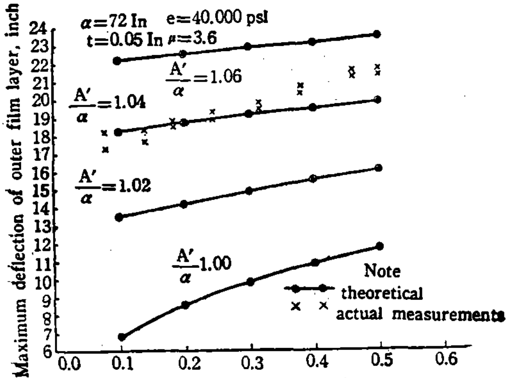


Fig 4. Relationship Between The Deflection Of The Outer-film Layer and The Pressure Between The Layer

Figure 4 shows the relationship between the deflection of the outer film layer and the pressure between the layers for several ratios of A to a. The curves represent predicted relationships and the X's represent actual measurements made on the building. Constants used for the calculations in this figure are those used in the preceeding calculation. There is close agreement between the measured and the predicted values.

Having selected all of the parameters associated with the upper layer of the roof it is necessary to next consider the lower roof layer. As the roof is applied as a single tube clamped along the gutters the thickness of the lower layer is the same as the upper layer. Furthermore, as the span between the gutters it is apparent that both the deflections and stresses in the lower layer be much less than in the upper layer. As the deflections are not as in the upper layer formula can be used to relate the inflation pressure, the stress in the film and the deflections below the plane of the supporting cables.

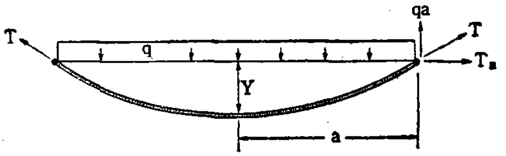


Fig. 5 Lading, Deflection and Forces in the Cable

Next, the design of the the supporting cables can be considered. Each cable span is loaded by the plastic film and is restrained by a tensile force T at each end. An analysis which determines the relationships between the geometry of the roof, the pressure between the roof layers, the tension in the cable, and the maximum deflection of the center of the cable follows.

A section of the cable is shown in Fig. 5. The load q in lb. per inch of cable can be computed by multiplying the pressure between the layers p by the cable spacing, 72 inches in this case. The other terms in the figure are:

a = one half the span length, inches
y = The midspan deflection, inches
T = The tensile force in the cable, pounds
T_H = The horizontal component of T

For the purpose of this analysis it will be assumed that y is small compared to a. In practice this can be achieved by placing an initial tension force T_o on the cable before the load q is applied.

Since each segment of cable is in equilibrium, the tension force T in the cable at the middle is T_H. The vertical component at the end is qa as shown in Fig. 5. Taking moments about the center of the cable and solving for T gives:

$$(13)\quad T_H=\frac{qa^2}{2y}$$

From Fig. 5 the magnitude of the tension force T at the ends is:

$$(14)\quad T=\left[T_H^2+(q\,a)^2\right]^{1/2}$$

Assume that the average tensile force in the cable causing it to stretch is the average of the force at the end and the force in the middle:

$$(15)\quad T_{AV}=\frac{T_H+T}{2}$$

Assume that before any load q is applied a tensile force T_o is applied and that $y=0$. Thus the length of the cable initially is 2a. After the load is applied the stretched length 1 can be approximated by

$$(16)\quad L=2a\left(1+\frac{2y^2}{3a^2}-\frac{2y^4}{5a^4}+\cdots\right)\qquad(5)$$

The increase in length △1 is due to the stretch caused by the increase in T_{AV} above T_O:

$$(17)\quad \triangle L=\frac{2a}{K}(T_{AV}-T_O)$$

If y is small compared to a the fourth order and higher order terms can be neglected giving:

$$(18)\quad \triangle L=\frac{4y^{z}}{3a}$$

Combining the above equations and solving for T_O yields:

$$(19)\quad T_O=\frac{qa^2}{4y}\left(1+\sqrt{\frac{y^2}{a^2}+1}\right)-\frac{2}{3}K\frac{y^2}{a^2}$$

Equation (19) gives the initial tension T_O which should be applied to the cable to limit the deflection y to a desired amount once the design parameters have been selected.

Consider as an example that it is desired that the maximum cable deflection y be 2 inches when the pressure between the layers is 0.2 inches of water, the normal operating pressure in this house. The cable spacing of 6 feet means that q will be 0.52 lb/inch of cable length. For this house a 5/16 inch diameter plastic coated steel cable was used and its initial "spring constant" K was measured and found to be 33,000 lb/inch. Using these values in equation (19) it is found that the initial tension on the cable T_O must be 658 lbs. Using formulas (13) and (14) the maximum cable tension T is found to be 676 lbs. when the pressure is applied.

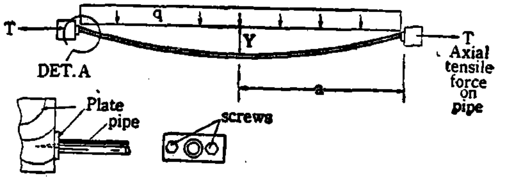


Fig. 6. Loading and Deflection of Pipe Roof Suport

An alternative to the use a cable to support the inner layer of plastic was also developed and used. Sections of 3/4 inch standard pipe were fastened between the gutters as shown in Fig. 6. The load q is due to the air pressure between the film layers and the weight of the pipe itself which is significant. The ends of the pipe are welded to a flat plate which is fastened to the gutter by two log screws as shown in detail A of Fig. 6. For the purpose of design it was assumed that this joint was completely rigid, i.e. the pipe could be considered to have built-in ends. Also, the gutters were assumed to be fixed in place so that there could be no displacement of the ends. Therefore, an axial force T can be developed in the pipe. The following equations for a tie rod with built-in ends and combined axial and uniform lateral loading are taken from Timoshenko (5) or developed from that analysis.

The bending moment at the ends of the pipe is given by:

(20) $M_0 = qa^2\left[\frac{u-\tanh u}{u^2\tanh u}\right]$

Where :

(21) $u = a\left(\frac{T}{EI}\right)^{1/2}$

The moment at the center of the pipe is :

(22) $M_1 = qa^2\left[\frac{\sinh u - u}{u^2 \sinh u}\right]$

For positive values of u the magnitude of M_0 is greater than M_1. The deflection at the center can be approximated by the equation :

(23) $y = \frac{\delta_0}{1+\frac{\mu}{4}}$

where :

(24) $\delta_0 = \frac{1}{384}\ \frac{qa^2}{EI}$

E is the elastic modulus of steel, I is the modulus of the pipe section and :

(25) $\alpha = \frac{4Ta^2}{\Pi^2 EI}$

In order to solve for the tensile force T, the term is computed by use of the following :

(26) $\alpha\left(1+\frac{\alpha}{4}\right)^2 = \frac{\delta_0{}^2 A}{4I}$

where A is the cross sectional area of the pipe.

Ⅲ. Experimental Results

Using this analysis, a design pressure of 0.4 inches of water and the properties of standard 3/4 inch pipe, the predicted midspan deflection of the pipe is found to be 0.825 inches and the maximum combined stress due to the bending moment M and the axial tensile force T is predicted to be 25,000 psi. This design stress is some what higher than the 20,000 psi usually used. However, in the actual structure the end conditions specified in the analysis cannot be met completely. There can be some rotation at the end connection and it is possible for the outer gutters to move a bit towards the center due to stretch in the cable leading diagonally down to the side anchors. Both of these factors will tend to increase the deflections and reduce the stress. In order to evaluate this situation the midspan deflection was recomputed based on three other end conditions: a) lateral deflection but hinged ends, b) rigid ends but no lateral force T, and c) hinged ends and no lateral force T. This last end condition predicts the largest deflection and smallest stress while the other two are intermediate when compared to the completely restrained condition. Actual measurements made on the structure show that deflections are less than those predicted by end conditions (c) but more than those predicted by the other end conditions. A wider house would be expected to have less lateral deflection of the gutters, especially in the interior spans, and the pipe to gutter connections could be made more rigid.

The wind loads on the building have also been analyzed on a preliminary basis. The results of the study so far indicate that more work needs to be done to understand the effects of wind on plastic covered greenhouses, especially with regard to the uplift foces on the roof.

Using the method of the Rutgers Farm building Standard(2) for calculating wind load pressures and assuming an 88 mph wind load, Q is 15 lb/sq. ft. and the following wind loadings are obtained :

Vertically up on the roof 11.5 lb/sq. ft.
on the windward vertical end 15 lb/sq. ft. inward
on the leeward vertical end 8.7 lb/sq.ft. outward
on the sloping side parallel to wind direction 8.7 lb/ sq.ft. outward

If these wind loads were applied to the structure there would be an increase in the stress in the outer layer of plastic equivalent to that produced by a static pressure between the layers of 2.21 inches of water which would almost certainly lift the building from its foundation. However, these effects have not been observed, either on large buildings or on the experimental house which did experience 55 mph winds.

The most probable explanatino for this apparant discrepancy between predictions and experience is that the predicted wind loads are much too high. Inflated plastic buildings have a very mobile outer layer. In high winds one can observe the shape of the outer layer continuously change in response to locally varying wind pressures. It would appear that this shape always tends to be such that the wind pressures normal to the plastic surface are minimized. Therefore, the shape factors to be used in predicting wind loads on inflated plastic buildings should be much smaller than for rigid buildings of similar shape. More work is needed to measure windloads under various conditions so that more realistic criteria can be determined for the design of these structures.

In the event that the snow load on the roof were to exceed the equivalent of 0.4 inches of water, some alteration of the design or action to remove the snow must be instituted. One solution would be to increase the air pressure between the layers to support the weight of the snow. This would require designing the structure for the indicated pressure. Another proposed solution is to pressurize the interior of the building forcing the inner roof layer up against the outer layer. This would essentially double the strength of the roof. More important, it would provide for more rapid heat transfer through the roof there by increasing the rate at which the snow could be melted. Not only does this remove the weight from the structure, but it also eliminates the snow cover which would otherwise shade the house. Assuming a heat transfer coefficient through the roof of 1 BTU/(HRft 20F) and an interior temperature of 75°F the snow could be melted at a rate of about 0.1 inches of water per hour, i.e. about 1 inch of snow per hour. In order to do this it is necessary the ventilation and heating systems be designed so that internal house pressure can be maintained with the heat on. The internal air pressure must be reduced as the snow load is melted off to avoid lifting the entire house.

During some tests of the structure it was discovered that the strength of the connections between the cable and the commercial anchor shown in Fig. 2c can be critical. The screw eye which fastens the turnbuckle to the 24 board was found to straighten out under a load of about 600 lbs. This problem is easily corrected by welding the loop of the eye shut. Once this is done the limiting factor is the cable turnbuckle connection which was found to slip at about 700 lbs. with a single cable clamp. The plastic coating on the cable decreases the holding can be removed from the end of the cable or cables cut to length with swaged can be utilized to eliminate this problem.

Also, the gutters which are loaded by the plastic films and the cables and supported by the poles were analyzed and it was found that the 44 lumber used in this experimental design is more than adequate for the maximum loadings. Further work will be done on the design of a rolled steel or extruded aluminum gutter and a fixture for connecting the gutter to the cables and the poles.

The apparent advantages of this experimental structure are :

1. The minimization of the structural components reduces shade within the greenhouse.
2. The module design simple components reduces labor and material costs for the structural frame.
3. The structural components can provide support for crops which require it.
4. The plastic cover can be easily and rapidly applied or removed without disturbing the crop planted within.

Replication of the 12′ module dimension can result in large areas being covered simply and economically. Bays 96′ long can utilize 100′ rolls of plastic tubing and any number of bays can be placed side by side.

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