論 文

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線形對稱 6點 給電 안테나의 임피이던스에 關하여

(Study on the impedances of 6 points symmetrically-fed linear antenna)

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要 約

길이에 비해서 그 반지름을 무시할 수 있는 線形多波長 안테나의 3쌍의 대칭 給電點에 各雙마다 다른 起電力을 급전하였을 경우의 電流分布式으로 부터 各 給電임피이던스와 相互임피이던스를 求하는 理論式을 구하고 그 給電임피이던스의 計算例를 提示하였다.

ABSTRACT

From the distribution equations of current on a 6 points symmetrically-fed linear antenne, equations of input impedances and mutual impedances were introduced. Although the practical calculation is very tideous, it can seen that it is calculable anyhow, as shown on an example.

1. 서 론

그림 1과 같이 세쌍의 대칭 급전점 1과 4, 2와 3, 3과 6에 각각 E_1, E_2, 및 E_3을 급전한 경우의 電流分布式[1]으로부터 各 雙給電點의 給電임피이던스와 雙給電點相互간의 相互임피이던스

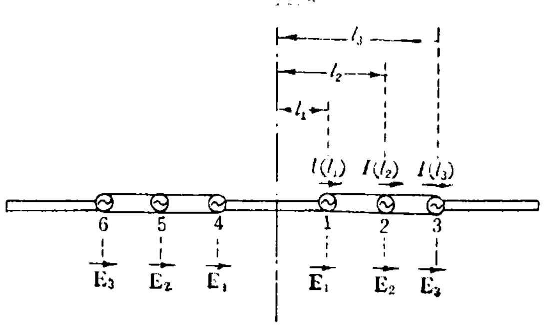

그림 1 6점급전선형 안테나

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式을 解析的으로 유도하였으며 한 예에 대한 급전임피이던스의 계산결과를 다이폴의 안테나의 그것과 비교검토하여 그 신빙성을 확인하였다.

2. 給電임피이던스

그림 1의 안테나에서 점1과 4에만 동일한 기전력 을 급전하였을때의 $o \sim l_1$, $l_1 \sim l_2$, $l_2 \sim l_3$, 및 $l_3 \sim l_4$ 부분의 전류를 각각 $I^a{}_1$, $I^a{}_2$, $I^a{}_3$ 및 $I^a{}_4$라 할때 이들은 문헌 (1)에 의하여

$$I^a{}_r(x) = j\frac{E_1}{30\Delta_0}\sum_{s=0}^{n-1}[\Delta_1 F_{rs}(x) - \Delta_4 H_{rs}(x) + \Delta_7 J_{rs}(x) - \Delta_{10} K_{rs}(x) + \Delta_{13} L_{rs}(x) - \Delta_{16} M_{rs}(x) + \Delta_{19} N_{rs}(x)] \cdots\cdots(1)$$

단$r=1, 2, 3$ 및 4이며 안테나상의 전류분포는 좌우가 軸對稱일것이므로 중앙에서 오른쪽 부분의 전류만을 나타냈다.

다음에 點2와 5에만 E_2를 급전하였을때의 전류 $I^b{}_r(x)$와 點 3과 6에만 E_3을 급전하였을때의 전류 $I^c{}_r(x)$도 동상 문헌에서

$$I^b{}_r(x)=\frac{jE_2}{30\Delta_o}\sum_{s=0}^{n-1}[-\Delta_2F_{rs}(x)+\Delta_5H_{rs}(x)-\Delta_8J_{rs}(x)+\Delta_{11}K_{rs}(x)-\Delta_{14}L_{rs}(x)$$
$$+\Delta_{17}M_{rs}(x)-\Delta_{20}N_{rs}(x)] \cdots\cdots(2)$$

$$I^c{}_r(x)=\frac{jE_3}{30\Delta_o}\sum_{s=0}^{n-1}[\Delta_3F_{rs}(x)-\Delta_6H_{rs}(x)+\Delta_9J_{rs}(x)-\Delta_{12}K_{rs}(x)+\Delta_{15}L_{rs}(x)$$
$$-\Delta_{18}M_{rs}(x)+\Delta_{21}N_{rs}(x)] \cdots\cdots(3)$$

단 Δ_o 는

$$\Delta_o=\begin{vmatrix} f_1 & h_1 & j_1 & k_1 & l_1 & m_1 & n_1 \\ f_2 & h_2 & j_1 & k_2 & l_2 & m_2 & n_2 \\ f_3 & h_3 & j_3 & k_3 & l_3 & m_3 & n_3 \\ f_4 & h_4 & i_4 & k_4 & l_4 & m_4 & n_4 \\ f_5 & h_5 & j_5 & k_5 & l_5 & m_5 & n_5 \\ f_6 & h_6 & j_6 & k_6 & l_6 & m_6 & n_6 \\ f_7 & h_7 & j_7 & k_7 & l_7 & m_7 & n_7 \end{vmatrix}$$

이며 $\Delta_1\sim\Delta_{21}$은 Δ_{ij} 를 Δ_o에서 제 i 행 j 열에 관한 餘因子行列式이라 할때

$$\left.\begin{array}{llll} \Delta_1=\Delta'_{11}, & \Delta_4=\Delta'_{12} & \Delta_7=\Delta'_{13} & \Delta_{10}=\Delta'_{14} \\ \Delta_2=\Delta'_{21}, & \Delta_5=\Delta'_{22} & \Delta_8=\Delta'_{23} & \Delta_{11}=\Delta'_{24} \\ \Delta_3=\Delta'_{31}, & \Delta_6=\Delta'_{32} & \Delta_9=\Delta'_{33} & \Delta_{12}=\Delta'_{34} \\ \Delta_{13}=\Delta'_{15} & \Delta_{16}=\Delta'_{16} & \Delta_{19}=\Delta'_{17} & \\ \Delta_{14}=\Delta'_{25} & \Delta_{17}=\Delta'_{26} & \Delta_{20}=\Delta'_{27} & \\ \Delta_{15}=\Delta'_{35} & \Delta_{18}=\Delta'_{36} & \Delta_{21}=\Delta'_{37} & \end{array}\right\} \cdots\cdots(4)$$

의 내용을 갖는다.

한편 문헌 (1)에 의해서

$$\left.\begin{array}{l} F_{10}(x)=\frac{\cos\beta x}{\Omega_1},\ H_{10}(x)=J_{10}(x)=K_{10}(x)=L_{10}(x)=M_{10}(x)=N_{10}(x)=0, \\ F_{20}(x)=0,\quad H_{20}(x)=\frac{\cos\beta x}{\Omega_2},\quad J_{20}(x)=\frac{\sin\beta|x|}{\Omega_2},\quad K_{20}(x)=L_{20}(x)=M_{20}(x)=N_{20}(x)=0, \\ F_{30}(x)=H_{30}(x)=J_{30}(x)=0,\ K_{30}(x)=\frac{\cos\beta x}{\Omega_3},\ L_{30}(x)=\frac{\sin\beta|x|}{\Omega_3},\ M_{30}(x)=N_{30}(x)=0, \\ F_{40}(x)=H_{40}(x)=J_{40}(x)=K_{40}(x)=L_{40}(x)=0,\ M_{40}(x)=\frac{\cos\beta x}{\Omega_4},\ N_{40}(x)=\frac{\sin\beta|x|}{\Omega_4} \end{array}\right\} \cdots\cdots(5)$$

$$\left.\begin{array}{l} F_{1,n+1}(x)=P_{11}\{F_{1,n}(x)\}+P_{12}\{F_{2,n}(x)\}+P_{13}\{F_{3,n}(x)\}+P_{14}\{F_{4,n}(x)\} \\ F_{2,n+1}(x)=P_{21}\{\quad\}+P_{22}\{\quad\}+P_{23}\{\quad\}+P_{24}\{\quad\} \\ F_{3,n+1}(x)=P_{31}\{\quad\}+P_{32}\{\quad\}+P_{33}\{\quad\}+P_{34}\{\quad\} \\ F_{4,n+1}(x)=P_{41}\{\quad\}+P_{42}\{\quad\}+P_{43}\{\quad\}+P_{44}\{\quad\} \\ H_{1,n+1}(x)=P_{11}\{H_{1,n}(x)\}+P_{12}\{H_{2,n}(x)\}+P_{13}\{H_{3,n}(x)\}+P_{14}\{H_{4,n}(x)\} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ N_{4,n+1}(x)=P_{41}\{N_{1,n}(x)\}+P_{43}\{N_{2,n}(x)\}+P_{43}\{N_{3,n}(x)\}+P_{44}\{N_{4,n}(x)\} \end{array}\right\} \cdots\cdots(6)$$

단 $n=1\sim4$

이므로 式 (1)~(3)의 Σ 기호내의 계산에서 近似的으로 $s=0$, 1에 대한 것만을 擇하고 안테나線上의 傳播定數를 $r=0+j\beta$, 각부분의 반지름을 $\rho_r(r\sim4)$, 각부분의 Ω 를 $\Omega_r=2ln\frac{2l_4}{\rho_r}$로 나타낸다면

$$F_{10}(x)+F_{11}(x)=\frac{\cos\beta x}{\Omega_1}+P_{11}\,[F_{10}(x)\,]=\frac{\cos\beta x}{\Omega_1}+P_{11}\,\frac{\cos\beta x}{\Omega_1}$$

$$=\frac{\cos\beta x}{\Omega_1}-\frac{1}{\Omega_1}\left(\frac{\cos\beta x}{\Omega_1}-\frac{\cos\beta l_1}{\Omega_1}\right)ln\frac{l_1{}^2-x^2}{l_4{}^2}-\frac{1}{\Omega_1}\int_{-l1}^{l1}\frac{\frac{\cos\beta\ \xi-\cos\beta l_1}{\Omega_1}e^{-j\beta r}-\frac{1}{\Omega_1}(\cos\beta x-\cos\beta l_1)}{r_1}d\xi$$

$$=\frac{\cos x}{\Omega_1}-\frac{1}{\Omega_1{}^2}\Big\{\cos\beta l_1\ ln\frac{l_1{}^2-x^2}{l_4{}^2}+\cos\beta x\cdot ln\frac{l_4{}^2}{l_1{}^2-x^2}-\cos\beta l_1\{E[\beta(x+l_1)]-E[\beta(l-x)]\}$$

$$+\frac{1}{2}e^{j\beta x}\cdot E[2\beta(x+l_1)]+\frac{1}{2}e^{-j\beta x}\cdot E[2\beta(l_i-x)]\Big\}\ \cdots\cdots\cdots\cdots(7)$$

$$H_{10}(x)+H_{11}(x)=P_{12}\,[H_{20}(x)]+P_{12}\,\frac{\cos\beta x}{\Omega_2}$$

$$=-\frac{1}{\Omega_1}\left(\frac{\cos\beta l_1}{\Omega_2}-\frac{\cos\beta l_2}{\Omega_2}\right)\left(ln\frac{l_2{}^2-x^2}{l_4{}^2}+\int_{-l1}^{l1}\frac{e^{-j\beta r}-1}{r_1}d\xi\right)$$

$$-\frac{1}{\Omega}\int_{-l2,\,l1}^{-l_1,\,l_2}\frac{\left(\frac{\cos\beta\xi}{\Omega_2}-\frac{\cos\beta l_2}{\Omega_2}\right)e^{-j\beta r_1}-\left(\frac{\cos\beta l_1}{\Omega_2}-\frac{\cos\beta l_2}{\Omega_2}\right)}{r_1}d\xi$$

$$=\frac{1}{\Omega_1\Omega_2}\Big\{\cos\beta l_1\cdot ln\frac{l_4{}^2}{l_1{}^2-x^2}+\cos\beta l_2\cdot ln\frac{l_2{}^2-x^2}{l_4{}^2}$$

$$+\cos\ \beta x\cdot ln\frac{l_1{}^2-x^2}{l_2{}^2-x_2}+\cos\beta l_1\cdot\{E[\beta(x+l_1)]+[\beta(l_1-x)]\}$$

$$-\cos\beta l_2\cdot\{E[\beta(x+l_2)]-E[\beta(l_2-x)]\}$$

$$+\frac{1}{2}e^{j\beta x}\cdot\{E[2\beta(x+l_2)]-E[2\,\beta(x+l_1)]\}+\frac{1}{2}e^{-j\beta x}\cdot\{E[2\beta(l_2-x)]-E[2\,\beta(l_1-x)]\}\Big\}\ \cdots\cdots(8)$$

$$J_{10}(x)+J_{11}(x)=P_{12}[J_{20}(x)]=P_{12}\left(\frac{\sin\beta|x|}{\Omega_2}\right)$$

$$=\frac{1}{\Omega_1\Omega_2}\Big\{\sin\beta l_1\cdot ln\frac{l_2{}^4}{l_1{}^2-x_2}+\sin\beta l_2\cdot ln\frac{l_2{}^2-x^2}{l_4{}^2}+\sin\beta x\cdot ln\frac{l_1-x}{l_1+x}\cdot\frac{l_2+x}{l_2-x}$$

$$+\sin\beta l_1\cdot\{E[\beta(x+l_1)]+E[\beta(l_1-x)]\}-\sin\beta l_2\{E[\beta(x+l_2)]+E[\beta(l_2-x)]\}$$

$$+\frac{j}{2}e^{jex}\cdot\Big\{E[2\beta(x+l_2)\Big\}-E[2\beta(x+l_1)]\Big\}$$

$$+\frac{j}{2}e^{-j\beta x}\Big\{E[2\beta(l_2-x)]-E[2\,\beta(l_1-x)]\Big\}\Big\}\ \cdots\cdots\cdots\cdots(9)$$

$$K_{10}(x)+K_{11}(x)=P_{13}[K_{30}(x)]=P_{13}\left(\frac{\cos\beta x}{\Omega_3}\right)$$

$$=-\frac{1}{\Omega_1}\left(\frac{\cos\beta l_2}{\Omega_3}-\frac{\cos\beta l_3}{\Omega_3}\right)\left(l_n\frac{l_3{}^2-x^2}{l_4{}^2}+\int_{-l2}^{l2}\frac{e^{-jer_1}-1}{r_1}d\xi\right)$$

$$-\frac{1}{\Omega_1}\int_{-l3,\,l2}^{-l2,\,l3}\frac{\left(\frac{\cos\xi}{\Omega_3}-\frac{\cos\beta l_3}{\Omega_3}\right)e^{j\beta r_1}-\left(\frac{\cos\beta l_2}{\Omega_3}-\frac{\cos\ \beta l_3}{\Omega_3}\right)}{r_1}d\xi$$

$$=\frac{1}{\Omega_1\Omega_3}\Big\{\cos\beta l_2\cdot l_n\frac{l_1{}^2}{l_2{}^2-x^2}+\cos\beta l_3\cdot l_n\frac{l_2{}^2-x^2}{l_4{}^2}+\cos\beta x\cdot l_n\frac{l_2{}^2-x^2}{l_3{}^2-x^2}$$

$$+\cos\ \beta l_2\Big\{E[\beta(x+l_2)]+E[\beta(l_2-x)]\Big\}-\cos\ \beta l_3\Big\{E[\beta(x+l_3)+E[\beta(l_2-x)]\Big\}$$

$$+\frac{1}{2}e^{j\beta x}\Big\{E[2\beta(x+l_3)]-E[2\beta(x+l_2)]\Big\}$$

$$+\frac{1}{2}e^{-j\beta x}\Big\{E[2\beta(l_3-x)]-E[2\beta(l_2-x_1]\Big\}\Big\}\cdots\cdots\cdots\cdots(10)$$

$$L_{10}(x)+L_{11}(x)=P_{13}[L_{30}(x)]=P_{13}\left(\frac{\sin\beta|x|}{\Omega_3}\right)$$

$$=\frac{1}{\Omega_1\Omega_3}\left\{\sin\beta l_2\cdot l_n\frac{l_4^2}{l_2^2-x^2}+\sin\beta l_3\cdot l_n\frac{l_2^2-x^2}{l_4}\right.$$

$$+\sin\beta x\cdot ln\frac{l_2-x}{l_2+x}\,\frac{l_3+x}{l_3-x}+\sin\beta l_2\left\{E[\beta(x+l_2)]+E[\beta(l_2-x)]\right.$$

$$-\sin\beta l_3\left\{E[\beta(x+l_3)]+E[\beta(l_3+x)]\right\}$$

$$+\frac{j}{2}e^{j\beta x}\left\{E[2\beta(x+l_3)]-E[2\beta(x_4l_2)]\right\}$$

$$+\frac{1}{2}e^{-j\beta x}\left\{E[2\beta(l_3-x)]-E[2\beta(l_2-x)]\right\} \qquad \cdots\cdots(11)$$

$$M_{10}(x)+M_{11}(x)=P_{14}[K_{40}(x)]=P_{14}\left(\frac{\cos\beta x}{\Omega_4}\right)$$

$$=-\frac{1}{\Omega_1}\left(\frac{\cos\beta l_3}{\Omega_4}-\frac{\cos\beta l_4}{\Omega_4}\right)\left(l_n\frac{l_4^2-x^2}{l_4^2}+\int_{-l3}^{l3}\frac{e^{-j\beta r1}-1}{r_1}d\xi\right)$$

$$-\frac{1}{\Omega_1}\int_{-l4,\,l3}^{-l3,\,l4}\frac{\left(\frac{\cos\beta\xi}{\Omega_4}-\frac{\cos\beta l_4}{\Omega_4}\right)e^{-j\beta r_1}-\left(\frac{\cos\beta l_3}{\Omega_4}-\frac{\cos\beta l_4}{\Omega_4}\right)}{r_1}d\xi$$

$$=\frac{1}{\Omega_1\Omega_4}\left(\cos\beta l_3\cdot l_n\frac{l_4^2}{l_2^2-x^2}+\cos\beta l_4\cdot l_n\frac{l_2^2-x^2}{l_4}+\cos\beta x\cdot l_n\frac{l_3^2-x^2}{l_4^2-x^2}\right.$$

$$+\cos\beta l_3\cdot\left\{E[\beta(x+l_3)]+E[\beta(l_3-x)]\right\}-\cos\beta l_4\cdot\left\{E[\beta(x+l_4)]+E[B(l_4-x)]\right\}$$

$$+\frac{1}{2}e^{j\beta x}\left\{E[2\beta(x+l_4)]-E[2\beta(x+l_3)]\right\}$$

$$\left.+\frac{1}{2}e^{-j\beta x}\left\{E[2\beta(l_4-x)]-E[2\beta(l_3-x)]\right\}\right) \qquad \cdots\cdots(12)$$

$$N_{10}(x)+N_{11}(x)=P_{14}[N_{40}(x)]=P_{14}\left(\frac{\sin\beta|x|}{\Omega_4}\right)$$

$$=\frac{1}{\Omega_1\Omega_4}\sin\beta l_3\cdot ln\frac{l_4^2-x^2}{l_3^2}+\sin\beta l_4\cdot l_n\frac{l_4^2-x^2}{l_4^2}+\sin\beta x\cdot l_n\frac{l_3-x}{l_3+x}\,\frac{l_4+x}{l_4-x}$$

$$+\sin\beta l_3\left\{E[\beta(x+l_3)]+E[\beta(l_3-x)]\right\}$$

$$-\sin\beta l_4\left\{E[\beta(x+l_4)]+E[\beta(l_4-x)]\right\}+\frac{j}{2}e^{-j\beta x}\left\{E[2\beta(x+l_4)]-E[2\beta(x+l_3)]\right.$$

$$\left.+\frac{j}{2}e^{-j\beta x}\left\{E[2\beta(l_4-x)]-E[2\beta(l_3-x)]\right\}\right) \qquad \cdots\cdots(13)$$

$$F_{20}(x)+F_{21}(x)=P_{21}[F_{10}(x)]=P_{21}\left(\frac{\cos\beta x}{\Omega_1}\right)$$

$$=-\frac{1}{\Omega_2}\int_{-l_1}^{l1}\frac{\left(\frac{\cos\beta\xi}{\Omega_1}-\frac{\cos\beta l_1}{\Omega_1}\right)e^{-j\beta x_1}}{r_2}d\xi$$

$$=\frac{1}{\Omega_1\Omega_2}\left(\cos\beta l_1\cdot l_n\frac{x+l_1}{x-l_1}+\cos\beta x\cdot l_n\frac{x-l_1}{x+l_1}\right.$$

$$+\cos\beta l_1\left\{E[\beta(x-l_1)]-E[\beta(x+l_1)]\right\}$$

$$\left.+\frac{1}{2}e^{j\beta x}\left\{E[2\beta(x+l_1)]-E[2\beta(x-l_1)]\right\}\right) \qquad \cdots\cdots(14)$$

$$H_{20}(x)+H_{21}(x)=\frac{\cos\beta x}{\Omega_2}+P_{22}[H_{20}(x)]=\frac{\cos\beta x}{\Omega_2}+P_{22}\left(\frac{\cos\beta x}{\Omega_2}\right)$$

$$=\frac{\cos\beta x}{\Omega_2}-\frac{1}{\Omega_2}\left(\frac{\cos\beta x}{\Omega_2}-\frac{\cos\beta l_2}{\Omega_2}\right)l_n\frac{l_2{}^2-x^2}{l_4{}^2}$$

$$-\frac{1}{\Omega_2}\int_{-l1}^{l1}\frac{\left(\frac{\cos\beta l_1}{\Omega_2}-\frac{\cos\beta l_2}{\Omega_2}\right)e^{-j\beta r_2}-\left(\frac{\cos\beta x}{\Omega_2}-\frac{\cos\beta l_2}{\Omega_2}\right)}{r_2}d\xi$$

$$-\frac{1}{\Omega_2}\int_{-l2,\ l1}^{-l1,\ l2}\frac{\left(\frac{\cos\beta\xi}{\Omega_2}-\frac{\cos\beta l_2}{\Omega_2}\right)e^{-j\beta r_2}-\left(\frac{\cos\beta x}{\Omega_2}-\frac{\cos\beta l_2}{\Omega_2}\right)}{r_2}d\xi$$

$$=\frac{\cos\beta x}{\Omega_2}+\frac{1}{\Omega_2{}^2}\Big(\cos\beta l_1\cdot l_n\frac{x-l_1}{x+l_1}+\cos\beta l_2\cdot l_n\frac{l_2{}^2-x^2}{l_4{}^2}$$

$$+\cos\beta x\cdot l_n\frac{l_4{}^2}{l_2{}^2-x^2}\cdot\frac{x+l_1}{x-l_1}+\cos\beta l_1\Big\{E[\beta x+l_1)]-E[\beta(x-l_1)]\Big\}$$

$$-\cos\beta l_2\Big\{E[\beta(x+l_2)]+E[\beta(l_2-x)]\Big\}$$

$$+\frac{1}{2}e^{j\beta x}\Big\{E[2\beta(x+l_2)]+E[2\beta(x-l_1)]-E[2\beta(x+l_1)]\Big\}$$

$$+\frac{1}{2}e^{-j\beta x}\ E[2\beta(l_2-x)]\Big)\qquad(15)$$

$$J_{20}(x)+J_{21}(x)=\frac{\sin\beta|x|}{\Omega_2}+P_{22}\left(\frac{\sin\beta|x|}{\Omega_2}\right)$$

$$=\frac{\sin\beta(x)}{\Omega_2}+\frac{1}{\Omega_2}\Big(\sin\beta l,\cdot l_n\frac{x-l_1}{x+l_1}+\sin\beta l_2\cdot l_n\frac{l_2{}^2-x^2}{l_4{}^2}$$

$$+\sin\beta|x|\cdot l_n\frac{l_4{}^2}{x^2-l_1{}^2}\ \frac{l_2+x}{l_2-x}+\sin\beta l_1\ \Big\{E[\beta(x+l_1)]-E[\beta(x-l_1)]$$

$$-\sin\beta l_2\cdot\Big\{E[\beta(x+l_2)]+E[\beta(l_2-x)]\Big\}$$

$$+\frac{j}{2}e^{j\beta x}\cdot\Big\{E[2\beta(x+l_2)]-E[2\beta(x+l_1)]-E[2\beta(x-l_1)]$$

$$+\frac{j}{2}e^{-j\beta x}\cdot E\ [2\beta(l_2-x)]\Big)\qquad(16)$$

$$K_{20}(x)+K_{21}(x)=P_{23}[K_{30}(x)]=P_{23}\left(\frac{\cos\beta x}{\Omega_3}\right)$$

$$=-\frac{1}{\Omega_2\Omega_3}\left(\cos\beta l_2-\cos\beta l_3\right)\left(l_n\frac{l_1{}^2-x^2}{l_4{}^2}+\int_{-l2}^{l2}\frac{e^{-j\beta r_2}-1}{r_1}d\xi\right)$$

$$-\frac{1}{\Omega_2\Omega_3}\int_{-l3,\ l2}^{-l2,\ l3}\frac{(\cos\beta\xi-\cos\beta l_3)e^{-j\beta r2}-\cos\beta l_2+\cos\beta l_3}{r_2}d\xi$$

$$=\frac{1}{\Omega_2\Omega_3}\ \cos\beta l_2\ \ l_n\frac{l_4{}^2}{l_2{}^2-x^2}+\cos\beta l_3\cdot l_n\frac{l_3{}^2-x^2}{l_4}+\cos\beta x\cdot l_n\frac{l_2{}^2-x^2}{l_3{}^2-x^2}$$

$$+\cos\beta l_2\Big\{E[\beta(x+l_2)]+E[\beta(l_2-x)]-\cos\beta l_3\{E[\beta(x+l_3])+E\ [\beta(l_3-x)]\Big\}$$

$$+\frac{1}{2}e^{j\beta x}\Big\{E[2\beta(x+l_3)]-E\ [2\beta(x+l_2)]\Big\}$$

$$+\frac{1}{2}e^{-j\beta x}\Big\{E[2\beta(l_3-x)]-E[2\ \beta(l_2-x)]\Big\}\Big)\qquad(17)$$

$$L_{20}(x)+L_{21}(x)=P_{23}[L_{30}(x)]=P_{23}\left(\frac{\sin\beta|x|}{\Omega_3}\right)$$

$$=\frac{1}{\Omega_2\Omega_3}\left(\sin\beta_2 l_2\cdot l_n\frac{l_4{}^2}{l_2{}^2-x^2}+\sin\beta l_3\cdot l_n\frac{l_3{}^2-x^2}{l_4{}^2}\right)$$

$$+\sin\beta x\cdot l_n\frac{l_2-x}{l_2+x}\,\frac{l_3+x}{l_3-x}+\sin\beta l_2\cdot\Big\{E[\beta(x+l_2)]+E[\beta(l_2-x)]\Big\}$$

$$-\sin\beta l_3\cdot\Big\{E[\beta(x+l_3)]+E[\beta(l_3-x)]\Big\}$$

$$+\frac{j}{2}e^{-j\beta x}\Big\{E[2\beta(l_3-x)]-E[2\beta(x+l_2)]$$

$$+\frac{j}{2}e^{-j\beta x}\Big\{E[2\beta(l_3-x)]-E[2\beta(l_2-x)]\Big\}\Big) \cdots\cdots(18)$$

$$M_{20}(x)+M_{21}(x)=P_{24}[M_{40}(x)]=P_{24}\left(\frac{\cos\beta x}{\Omega_4}\right)$$

$$=-\frac{1}{\Omega_2\Omega_4}\Big(\cos\beta l_3-\cos\beta l_4\Big)\Big(l_n\frac{l_4^2-x^2}{l_4^2}+\int_{-l3}^{l3}\frac{e^{j\beta r1}-1}{r_2}d\xi\Big)$$

$$-\frac{1}{\Omega_2\Omega_4}\int_{-l_4,\,l_3}^{-l_3,\,l_4}\frac{(\cos\beta\xi-\cos\beta l_4)e^{-j\beta r2}-(\cos\beta l_3-\cos\beta l_4)}{r_2}d\xi$$

$$=\frac{1}{\Omega_2\Omega_4}\Big(\cos\beta l_3\cdot l_n\frac{l_4^2}{l_3^2-x^2}+\cos\beta l_4\cdot ln\frac{l_4^2-x^2}{l_4^2}$$

$$+\cos\beta x\cdot l_n\frac{l_3^2-x^2}{l_4^2-x^2}+\cos\beta l_3\cdot\Big\{E[\beta(x+l_3)]+E[\beta(l_3-x)]\Big\}$$

$$-\cos\beta l_4\Big\{E[\beta(l_4-x)]+E[\beta(x+l_3)]\Big\}$$

$$+\frac{1}{2}e^{j\beta x}\Big\{E[2\beta(x+l_4)]-E[2\beta(x+l_3)]\Big\}$$

$$+\frac{1}{2}e^{-j\beta x}\Big\{E[2\beta(l_4-x)]-E[2\beta(l_3-x)]\Big\}\Big) \cdots\cdots(19)$$

$$N_{20}(x)+N_{21}(x)=P_{24}[N_{40}(x)]=P_{24}\left(\frac{\sin\beta|x|}{\Omega_4}\right)$$

$$=\frac{1}{\Omega_2\,\Omega_4}\ \sin\beta l_3\cdot l_n\frac{l_4^2}{l_3^2-x^2}+\sin\beta l_4\cdot l_n\frac{l_4^2-x^2}{l_4^2}$$

$$=\sin\beta x\cdot l_n\frac{l_3-x}{l_3+x}\cdot\frac{l_4+x}{l_4-x}+\sin\beta l_3\cdot\Big\{E[\beta(x+l_3)]+E[\beta(l_3-x)]\Big\}$$

$$-\sin\beta l_4\cdot\Big\{E[\beta(x+l_4)]+E[\beta(l_4-x)]\Big\}+\frac{j}{2}e^{j\beta x}\cdot\Big\{E[2\beta(x+l_4)]-E[2\beta(x+l_3)]\Big\}$$

$$+\frac{j}{2}e^{-j\beta x}\Big\{E[2\beta(l_4-x)]-E[2\beta(l_3-x)]\Big\}\Big) \cdots\cdots(20)$$

$$F_{30}(x)+F_{31}(x)=P_{31}[F_{10}(x)]=P_{31}\left(\frac{\cos\beta x}{\Omega_1}\right)=-\frac{1}{\Omega_3}\int_{-l1}^{l1}\frac{\left(\frac{\cos\xi}{\Omega_1}-\frac{\cos\beta l_1}{\Omega_1}\right)e^{-j\beta r_3}}{r_3}d\xi$$

$$=\frac{1}{\Omega_1\,\Omega_3}\Big(\cos\beta l_1\cdot l_n\frac{x+l_1}{x-l_1}+\cos\beta x\cdot l_n\frac{x-l_1}{x+l_1}$$

$$+\cos\beta l_1\Big\{E[\beta(x-l_1)]-E[\beta(x+l_1)]\Big\}+\frac{1}{2}e^{j\beta x}\Big\{E[2\beta(x+l_1)]-E[2\beta(x-l_1)]\Big\}\Big) \cdots\cdots(21)$$

$$H_{30}(x)+H_{31}(x)=P_{32}[H_{20}(x)]=P_{32}\left(\frac{\cos\beta x}{\Omega_2}\right)$$

$$=-\frac{1}{\Omega_3}\int_{-l1}^{l1}\frac{\left(\frac{\cos\beta l_1}{\Omega_2}-\frac{\cos\beta l_2}{\Omega_2}\right)e^{-j\beta r_3}}{r_3}d\xi-\frac{1}{\Omega_3}\int_{-l_2,\,l_1}^{-l_1,\,l_2}\frac{\left(\frac{\cos\beta\xi}{\Omega_2}-\frac{\cos\beta l_2}{\Omega_2}\right)e^{-j\beta r_3}}{r_3}d\xi$$

$$=\frac{1}{\Omega_2\,\Omega_3}\Big(\cos\beta l_1\cdot l_n\frac{x-l_1}{x+l_2}+\cos\beta l_2\cdot l_n\frac{x+l_2}{x-l_2}+\cos\beta x\cdot l_n\frac{x-l_2}{x-l_2}\cdot\frac{x+l_1}{x-l_1}$$

$$+\cos\beta l_1\Big\{E[\beta(x+l_1)]-E[\beta(x-l_1)]\Big\}+\cos\beta l_2\cdot\Big\{E[\beta(x-l_2)]-E[\beta(x+l_2)]\Big\}$$

$$+\frac{1}{2}e^{j\beta x}\Big\{E[2\beta(x+l_2)-E[2\beta(x-l_2)]-E[2\beta(x+l_1)]+E[2\beta(x-l_1)]\Big\}\Big) \cdots\cdots(22)$$

$$J_{30}(x)+J_{31}(x)=P_{32}[J_{20}(x)]=P_{32}\Big(\frac{\sin\beta|x|}{\Omega_2}\Big)=\frac{1}{\Omega_2\ \Omega_3}\Big(\sin\beta l_1\cdot l_n\frac{x-l_1}{x+l_1}$$

$$+sin\,\beta l_2\cdot l_n\frac{x+l_2}{x-l_2}+\sin\beta x\cdot l_n\frac{x^2-l_2{}^2}{x^2-l_1{}^2}+\sin\beta l_1\Big\{E[\beta(x+l_1)]-E[\beta(x-l_1)]\Big\}$$

$$+\sin\beta l_2\cdot\Big\{E[\beta(x-l_2)]-E[\beta(x+l_2)]\Big\}+\frac{j}{2}e^{j\beta x}\cdot\Big\{E\{2\beta(x+l_2]+E[2\beta(x-l_2)]$$

$$-E[2\beta(x+l_1)]-E[2\beta(x-l_1)]\Big\}\Big)\cdots\cdots(23)$$

$$\mathrm{K}_{30}(x)+\mathrm{K}_{31}(x)=\frac{\cos\beta x}{\Omega_3}+P_{33}\Big(\mathrm{K}_{30}(x)\Big)=\frac{\cos\beta x}{\Omega_3}+P_{33}\Big(\frac{\cos\beta x}{\Omega_3}\Big)$$

$$=\frac{\cos\beta x}{\Omega_3}-\frac{1}{\Omega_3{}^2}(\cos\beta x-\cos\beta l_3)\cdot l_n\frac{l_3{}^2-x^2}{l_4{}^2}$$

$$-\frac{1}{\Omega_3{}^2}\int_{-l2}^{l2}\frac{(\cos\beta l_2-\cos\beta l_3)e^{-j\beta r_3}-(\cos\beta x-\cos\beta l_3)}{r_3}d\xi$$

$$-\frac{1}{\Omega_3{}^2}\int_{-l3,\,l2}^{-l2,\,l3}\frac{(\cos\beta\xi-\cos\beta l_3)e^{-j\beta r_3}-(\cos\beta x-\cos\beta l_3)}{r_3}d\xi$$

$$=\frac{\cos\beta x}{\Omega_3}+\frac{1}{\Omega_3{}^2}\Big(\cos\beta l_2\cdot l_n\frac{x-l_2}{x+l_2}+\cos\beta l_3\cdot l_n\frac{l_3{}^2-x^2}{l_4{}^2}$$

$$+\cos\beta x\cdot l_n\frac{x+l_2}{x-l_2}\cdot\frac{l_4{}^2}{l_3{}^2-x^2}+\cos\beta l_2\Big\{E[\beta(x+l_2)]-E[\beta(x-l_2)]\Big\}$$

$$-\cos\beta l_3\cdot\Big\{E[\beta(x+l_3)+E[\beta(l_3-x)]\Big\}+\frac{1}{2}e^{j\beta x}\cdot\Big\{E[2\beta(x+l_3)]-E[2\beta(x+l_2)]$$

$$+E[2\beta(x-l_2)]\Big\}+\frac{1}{2}e^{-j\beta x}\cdot E[2\beta(l_3-x)]\cdots\cdots(24)$$

$$L_{30}(x)+L_{31}(x)=\frac{\sin\beta|x|}{\Omega_3}+P_{33}[L_{30}(x)]=\frac{\sin\beta|x|}{\Omega_3}+P_{33}\Big(\frac{\sin\beta|x|}{\Omega_3}\Big)$$

$$=\frac{\sin\beta|x|}{\Omega_3}+\frac{1}{\Omega_3{}^2}\Big(\sin\beta l_2\cdot l_n\frac{x-l_2}{x+l_2}+\sin\beta l_3\cdot l_n\frac{l_3{}^2-x^2}{l_4{}^2}+\sin\beta(x)\cdot ln\frac{l_4{}^2}{x^2-l_2{}^2}$$

$$+\sin\beta l_2\cdot\Big\{E[\beta(x+l_2)]-E[\beta(x-l_2)]\Big\}-\sin\beta l_3\cdot\Big\{E[\beta(l_3+x)]+E[\beta(l_3-x)]\Big\}$$

$$+\frac{j}{2}e^{j\beta x}\cdot\Big\{E[2\beta(x+l_3)]-E[2\beta(x+l_2)]-E[2\beta(x-l_2)]\Big\}$$

$$+\frac{j}{2}e^{-j\beta x}\cdot E[2\beta(l_3-x)]\Big)\cdots\cdots(25)$$

$$M_{30}(x)+M_{31}(x)=P_{34}[M_{40}(x)]=P_{34}\Big(\frac{\cos\beta x}{\Omega_4}\Big)$$

$$=-\frac{1}{\Omega_3\ \Omega_4}(\cos\beta l_3-\cos\beta l_4)(l_n\frac{l_4{}^2-x^2}{l_4{}^2})+\int_{-l3}^{l3}\frac{e^{-j\beta r_3}-1}{r^3}d\xi$$

$$-\frac{1}{\Omega_3\ \Omega_4}\int_{-l4,\,l3}^{-3l,\,l4}\frac{(\cos\beta\xi-\cos\beta l_4)e^{-j\beta r_3}-(\cos\beta l_3-\cos\beta l_4)}{r^3}d\xi$$

$$=\frac{1}{\Omega_3\ \Omega_4}\Big(\cos\beta l_3\cdot l_n\frac{l_4{}^2}{l_3{}^2-x^2}+\cos\beta l_4\cdot l_n\frac{l_4{}^2-x^2}{l_4{}^2}+\cos\beta x\cdot l_n\frac{l_3{}^2-x^2}{l_4{}^2-x^2}$$

$$+\cos\beta l_3\cdot\Big\{E[\beta(l_3+x)]+E[\beta(l_3-x]\Big\}-\cos\beta l_4\cdot\Big\{E[\beta(l_4+x)]+E[\beta(l_4-x)]\Big\}$$

$$+\frac{1}{2}e^{j\beta x}\cdot\Big\{E[2\beta(x+l_4)]-E[2\beta(x+l_3)]$$

$$+\frac{1}{2}e^{-j\beta x}\cdot\left\{E[2\beta(l_4-x)]-E[2\beta(l_3-x)]\right\}\Big)\cdots\cdots(26)$$

$$N_{30}(x)+N_{31}(x)=P_{34}[N_{40}(x)]=P_{34}\left(\frac{\sin\beta(x)}{\Omega_4}\right)=\frac{1}{\Omega_3\cdot\Omega_4}\Big(\sin\beta l_3\cdot ln\frac{l_4^2}{l_3^2-x^2}$$

$$+\sin\beta l_4\cdot l_n\frac{l_4^2-x^2}{l_4^2}+\sin\beta x\cdot l_n\frac{l_3-x}{l_3+x}\cdot\frac{l_4+x}{l_4-x}$$

$$+\sin\beta l_3\cdot\left\{E[\beta(x+l_3)]+E[\beta(l_3-x)]-\sin\beta l_4\cdot\left\{E[\beta(x+l_4)]+E[\beta(l_4-x)]\right\}\right.$$

$$+\frac{j}{2}e^{j\beta x}\cdot\left\{E[2\beta(x+l_4)]-E[2\beta(x+l_3)]\right\}$$

$$+\frac{j}{2}e^{j\beta x}\cdot\left\{E[2\beta(l_4-x)]-E[2\beta(l_3-x)]\right\}\Big)\cdots\cdots(27)$$

$$F_{40}(x)+F_{41}(x)=P_{41}[F_{10}(x)]-P_{41}\left(\frac{\cos\beta x}{\Omega_1}\right)=-\frac{1}{\Omega_1\ \Omega_4}\int_{-l1}^{l1}\frac{(\cos\beta\xi-\cos\beta l_1)e^{-j\beta r_4}}{r_4}d\xi$$

$$=\frac{1}{\Omega_1\ \Omega_4}\Big(\cos\beta l_1\cdot l_n\frac{x+l_1}{x-l_1}+\cos\beta x\cdot l_n\frac{x-l_1}{x-l_1}+\cos\beta l_1\left\{E[\beta(x-l_1)]-E[\beta(x+l_1)]\right\}$$

$$+\frac{1}{2}e^{j\beta x}\cdot\left\{E[2\beta(x+l_1)]-E[2\beta(x-l_1)]\right\}\Big)\cdots\cdots(28)$$

$$H_{40}(x)+H_{41}(x)=P_{42}[H_{20}(x)]=P_{42}\left(\frac{\cos\beta x}{\Omega_2}\right)=-\frac{1}{\Omega_2\ \Omega_4}\int_{-l1}^{l1}\frac{(\cos\beta l_1-\cos\beta l_2)e^{-j\beta r_4}}{r_4}d\xi$$

$$-\frac{1}{\Omega_2\ \Omega_4}\int_{-l2,\ l1}^{-l1,\ l2}\frac{(\cos\beta\xi-\cos\beta l_2)e^{-j\beta r_4}}{r_4}d\xi=\frac{1}{\Omega_2\ \Omega_4}\Big(\cos\beta l_1\cdot l_n\frac{x-l_1}{x+l_1}$$

$$+\cos\beta l_2\cdot l_n\frac{x+l_2}{x-l_2}+\cos\beta x\cdot l_n\frac{x+l_1}{x-l_1}\cdot\frac{x-l_2}{x+l_2}+\cos\beta l_1\cdot\left\{E[\beta(x+l_1)]+E[\beta(x-l_1)]\right\}$$

$$+\cos\beta l_2\cdot\left\{E[\beta(x-l_2)]-E[\beta(x+l_2)]\right\}+\frac{1}{2}e^{j\beta x}\cdot\Big\{E[2\beta(x+l_2)]$$

$$-E[2\beta(x+l_1)]+E[2\beta(x-l_1)]-E[2\beta(x-l_2)]\Big\}\Big)\cdots\cdots(29)$$

$$J_{40}(x)+J_{41}(x)=P_{42}[J_{20}(x)]=P_{42}\left(\frac{\sin\beta|x|}{\Omega_2}\right)=\frac{1}{\Omega_2\ \Omega_4}\Big(\sin\beta l_1\cdot l_n\frac{x-l_1}{x+l_1}$$

$$+\sin\beta l_2\cdot l_n\frac{x+l_2}{x-l_2}+\sin\beta x\cdot l_n\frac{x^2-l_2^2}{x^2-l_1^2}+\sin\beta l_1\cdot\left\{E\,\beta(x+l_1)]-E[\beta(x-l_1)]\right\}$$

$$+\sin\beta l_2\cdot\left\{E[\beta(x-l_2)]-E[\beta(x+l_2)]\right\}+\frac{j}{2}e^{j\beta x}\Big\{E[2\beta(x+l_2)]-E[2\beta(x+l_1)]$$

$$+E[2\beta(x-l_2)]-E[2\beta(x-l_1)]\Big\}\Big)\cdots\cdots(30)$$

$$K_{40}(x)+K_{41}(x)=P_{43}[K_{30}(x)]=P_{43}\left(\frac{\cos\beta x}{\Omega_3}\right)=-\frac{1}{\Omega_3\ \Omega_4}\int_{-l2}^{l2}\frac{(\cos\beta l_2-\cos\beta l_3)e^{-j\beta r_4}}{r_4}d\xi$$

$$-\frac{1}{\Omega_3\ \Omega_4}\int_{-l3,\ l2}^{-l2,\ l3}\frac{(\cos\xi-\cos\beta l_3)e^{-j\beta r_4}}{r_4}d\xi$$

$$=\frac{1}{\Omega_3\ \Omega_4}\Big(\cos\beta l_2\cdot l_n\frac{x-l_2}{x+l_2}+\cos\beta l_3\cdot l_n\frac{x+l_3}{x-l_3}+\cos\beta x\cdot l_n\frac{x-l_3}{x+l_3}\cdot\frac{x+l_2}{x-l_2}$$

$$+\cos\beta l_2\cdot\left\{E[\beta(x+l_2)]-E[\beta(x-l_2)]\right\}+\cos\beta l_3\left\{E[\beta(x-l_3)]-E[\beta(x+l_3)]\right\}$$

$$+\frac{1}{2}e^{j\beta x}\cdot\left\{E[2\beta(x+l_3)]-E[2\beta(x+l_3)]-E[2\beta(x+l_2)]+E[2\beta(x-l_2)]\right\}\Big)\cdots\cdots(31)$$

$$L_{40}(x)+L_{41}(x)=P_{43}[L_{30}(x)]=P_{43}\left(\frac{\sin\beta|x|}{\Omega_3}\right)=\frac{1}{\Omega_3\ \Omega_4}\Big(\sin\beta l_2\cdot l_n\frac{x-l_2}{x+l_2}$$

$$+\sin\beta l_3\cdot l_n\frac{x+l_3}{x-l_3}+\sin\beta x\cdot l_n\frac{x^2-l_3^2}{x^2-l_2^2}+\sin\beta_2\cdot\left\{E[\beta(x+l_2)]-E[\beta(x-l_2)]\right\}$$

$$+\sin\beta l_3\cdot\Big\{E[\beta(x-l_3)-E[\beta(x+l_3)]\Big\}+\frac{j}{2}e^{j\beta x}\cdot\Big\{E[2\beta(x+l_3)]+E[2\beta(x-l_3)]$$

$$-E[2\beta(x+l_2)]-E[2\beta(x-l_2)]\Big\}\Big\} \quad\cdots\cdots(32)$$

$$M_{40}(x)+M_{41}(x)=\frac{\cos\beta x}{\Omega_4}+P_{44}\Big\{M_{40}(x)\Big\}=\frac{\cos\beta x}{\Omega_4}+P_{44}\Big(\frac{\cos\beta x}{\Omega_4}\Big)$$

$$=\frac{\cos\beta x}{\Omega_4}-\frac{1}{\Omega_4^2}(\cos\beta x-\cos\beta l_4)\cdot l_n\frac{l_4^2-x^2}{l_4^2}$$

$$-\frac{1}{\Omega_4^2}\int_{-l3}^{l3}\frac{(\cos\beta l_3-\cos\beta l_4)e^{-j\beta r_4}-(\cos\beta x-\cos\beta l_4)}{r_4}d\xi$$

$$-\frac{1}{\Omega_4^2}\int_{-l4,\,l3}^{-l3,\,l4}\frac{(\cos\beta\xi-\cos\beta l_4)e^{-j\beta r_4}-(\cos\beta x-\cos\beta l_4)}{r_4}d\xi$$

$$=\frac{\cos\beta x}{\Omega_4}+\frac{1}{\Omega_4^2}\Big(\cos\beta l_3\cdot l_n\frac{x-l_3}{x+l_3}+\cos\beta l_4\cdot l_n\frac{l_4^2-x^2}{l_4^2}$$

$$+\cos\beta x\cdot l_n\frac{l_4^2}{l_4^2-x^2}\cdot\frac{x+l_3}{x-l_3}+\cos\beta l_3\Big\{E[\beta(x+l_3)]-E[\beta(x-l_3)]$$

$$-\cos\beta l_4\cdot\Big\{E[\beta(x+l_4)]+E[\beta(l_4-x)]\Big\}+\frac{1}{2}e^{j\beta x}\Big\{E[2\beta(x+l_4)]+E[2\beta(x-l_3)-E[2\beta(x+l_3)]\Big\}$$

$$+\frac{1}{2}e^{-j\beta x}\cdot E[2\beta(l_4-x)] \quad\cdots\cdots(33)$$

$$N_{40}(x)+N_{41}(x)=\frac{\sin\beta|x|}{\Omega_4}+P_{44}\Big\{N_{40}|x|\Big\}=\frac{\sin\beta|x|}{\Omega_4}+P_{44}\Big(\frac{\sin\beta|x|}{\Omega_4}\Big)$$

$$=\frac{\sin\beta|x|}{\Omega_4}+\frac{1}{\Omega_4^2}\Big(\sin\beta l_3\cdot l_n\frac{x-l_3}{x+l_3}+\sin\beta l_4\cdot l_n\frac{l_4^2-x^2}{l_4^2}$$

$$+\sin\beta|x|\cdot l_n\frac{l_4^2}{x^2-l_3^2}\cdot\frac{l_4+x}{l_4-x}+\sin\beta l_3\cdot\Big\{E[\beta(x+l_3)]-E[\beta(x-l_3)]\Big\}$$

$$-\sin\beta l_4\cdot\Big\{E[\beta(x+l_4)]+E[\beta(l_4-x)]\Big\}+\frac{j}{2}e^{j\beta x}\Big\{E[2\beta(x+l_4)-E[2\beta(x+l_3)]$$

$$-E[2\beta(x-l_3)]\Big\}+\frac{j}{2}e^{-j\beta x}\cdot E[2\beta(l_4-x)]\Big) \quad\cdots\cdots(34)$$

다음에 $\rho_1\doteqdot\rho_2\doteqdot\rho_3\doteqdot\rho_4\doteqdot\rho$, 따라서 $\Omega_1\doteqdot\Omega_2\doteqdot\Omega_3\doteqdot\Omega_4\doteqdot\Omega\gg1$ 이라하면

$$f_1=\sin\beta l_1,\ h_1=\sin\beta l_1,\ j_1=-\cos\beta l_1,\ k_1=l_1=m_1=n_2=0 \quad\cdots\cdots(35)$$

$$f_2=0,\ h_2=-\sin\beta l_2,\ j_2=\cos\beta l_2,\ k=\sin\beta l_2,\ l_2=-\cos\beta l_2,\ m_2=n_2=0 \quad\cdots\cdots(36)$$

$$f_3=h_3=j_3=0,\ k_3=-\sin\beta l_3,\ l_3=\cos\beta l_3\ \ m_3=\sin\beta l_3,\ n_3=-\cos\beta l_3 \quad\cdots\cdots(37)$$

$$\left.\begin{aligned}
&f_4=\sum_{s=0}^{n}F_{4s}(l_4)\doteqdot F_{40}(l_4)+F_{41}(l_4)=\frac{f_{44}}{\Omega_1\Omega_4}\doteqdot\frac{f_{44}}{\Omega^2}\\
&h_4\doteqdot H_{40}(l_4)+H_{41}(l_4)=\frac{h_{44}}{\Omega_2\Omega_4}\doteqdot\frac{h_{44}}{\Omega^2}\\
&j_4\doteqdot J_{40}(l_4)+J_{41}(l_4)=\frac{j_{44}}{\Omega_2\Omega_4}\doteqdot\frac{j_{44}}{\Omega^2}\\
&k_4\doteqdot K_{40}(l_4)+K_{41}(l_4)\frac{k_{44}}{\Omega_3\Omega_4}\doteqdot\frac{k^{44}}{\Omega^2}\\
&l_4\doteqdot L_{40}(l_4)\ L_{41}(l_4)=\frac{l_{44}}{\Omega_3\Omega_4}\doteqdot\frac{l_{44}}{\Omega^2}\\
&m_4\doteqdot M_{40}(l_4)+M_{41}(l_4)=\frac{\cos\beta l_4}{\Omega_4}+\frac{m_{44}}{\Omega_4^2}\doteqdot\frac{\cos\beta l_4}{\Omega}+\frac{m_{44}}{\Omega^2}\\
&n_4\doteqdot N_{40}(l_4)+N_{41}(l_4)=\frac{\sin\beta l_4}{\Omega_4}+\frac{n_{44}}{\Omega_4^2}\doteqdot\frac{\sin\beta l_4}{\Omega}+\frac{n_{44}}{\Omega^2}
\end{aligned}\right\} \quad\cdots\cdots(38)$$

$$
\left.\begin{aligned}
f_5&=\sum_{s=0}^{n}\left\{F_{2s}(l_1)-F_{1s}(l_1)\right\}\doteqdot F_{20}(l_1)+F_{21}(l_1)-F_{10}(l_1)-F_{11}(l_1)\\
&=F_{20}(l_1)-F_{10}(l_1)+F_{21}(l_1)-F_{11}(l_1)=0-\frac{\cos\beta l_1}{\Omega_1}+\frac{f_{21}}{\Omega_1\Omega_2}-\frac{f_{12}}{\Omega_1{}^2}\doteqdot-\frac{\cos\beta l_1}{\Omega}\\
h_5&\doteqdot H_{20}(l_1)-H_{10}(l_1)+H_{21}(l_1)-H_{11}(l_1)=\frac{\cos\beta l_1}{\Omega_2}+\frac{h_{21}}{\Omega_2{}^2}-\frac{h_{12}}{\Omega_1\Omega_2}\doteqdot\frac{\cos\beta l_1}{\Omega}\\
j_5&\doteqdot J_{20}(l_1)-J_{10}(l_1)+J_{21}(l_1)-J_{11}(l_1)=\frac{\sin\beta l_1}{\Omega_2}+\frac{j_{21}}{\Omega^2}-\frac{j_{12}}{\Omega_1\Omega_2}\doteqdot\frac{\sin\beta l_1}{\Omega}\\
k_5&\doteqdot \mathrm{K}_{20}(l_1)-\mathrm{K}_{10}(l_1)+\mathrm{K}_{21}(l_1)-\mathrm{K}_{11}(l_1)=\frac{k_{21}}{\Omega_2\Omega_3}-\frac{k_{12}}{\Omega_1\Omega_3}\doteqdot 0\\
l_5&\doteqdot L_{20}(l_1)-L_{10}(l_1)+L_{21}(l_1)-L_{11}(l_1)=\frac{l_{21}}{\Omega_2\Omega_3}-\frac{l_{12}}{\Omega_1\Omega_3}\doteqdot 0\\
m_5&\doteqdot M_{20}(l_1)-M_{10}(l_1)+M_{21}(l_1)-M_{11}(l_1)=\frac{m_{21}}{\Omega_2\Omega_4}-\frac{m_{12}}{\Omega_1\Omega_4}\doteqdot 0\\
n_5&\doteqdot N_{20}(l_1)-N_{10}(l_1)+N_{21}(l_1)-N_{11}(l_1)=\frac{m_{21}}{\Omega_2\Omega_4}-\frac{m_2}{\Omega_1\Omega_4}\doteqdot 0
\end{aligned}\right\}\cdots\cdots(39)
$$

$$
\left.\begin{aligned}
f_6&=\sum_{s=0}^{n}\left\{F_{3s}(l_1)-F_{2s}(l_2)\right\}\doteqdot F_{30}(l_2)-F_{20}(l_2)+F_{31}(l_2)-F_{21}(l_2)=\frac{f_{32}}{\Omega_1\Omega_3}-\frac{f_{23}}{\Omega_1\Omega_2}\doteqdot 0\\
h_6&\doteqdot H_{30}(l_2)-H_{20}(l_2)+H_{31}(l_2)-H_{21}(l_2)=-\frac{\cos\beta l_2}{\Omega_2}-\frac{h_{32}}{\Omega_2\Omega_3}-\frac{h_{23}}{\Omega_2{}^2}\doteqdot-\frac{\cos\beta l^2}{\Omega}\\
j_6&=J_{30}(l^2)-J_{20}(l_2)+J_{31}(l_2)-J_{21}(l_2)=-\frac{\sin\beta l_2}{\Omega_2}-\frac{j_{32}}{\Omega_2\Omega_3}-\frac{j_{23}}{\Omega_2{}^2}\doteqdot-\frac{\sin\beta l_2}{\Omega}\\
k_6&\doteqdot \mathrm{K}_{30}(l_2)-\mathrm{K}_{20}(l_2)+\mathrm{K}_{31}(l_2)-\mathrm{K}_{21}(l_2)=\frac{\cos\beta l_2}{\Omega_3}+\frac{k_{32}}{\Omega_3{}^2}-\frac{k_{23}}{\Omega_2\Omega_3}\doteqdot\frac{\cos\beta l_2}{\Omega}\\
l_6&\doteqdot L_{30}(l_2)+L_{20}(l_2)+L_{31}(l_2)-L_{21}(l_2)=\frac{\sin\beta l_2}{\Omega_3}+\frac{l_{32}}{\Omega_3{}^2}-\frac{l_{23}}{\Omega_2\Omega_3}\doteqdot\frac{\sin\beta l_2}{\Omega}\\
m_6&\doteqdot M_{30}(l_2)-M_{20}(l_2)+M_{31}(l_2)-M_{21}(l_2)=\frac{m_{32}}{\Omega_3\Omega_4}-\frac{m_{23}}{\Omega_2\Omega_4}\doteqdot 0\\
n_6&\doteqdot N_{30}(l_2)-N_{20}(l_2)+N_{31}(l_2)-N_{21}(l_2)=\frac{n_{32}}{\Omega_3\Omega_4}-\frac{n_{23}}{\Omega_2\Omega_4}\doteqdot 0
\end{aligned}\right\}\cdots\cdots(40)
$$

$$
\left.\begin{aligned}
f_7&=\sum_{s=0}^{n}\left\{F_{4s}(l_3)-F_{3s}(l_3)\right\}\doteqdot F_{40}(l_3)-F_{30}(l_3)+F_{41}(l_3)-F_{31}(l_3)=\frac{f_{43}}{\Omega_1\Omega_4}-\frac{f_{34}}{\Omega_1\Omega_3}\doteqdot 0\\
h_7&\doteqdot H_{40}(l_3)-H_{30}(l_3)+H_{41}(l_3)-H_{31}(l_3)=\frac{h_{43}}{\Omega_2\Omega_4}-\frac{h_{34}}{\Omega_2\Omega_3}\doteqdot 0\\
j_7&\doteqdot J_{40}(l_3)-J_{30}(l_3)+J_{41}(l_3)-J_{31}(l_3)=\frac{j_{43}}{\Omega_2\Omega_4}\quad\frac{j_{43}}{\Omega_2\Omega_3}\doteqdot 0\\
k_7&\doteqdot K_{40}(l_3)-K_{30}(l_3)+K_{41}(l_3)-K_{31}(l_3)=-\frac{\cos\beta l_3}{\Omega_3}+\frac{k_{43}}{\Omega_3\Omega_4}-\frac{k_{34}}{\Omega_3{}^2}\doteqdot-\frac{\cos\beta l_3}{\Omega}\\
l_7&\doteqdot L_{40}(l_3)-L_{30}(l_3)+L_{41}(l_3)+L_{41}(l_3)=-\frac{\sin\beta l_3}{\Omega_3}+\frac{l_{43}}{\Omega_3\Omega_4}-\frac{l_{34}}{\Omega_3{}^2}\doteqdot-\frac{\sin\beta l_3}{\Omega}\\
m_7&\doteqdot M_{40}(l_3)-M_{30}(l_3)+M_{41}(l_3)-M_{31}(l_3)=\frac{\cos\beta l_3}{\Omega_4}+\frac{m_{43}}{\Omega_4{}^2}-\frac{m_{34}}{\Omega_3\Omega_4}\doteqdot\frac{\cos\beta l_3}{\Omega}\\
n_7&\doteqdot N_{40}(l_3)-N_{30}(l_3)+N_{41}(l_3)-N_{31}(l_3)=\frac{\sin\beta l_3}{\Omega_4}+\frac{n_{43}}{\Omega_4{}^2}-\frac{n_{34}}{\Omega_3\Omega_4}\doteqdot\frac{\sin\beta l_3}{\Omega}
\end{aligned}\right\}\cdots\cdots(41)
$$

처럼 쓸 수 있으므로 式 (1)~(3)에 依하여

$$
\begin{aligned}
I_1{}^a(l_1)=j\frac{E_1}{30\Delta_0}\Bigg(&\Delta_1\sum_{s=0}^{n-1}F_{1s}(l_1)-\Delta_4\sum_{s=0}^{n-1}H_{1s}(l_1)+\Delta_7\sum_{s=0}^{n-1}J_{1s}(l_1)-\Delta_{10}\sum_{s=0}^{n-1}K_{1s}(l_1)\\
&+\Delta_{13}\sum_{s=0}^{n-1}L_{1s}(l_1)-\Delta_{16}\sum_{s=0}^{n-1}M_{1s}(l_1)+\Delta_{19}\sum_{s=0}^{n-1}N_{1s}(l_1)\Bigg)
\end{aligned}
$$

$$=j\frac{E_1}{30\Delta_0}\left(\Delta_1 f_8-\Delta_4 h_8+\Delta_7 j_8-\Delta_{10}k_8+\Delta_{13}l_8-\Delta_{16}m_8+\Delta_{19}n_8\right)$$

$$=j\frac{E_1}{30\Delta_0}\cdot\Delta_a,$$

$$\Delta_a=\begin{vmatrix} f_2 & h_2 & j_2 & k_2 & l_2 & m_2 & n_2\\ f_3 & h_3 & j_3 & k_3 & l_3 & m_3 & n_2\\ f_4 & h_4 & j_4 & k_4 & l_4 & m_4 & n_4\\ f_5 & h_5 & j_5 & k_5 & l_5 & m_5 & n_5\\ f_6 & h_6 & j_6 & k_6 & l_6 & m_6 & n_6\\ f_7 & h_7 & j_7 & k_7 & l_7 & m_7 & n_7\\ f_8 & h_8 & j_8 & k_8 & l_5 & m_8 & n_8 \end{vmatrix} \quad\cdots\cdots(42)$$

$$I_2^b(l_2)=\frac{jE_2}{30\Delta_0}\left(-\Delta_2\sum_{s=0}^{n-1}F_{2s}(l_2)+\Delta_5\sum_{s=0}^{n-1}H_{2s}(l_2)-\Delta_s\sum_{s=0}^{n-1}J_{2s}(l_2)+\Delta_{11}\sum_{s=0}^{n-1}K_{2s}(l_2)\right.$$
$$\left.-\Delta_{14}\sum_{s=0}^{n-1}L_{2s}(l_2)+\Delta_{17}\sum_{s=0}^{n-1}M_{2s}(l_2)-\Delta_{20}\sum_{s=0}^{n-1}N_{2s}(l_2)\right)$$

$$=\frac{jE_2}{30\Delta_0}\left(-\Delta_2 f_9+\Delta_5 h_9-\Delta_8 j_9+\Delta_{11}k_9-\Delta_{14}l_9+\Delta_{17}m_q-\Delta_{20}n_9\right)$$

$$=-j\frac{E_2}{30\Delta_0}\Delta_b,$$

$$\Delta_b=\begin{vmatrix} f_1 & h_1 & j_1 & k_1 & l_1 & m_1 & n_1\\ f_3 & h_3 & j_3 & k_3 & l_3 & m_3 & n_3\\ f_4 & h_4 & j_4 & k_4 & l_4 & m_4 & n_4\\ f_5 & h_5 & j_5 & k_5 & l_5 & m_5 & n_5\\ f_6 & h_6 & j_6 & k_6 & l_6 & m_6 & n_6\\ f_7 & h_7 & j_7 & k_7 & l_7 & m_7 & n_7\\ f_9 & h_9 & j_9 & k_9 & l_9 & m_9 & n_9 \end{vmatrix} \quad\cdots\cdots(43)$$

$$I_3^c(l_3)=\frac{jE_3}{30\Delta_0}\left(\Delta_3\sum_{s=0}^{n-1}F_{3s}(l_3)-\Delta_6\sum_{s=0}^{n-1}H_{3s}(l_3)+\Delta_9\sum_{s=0}^{n-1}J_{3s}(l_3)-\Delta_{12}\sum_{s=0}^{n-1}K_{3s}(l_3)\right.$$
$$\left.+\Delta_{15}\sum_{s=0}^{n-1}L_{3s}(l_3)-\Delta_{18}\sum_{s=0}^{n-1}M_{3s}(l_3)-\Delta_{21}\sum_{s=0}^{n-1}N_{3s}(l_3)\right)$$

$$=\frac{jE_3}{30\Delta_0}\left(\Delta_3 f_{10}-\Delta_6 h_{10}+\Delta_9 j_{10}-\Delta_{12}k_{10}+\Delta_{15}l_{10}-\Delta_{18}m_{10}+\Delta_{21}n_{10}\right)$$

$$=\frac{jE_3}{30\Delta_0}\Delta c,$$

$$\Delta_c=\begin{vmatrix} f_1 & h_1 & j_1 & k_1 & l_1 & m_1 & n_1\\ f_2 & h_2 & j_2 & k_2 & l_2 & m_2 & n_2\\ f_4 & h_4 & j_4 & k_4 & l_4 & m_4 & n_4\\ f_5 & h_5 & j_5 & k_5 & l_5 & m_5 & n_5\\ f_6 & h_6 & j_6 & k_6 & l_6 & m_6 & n_6\\ f_7 & h_7 & j_7 & k_7 & l_7 & m_7 & n_7\\ f_{10} & h_{10} & j_{10} & k_{10} & l_{10} & m_{10} & n_{10} \end{vmatrix} \quad\cdots\cdots(44)$$

Δ_0, Δ_a, Δ_b, 및 Δ_c에 $f_1 \sim n_{10}$의 값을 代入함으로써 附錄 I에 의하여

$$\Delta_0 = -\frac{1}{\Omega^4}\left\{\cos\beta l_4 + \frac{\alpha_1}{\Omega}\right\} \quad \cdots\cdots(45)$$

$$\alpha_1 = f_{44} + h_{44} + k_{44} + m_{44} \quad \cdots\cdots(46)$$

$$\Delta_a = \frac{1}{\Omega^5}\left\{\cos\beta l \cdot \sin(l_4 - l_1) + \frac{\beta_a}{\Omega}\right. \quad \cdots\cdots(47)$$

$$\beta_a = \cos\beta l_1[-\sin\beta l_1(h_{44}+k_{44}+m_{44}) + \cos\beta l_1(j_{44}+l_{44}+n_{44}) + \sin\beta l_4(h_{12}+k_{12}+m_{12}) - \cos\beta l_4(l_{12}+n_{12})] + \sin\beta(l_4-l_1)\ f_{12} \quad \cdots\cdots(48)$$

$$\Delta_b = -\frac{1}{\Omega^5}\left\{\cos\beta l_2 \sin\beta(l_4-l_2) + \frac{\beta_b}{\Omega}\right\} \quad \cdots\cdots(49)$$

$$\beta_b = \cos\beta l_2[-\sin\beta l_2 \cdot (k_{44}+m_{44}) + \cos\beta l_2(l_{44}+n_{44}) + \sin\beta l_4(k_{23}+m_{23}) - \cos\beta l_4(l_{23}+n_{23}) + \sin\beta(l_4-l_2)(f_{23}+h_{23}) \quad \cdots\cdots(50)$$

$$\Delta_c = \frac{1}{\Omega^5}\left\{\cos\beta l_3 \sin\beta(l_4-l_3) + \frac{\beta_c}{\Omega}\right. \quad \cdots\cdots(51)$$

$$\beta_c = \cos\beta l_3(\cos\beta l_3 \cdot n_{44} - \sin\beta l_3 m_{44}) + \sin\beta(l_4-l_3)(f_{34}+h_{34}+k_{34}) \quad \cdots\cdots(52)$$

따라서 給電點 1과 4의 給電 임피이던스 Z_A, 給電點 2와 5의 給電 임피던스 Z_B 및 給電點 3과 6의 給電 임피이던스 Z_C는 各各式 (42), (43) 및 (44)로 부터[2]

$$Z_A = \frac{E_1}{I_1^a(l_1)} = \frac{30\,\Delta_0}{j\Delta_a} = j30\Omega\frac{\cos\beta l_4 + \frac{\alpha_1}{\Omega}}{\cos\beta l_1 \cdot \sin\beta(l_4-l_1) + \frac{\beta_a}{\Omega}} \quad \cdots\cdots(53)$$

$$Z_B = \frac{E_2}{I_2^b(l_2)} = \frac{30\,\Delta_0}{-j\Delta_b} = j30\Omega\frac{\cos\beta l_4 + \frac{\alpha_1}{\Omega}}{\cos\beta l_2 \cdot \sin\beta(l_4-l_2) + \frac{\beta_b}{\Omega}} \quad \cdots\cdots(54)$$

$$Z_C = \frac{E_3}{I_3^c(l_3)} = \frac{30\,\Delta_0}{j\Delta_C} = j30\Omega \cdot \frac{\cos\beta l_4 + \frac{\alpha_1}{\Omega}}{\cos\beta l_3 \cdot \sin\beta(l_4-l_3) + \frac{\beta_C}{\Omega}} \quad \cdots\cdots(55)$$

3. 相互 임피이던스

Ynm로 그림 1의 給電點 n과 m 사이의 相互 아드미턴스를 나타내기로 한다면 式 (1)~(3)으로부터 $E_1 \sim E_3$이 공급되었을때의 l_1, l_2, 및 l_3 點이 電流 $I(l_1)$, $I(l_2)$, 및 $I(l_3)$의 各各

$$I(l_1) = I_1^a(l_1) + I_1^b(l_1) + I_1^c(l_1) = I_2^a(l_1) + I_2^b(l_1) + I_2^c(l_1) = E_1(y_{11}+y_{14}) + E_2(y_{12}+y_{15}) + E_3(y_{13}+y_{16}) \quad \cdots\cdots(56)$$

$$I(l_2) = I_2^a(l_1) + I_2^b(l_2) + I_2^c(l_1) = I_3^a(l_2) + I_3^b(l_2) + I_3^c(l_2) = E_1(y_{21}+y_{24}) + E_2(y_{22}+y_{25}) + E_3(y_{23}+y_{26}) \quad \cdots\cdots(57)$$

$$I(l_3) = I_3^a(l_3) + I_3^b(l_3) + I_3^c(l_3) = I_4^a(l_3) + I_4^b(l_3) + I_4^c(l_3) = E_1(y_{31}+y_{34}) + E_2(y_{32}+y_{35}) + E_3(y_{33}+y_{36}) \quad \cdots\cdots(58)$$

式과 같다. 따라서 (56)~(58)式에서 $E_2 = E_3 = 0$이라고 놓았을때의 l_2點의 電流와 $E_1 = E_3 = 0$이라고 놓았을 때의 l_1點의 電流로부터

$$\frac{E_1}{I_2^a(l_2)} = \frac{E_1}{I_3^a(l_3)} = \frac{1}{y_{21}+y_{24}} = \frac{E_2}{I_1^b(l_1)} = \frac{1}{y_{12}+y_{15}} = Z_{12} \quad \cdots\cdots(59)$$

$E_2 = E_3 = 0$ 일때의 l_3點이 電流와 $E_1 = E_2 = 0$ 일때의 l_1點의 電流에서

$$\frac{E_1}{I_3^a(l_3)} = \frac{E_1}{I_4^a(l_1)} = \frac{1}{y_{31}+y_{34}} = \frac{E_3}{I_1^c(l_1)} = \frac{1}{y_{13}+y_{16}} = Z_{13} \quad \cdots\cdots(60)$$

$E_1=E_2=0$ 일때의 l_3點의 電流와 $E_1=E_2=0$ 일때의 l_2點의 電流에서

$$\frac{E_2}{I_3^b(l_3)}=\frac{E_2}{I_4^b(l_3)}=\frac{1}{y_{22}+y_{25}}=\frac{E_3}{I_2^c(l_2)}=\frac{1}{y_{23}+y_{26}}=Z_{23} \qquad (61)$$

한편 (1)式으로부터

$$I_2^a(l_2)=j\frac{E_1}{30\Delta_0}\Big\{\Delta_1\sum_{s=0}^{n-1}F_{2s}(l_2)-\Delta_4\sum_{s=0}^{n-1}H_{2s}(l_2)+\Delta_7\sum_{s=0}^{n-1}J_{2s}(l_2)-\Delta_{10}\sum_{s=}^{n-1}K_{2s}(l_2)$$
$$+\Delta_{13}\sum_{s=0}^{n-1}L_{2s}(l_2)-\Delta_{11}\sum_{s=}^{n-1}M_{2s}(l_2)+\Delta_{19}\sum_{s=0}^{n-1}N_{2s}(l_2)\Big\}$$
$$=j\frac{E_1}{30\Delta_0}\Big\{\Delta_1 f_9-\Delta_4 h_9+\Delta_7 j_9-\Delta_{10}\ k_9+\Delta_{13}\cdot l_9-\Delta_{16}\ m_9+\Delta_{19} n_9\Big\}$$
$$=j\frac{E}{30\Delta_0}\Delta y_{12},$$

$$\Delta y_{12}=\begin{vmatrix} f_2 & h_2 & j_2 & k_2 & f_2 & m_2 & n_2 \\ f_3 & h_3 & j_3 & k_3 & l_3 & m_3 & n_3 \\ f_4 & h_4 & j_4 & k_4 & l_4 & m_4 & n_4 \\ f_5 & h_5 & j_5 & k_5 & l_5 & m_5 & n_5 \\ f_6 & h_6 & j_6 & k_6 & l_6 & m_6 & n_6 \\ f_7 & h_7 & j_7 & k_7 & l_7 & m_7 & n_7 \\ f_9 & h_9 & j_9 & k_9 & l_9 & m_9 & n_9 \end{vmatrix} \qquad (62)$$

$$I_4^a(l_3)=j\frac{E_1}{30\Delta_0}\Big\{\Delta_1\sum_{s=0}^{n-1}F_{4s}(l_3)-\Delta_4\sum_{s=0}^{n-1}H_{4s}(l_3)+\Delta_7\sum_{s=0}^{n-1}J_{4s}(l_3)-\Delta_{10}\sum_{s=0}^{n-1}K_{4s}(l_3)$$
$$+\Delta_{13}\sum_{s=0}^{n-1}L_{4s}(l_3)-\Delta_{16}\sum_{s=0}^{n-1}M_{4s}(l_3)+\Delta_{19}\sum_{s=0}^{n-1}N_{4s}(l_3)\Big\}$$
$$=j\frac{E_1}{30\Delta_0}\Big\{\Delta_1 f_{11}-\Delta_4 h_{11}+\Delta_7 j_{11}-\Delta_{10} k_{11}+\Delta_{13} l_{11}-\Delta_{16} m_{11}+\Delta_{19} n_{11}\Big\}$$
$$=j\frac{E_1}{30\Delta_0}\cdot\Delta y_{13},$$

$$\Delta y_{13}=\begin{vmatrix} f_2 & h_2 & j_2 & k_2 & l_2 & m_2 & n_2 \\ f_3 & h_3 & j_3 & k_3 & l_3 & m_3 & n_3 \\ f_4 & h_4 & j_4 & k_4 & l_4 & m_4 & n_4 \\ f_5 & h & j_5 & k_5 & l_5 & m_5 & n_5 \\ f_6 & h_6 & j_6 & k_6 & l_6 & m_6 & n_6 \\ f_7 & h_7 & j_7 & k_7 & l_7 & m_7 & n_7 \\ f_{11} & h_{11} & j_{11} & k_{11} & l_{11} & m_{11} & n_{11} \end{vmatrix} \qquad (63)$$

$$I_4^b(l_3)=\frac{jE_2}{30\Delta_0}\Big\{-\Delta_2\sum_{s=0}^{n-1}F_{4s}(l_3)+\Delta_5\sum_{s=0}^{n-1}H_{4s}(l_3)-\Delta^8\sum_{s=0}^{n-1}J_{4s}(l_3)+\Delta_4\sum_{s=0}^{n-1}K_{4s}(l_3)$$
$$-\Delta_{4s}\sum_{s=0}^{n-1}L_{4s}(l_3)+\Delta_{17}\sum_{s=0}^{n-1}M_{4s}(l_3)-\Delta_{20}\sum_{s=0}^{n-1}N_{4s}(l^3)\Big\}$$
$$=\frac{jE_2}{30\Delta}\Big\{-\Delta_2 f_{11}+\Delta_5 h_{11}-\Delta_8 j_{11}+\Delta_{11} k_{11}-\Delta_{14} l_{11}+\Delta_{17} m_{11}-\Delta_{20}\, n_{11}\Big\}$$

$$=\frac{jE_2}{30\Delta_0}\cdot\Delta y_{23},$$

$$\Delta y_{23}=\begin{vmatrix} f_1 & h_1 & j_1 & k_1 & l_1 & m_1 & n_1 \\ f_3 & h_3 & j_3 & k_3 & l_3 & m_3 & n_3 \\ f_4 & h_4 & j_4 & k_4 & l_4 & m_4 & n_4 \\ f_5 & h_5 & j_5 & k_5 & l_5 & m_5 & n_5 \\ f_6 & h_6 & j_6 & k_6 & l_6 & m_6 & n_6 \\ f_7 & h_7 & j_7 & k_7 & l_7 & m_7 & n_7 \\ f_{11} & h_{11} & j_{11} & k_{11} & l_{11} & m_{11} & n_{11} \end{vmatrix} \qquad \cdots\cdots(64)$$

이며 Δy_{12}, Δy_{13}, 및 Δ_{23}의 값들은 附錄 II에 의하여

$$\Delta y_{12}=\frac{1}{\Omega^5}\left\{\cos\beta l_1\cdot\sin\beta(l_4-l_2)+\frac{\beta_{12}}{\Omega}\right\} \qquad \cdots\cdots(65)$$

$$\beta_{12}=\cos\beta l_1[-\sin\beta l_2(h_{44}+k_{44}+m_{44})+\cos\beta l_2(j_{44}+l_{44}+n_{44}) +\sin\beta l_4(h_{23}+k_{23}+m_{23})-\cos\beta l_4(j_{23}+l_{23}+n_{23})] -\sin\beta l(l_4-l_1)\cdot f_{44}+\sin\beta(l_4-l_1)\cdot f_{23} \qquad \cdots\cdots(66)$$

$$\Delta y_{13}=\frac{1}{\Omega^5}\left\{\cos\beta l_1\cdot\sin\beta(l_4-l_3)+\frac{\beta_{13}}{\Omega}\right\} \qquad \cdots\cdots(67)$$

$$\beta_{13}=\cos\beta l_1[-\sin\beta l_3(h_{44}+k_{44}+m_{44})+\cos\beta l_3(j_{44}+l_{44}+n_{44}) +\sin\beta l_4\cdot(h_{43}+k_{43}+m_{43})-\cos\beta l_3(j_{43}+l_{43}+n_{43})] -\sin\beta(l_3-l_1)\cdot f_{44}+\sin\beta(l_4-l_1)\cdot f_{43} \qquad \cdots\cdots(68)$$

$$\Delta y_{23}=-\frac{1}{\Omega^5}\left\{\cos\beta l_2\ \sin\beta(l_4-l_3)+\frac{\beta_{23}}{\Omega}\right\} \qquad \cdots\cdots(69)$$

$$\beta_{23}=\cos\beta l_2[-\sin\beta l_3(k_{44}+m_{44})+\cos\beta l_3(l_{44}+n_{44})+\sin\beta l_4(k_{43}+m_{43})-\cos\beta l_4(l_{43}+n_{43})] -\sin\beta(l_3+l_3)\cdot(f_{44}+h_{44})+\sin\beta(l_4-l_2)\cdot(f_{43}+h_{43}) \qquad \cdots\cdots(70)$$

따라서 點 1과 4에 E_1, 2와 5에 E_2, 3과 6에 E_3을 給電하였을때의 給電點 1과 2, 1과 3, 및 2와 3 사이의 相互 임피이던스를 各各 Z_{12}, Z_{13} 및 Z_{23}이라 하면 (62)~(64)式으로 부터

$$Z_{12}=\frac{E_1}{I_2^a(l_2)}=\frac{E}{j\frac{E_1}{30\Delta_0}\Delta y_{12}}=-j\frac{30\Delta_0}{\Delta y_{12}}=j30\cdot\Omega\cdot\frac{\cos\beta l_4+\frac{\alpha_1}{\Omega}}{\cos\beta l_1\cdot\sin\beta(l_4-l_2)+\frac{\beta_{12}}{\Omega}} \qquad \cdots\cdots(71)$$

$$Z_{13}=\frac{E}{I_4^a(l_2)}=\frac{E_1}{j\frac{E_1}{30\Delta_0}\Delta y_{13}}=-j\frac{30\Delta_0}{\Delta y_{13}}=j30\cdot\Omega\cdot\frac{\cos\beta l_4+\frac{\alpha_1}{\Omega}}{\cos\beta l_1\cdot\sin\beta(l_4-l_3)+\frac{\beta_{13}}{\Omega}} \qquad \cdots\cdots(72)$$

$$Z_{23}=\frac{E_2}{I_4^b(l_3)}=\frac{E}{j\frac{E_2}{30\Delta_0}\Delta y_{23}}=-j\frac{30\Delta_0}{\Delta y_{23}}=-j30\cdot\Omega\cdot\frac{\cos\beta l_4+\frac{\alpha_1}{\Omega}}{\cos\beta l_2-\sin\beta(l_4-l_3)+\frac{\beta_{23}}{\Omega}} \qquad \cdots\cdots(73)$$

4. 임피이던스의 數値計算

(53)~(55) 및 (71)~(73)式의 임피이던스들을 計算하기 위하여 式(46)의 α_1과 式(50)의 β_b 內의 各要素들의 內容을 整理하여보면 다음과 같다.

$$\alpha_1=f_{44}+h_{44}+k_{44}+m_{44}$$

$$=-\cos\beta l_4\cdot E(2\beta l_4)+\frac{1}{2}e^{j\beta l4}\cdot E(4\beta l_4) \qquad \cdots\cdots(74)$$

$$k_{44}+m_{44}=\cos\beta l_2\cdot ln\frac{l_4-l_2}{l_4+l_2}+\cos\beta l_{40} ln\frac{l_4+l_2}{l_4-l_2}+\cos\beta l\left\{E[\beta(l_4+l_2)]-E[\beta(l_4-l_2)]\right\}$$

$$-\cos\beta l_4\cdot E(2\beta l_4)+\frac{1}{2}e^{j\beta l_4}\cdot\left\{E(4\beta l_4)-E[2\beta(l_4+l_2)]+E[2\beta(l_4-l_2)]\right\} \quad\cdots\cdots(75)$$

$$l_{44}+n_{44}=\sin\beta l_2\cdot ln\frac{l_4-l_2}{l_4+l_2}+\sin\beta l_4\cdot ln\frac{(2l_4)^2}{l_4^2-l_2^2}+\sin\beta l_2\left\{E[(\beta(l_4+l_2)]-E[\beta(l_4-l_2)]\right\}$$

$$-\sin\beta l_4\cdot E(2\beta l_4)+\frac{j}{2}e^{j\beta l_4}\cdot\left\{E[4\beta l_4)-E[2\beta(l_4+l_2)]-E[2\beta(l_4-l_2)]\right\} \quad\cdots\cdots(76)$$

$$k_{23}+m_{23}=\cos\beta l_2\cdot ln\frac{l_4^2}{l_4^2-l_2^2}+\cos\beta l_4\cdot ln\frac{l_4^2-l_2^2}{l_4^2}+\cos\beta l_2\cdot E(2\beta l_2)$$

$$-\cos\beta l_4\left\{E[\beta(l_4+l_2)]+E[\beta(l_4-l_2)]\right\}+\frac{1}{2}e^{j\beta l_2}\cdot\left\{E[2\beta(l_4+l_2)]-E(4\beta l_2)\right\}$$

$$+\frac{1}{2}e^{-j\beta l_2}\cdot E[2\beta(l_4-l_2)] \quad\cdots\cdots(77)$$

$$l_{23}+n_{23}=\sin\beta l_2\cdot ln\frac{l_4^2}{(2l_2)^2}\cdot\frac{l_4+l_2}{l_4-l_2}+\sin\beta l_4\cdot ln\frac{l_4^2-l_2^2}{l_4^2}+\sin\beta l_2\cdot E(2\beta l_2)$$

$$-\sin\beta l_4\cdot\left\{E[\beta(l_4+l_2)]+E[\beta(l_4-l_2)]\right\}+\frac{j}{2}e^{+j\beta l_2}\cdot\left\{E[2\beta(l_4+l_2)-E(4\beta l_2)\right\}$$

$$+\frac{j}{2}e^{-j\beta l_2}\cdot E[2\beta(l_4-l_2)] \quad\cdots\cdots(78)$$

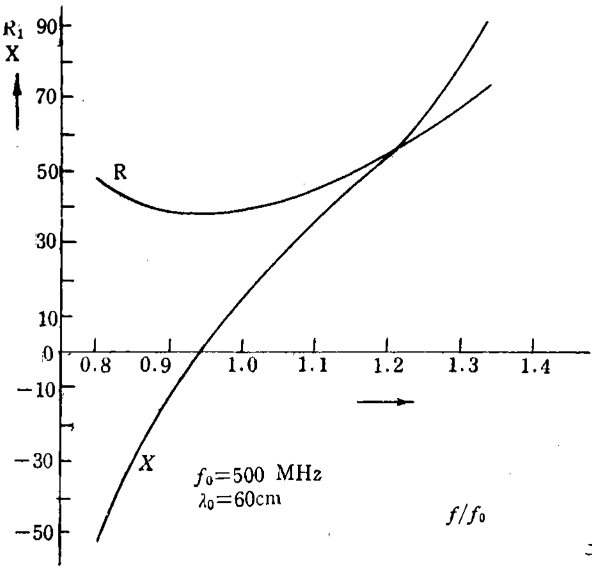
$$f_{23}+h_{23}=-\cos\beta l_2\cdot E(2\beta l_2)+\frac{1}{2}e^{j\beta l}\cdot E(4\beta l_2) \quad\cdots\cdots(79)$$

위의 관계式들을 사용하여서 $\rho=0.8$[cm], $2l_4=150$[cm]인 導體棒의 對稱點 2, 5(그림 1 참조)에만 同一한 起電力을 供給하였을때의 給電 임피이던스 Z_B를 (54)式에 依하여 計算하여본다.

우선 Ω는

$$\Omega=2ln\frac{2l_4}{\rho}=2\times2.30259\times\log_{10}\frac{150}{0.8}\fallingdotseq10.47$$

이며 $f_0=500MH_z$에 대해서는 $\lambda_0=60$[cm]이므로 $l_4=\frac{5}{4}\lambda_0$가 되고 周波數를 $0.80f_0\sim1.25f_0$ 범위에서 變化했을때의 $Z_z=R+jX$의 값은 그림 2와 같이 變化하며 f_0 무근에서는 다이폴 안테나의 給電임피이



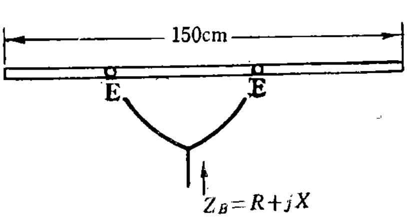


그림 2. Z_B의 주파수 특성

던스와 比例關係에 있는 폴딛드 다이폴 안테나의 給電임피이던스 變化[4]와 同一한 경향임을 알 수 있다. 따라서 Z_B 뿐만 아니라 같은 解析節次에 의해서 求해질 다른 給電 임피이던스 및 相互 임피이던스 式들도 틀림이 없을 것으로 생각된다.

5. 結 論

이 論文에 依하여 線形多波長 導線에서 中央點에 對하여 서로 對稱인 한雙의 給電點에 給電하였을때의 給電 임피이던스는 물론 또 다른 任意의 한雙의 給電點과의 相互임피이던스 計算이 처음으로 可能하게 되었다. 電子計算機를 使用한다면 여러 경우에 대한 임피이던스의 주파수특성을 얻을 수 있을 것이며 測定機器未備로 아직 充分한 實驗測定은 하지 못하였으나 앞으로 線形多波長 안테나의 解析 및 設計에 緊要하게 使用될 것으로 생각한다.

이 硏究는 高麗大學校硏究費로 이뤄 졌으며 이 기회에 방대한 계산을 도와준 高麗大學校 電子科 金仁洙, 李忠根 兩君에게 감사한다.

參考文獻

(1) 朴檉基: 六點 給電안테나의 電流分布, 大韓電子工學會誌, 6 卷 3 號, 1969

(2) K. Nagai : Five Points Fed Linear Antennas Rep. Res. Inst. Elect. Comm. Tohoku Univ. Vol. 15 No. 2 1963

(3) 日本海軍技術硏究所報告 Si(x) 表

(4) Stanford Research Institute Tech. Rep. C(x) 表

(5) 內田, 虫明: 超短波空中線, Corona 社, 1954

附　錄 I

$$\Omega_5 \Delta_0 = \begin{vmatrix} -\sin\beta l_1 & \sin\beta l_1 & -\cos\beta l_1 & \bigcirc & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & -\sin\beta l_2 & \cos\beta l_2 & \sin\beta l_2 & -\cos\beta l_2 & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & -\sin\beta l_3 & \cos\beta l_3 & \sin\beta l_3 & -\cos\beta l_3 \\ f_{44} & h_{44} & j_{44} & k_{44} & l_{44} & \Omega\cos\beta l_4+\mathrm{m}_{44} & \Omega\sin\beta l_4+\mathrm{n}_{44} \\ -\cos\beta l_1 & \cos\beta l_1 & \sin\beta l_1 & \bigcirc & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & -\cos\beta l_2 & -\sin\beta l_2 & \cos\beta l_2 & \sin\beta l_2 & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & -\cos\beta l_3 & -\sin\beta l_3 & \cos\beta l_3 & \sin\beta l_3 \end{vmatrix}$$

$$= -\Omega\cos\beta l_4 - (f_{44}+h_{44}+k_{44}+m_{44})$$

$$\Omega^7 \Delta_a = \begin{vmatrix} \bigcirc & -\sin\beta l_2 & \cos\beta l_2 & \sin\beta l_2 & -\cos\beta l_2 & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & \sin\beta l_3 & -\cos\beta l_3 & -\sin\beta l_3 & \cos\beta l_3 \\ f_{4} & h_{44} & j_{44} & k_{44} & l_{44} & \Omega\cos\beta l_4+\mathrm{m}_{44} & \Omega\sin\beta l_4+\mathrm{n}_{44} \\ \cos\beta l_1 & -\cos\beta l_1 & -\sin\beta l_1 & \bigcirc & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & \cos\beta l_1 & \sin\beta l_2 & -\cos\beta l_2 & -\sin\beta l_2 & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & \cos\beta l_3 & \sin\beta l_3 & -\cos\beta l_3 & -\sin\beta l_3 \\ \Omega\cos\beta l_1+f_{12} & h_{12} & j_{12} & k_{12} & l_{12} & m_{12} & n_{12} \end{vmatrix}$$

$$= \Omega^2\cos\beta l_1 \; \sin\beta(l_4-l_1) + \Omega\Big\{\cos\beta l_1[\cos\beta l_1(j_{44}+l_{44}+n_{44})$$

$$-\sin\beta l_1(h_{44}+k_{44}-m_{44}) + \sin\beta l_4(h_{12}+k_{12}+m_{12})$$

$$-\cos\beta l_4(l_{12}+n_{12}) - \sin\beta(l_4-l_1)\cdot f_{12}\Big\}$$

$$\Omega^7 \Delta_b = \begin{vmatrix} -\sin\beta l_1 & \sin\beta l_1 & -\cos\beta l_1 & \bigcirc & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & \sin\beta l_3 & -\cos\beta l_3 & -\sin\beta l_3 & \cos\beta l_3 \\ f_{44} & h_{44} & j_{44} & k_{44} & l_{44} & \Omega\cos\beta l_4+\mathrm{m}_{44} & \Omega\sin\beta l_4+\mathrm{n}_{44} \\ \cos\beta l_1 & -\cos\beta l_1 & -\sin\beta l_1 & \bigcirc & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & \cos\beta l_2 & \sin\beta l_2 & -\cos\beta l_2 & -\sin\beta l_2 & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & \cos\beta l_3 & \sin\beta l_3 & -\cos\beta l_3 & -\sin\beta l_3 \\ f_{23} & \Omega\cos\beta l_2+h_{23} & \Omega\sin\beta l_2+j_{23} & k_{23} & l_{23} & m_{23} & n_{23} \end{vmatrix}$$

$$= -\Omega^2\cos\beta l_2 \; \sin\beta(l_4-l_2) - \Omega\Big\{\cos\beta l_2[-\sin\beta l_2+(k_{44}+m_{44})$$

$$+\cos\beta l_2\cdot(l_{44}+n_{44}) + \sin\beta l_4\cdot(k_{23}+m_{23}) - \cos\beta l_4\cdot(l_{23}+n_{23}))]$$

$$+\sin\beta(l_4-l_2)\cdot(f_{23}+h_{23})\Big\}$$

$$\Omega^7 \Delta_c = \begin{vmatrix} -\sin\beta l_1 & \sin\beta l_1 & -\cos\beta l_1 & \bigcirc & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & -\sin\beta l_2 & \cos\beta l_2 & \sin\beta l_2 & -\cos\beta l_2 & \bigcirc & \bigcirc \\ f_{44} & h_{44} & j_{44} & k_{44} & l_{44} & \Omega\cos\beta l_4+m_{44} & \Omega\sin\beta l_4+n_{44} \\ \cos\beta l_1 & -\cos\beta l_1 & -\sin\beta l_1 & \bigcirc & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & \cos\beta l_2 & \sin\beta l_2 & -\cos\beta l_2 & -\sin\beta l_2 & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & \cos\beta l_3 & \sin\beta l_3 & -\cos\beta l_3 & -\sin\beta l_3 \\ f_{34} & h_{34} & j_{34} & \Omega\cos\beta l_3+k_{34} & \Omega\sin\beta l_3+l_{34} & m_{34} & n_{34} \end{vmatrix}$$

$$\fallingdotseq \Omega^2 \; \cos\beta l_3\cdot\sin\beta(l_4-l_{31}) + \Omega[\cos\beta l_3(\cos\beta l_3\cdot n_{44} - \sin\beta l_3\cdot m_{44})$$

$$+\sin\beta(l_4-l_3)\cdot(f_{34}+h_{34}+k_{34})]$$

附 錄 Ⅱ

$$\Omega^6\,\Delta y_{12}=\begin{vmatrix} \bigcirc & -\sin\beta l_2 & \cos\beta l_2 & \sin\beta l_2 & -\cos\beta l_2 & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & -\sin\beta l_3 & \cos\beta l_3 & \sin\beta l_3 & -\cos\beta l_3 \\ f_{44} & h_{44} & j_{44} & k_{44} & l_{44} & \Omega\cos\beta l_4+m_{44} & \Omega\sin\beta l_4+n_{44} \\ -\cos\beta l_1 & \cos\beta l_1 & \sin\beta l_1 & \bigcirc & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & -\cos\beta l_2 & -\sin\beta l_2 & \cos\beta l_2 & \sin\beta l_3 & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & -\cos\beta l_3 & -\sin\beta l_3 & \cos\beta l_3 & \sin\beta l_3 \\ f_{23} & \Omega\cos\beta l_2+h_{23} & \Omega\cos\beta l_2+j_{23} & k_{23} & l_{23} & m_{23} & n_{23} \end{vmatrix}$$

$$=\Omega^2\cos\beta l_1\ \sin\beta(l_4-l_2)+\Omega\Big\{\cos\beta l_1[\cos\beta l_2\cdot(j_{44}+l_{44}+n_{44})-\sin\beta l_2\cdot(h_{44}+k_{44}+m_{44})$$
$$-\cos\beta l_4\cdot(j_{23}+l_{23}+n_{23})+\sin\beta l_4\cdot(h_{23}+k_{23}+m_{23})]+\sin\beta(l_4-l_1)\cdot f_{23}$$
$$-\sin\beta(l_2-l_1)\cdot f_{44}\Big\}$$

$$\Omega^7\,\Delta y_{13}=\begin{vmatrix} \bigcirc & -\sin\beta l_2 & \cos\beta l_2 & \sin\beta l_2 & -\cos\beta l_2 & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & -\sin\beta l_3 & \cos\beta l_3 & \sin\beta l_3 & -\cos\beta l_3 \\ f_{44} & h_{44} & j_{44} & k_{44} & l_{44} & \Omega\cos\beta l_4+m_{44} & \Omega\sin\beta l_4+n_{44} \\ -\sin\beta l_1 & \cos\beta l_1 & \sin\beta l_1 & \bigcirc & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & -\cos\beta l_2 & -\sin\beta l_2 & \cos\beta l_2 & \sin\beta l_2 & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & -\cos\beta l_3 & -\sin\beta l_3 & \cos\beta l_3 & \sin\beta l_3 \\ f_{43} & h_{43} & j_{43} & k_{43} & l_{43} & \Omega\cos\beta l_3+m_{43} & \Omega\sin\beta l_3+n_{43} \end{vmatrix}$$

$$=\Omega^2\cos\beta l_1\cdot\sin\beta(l_4-l_3)+\Omega\Big\{\cos\beta l_1\cdot[\cos\beta l_3\cdot(j_{44}+l_{44}+n_{44})$$
$$-\sin\beta l_3\cdot(h_{44}+k_{44}+m_{44})-\cos\beta l_4\cdot(j_{43}+l_{43}+n_{43})+\sin\beta l_4\cdot(h_{43}+k_{43}+m_{43})]$$
$$+\sin\beta(l_4-l_1)\cdot f_{43}-\sin\beta(l_3-l_1)\cdot f_{44}\Big\}$$

$$\begin{vmatrix} -\sin\beta l_1 & \sin\beta l_1 & -\cos\beta l_1 & \bigcirc & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & -\sin\beta l_3 & \cos\beta l_3 & \sin\beta l_3 & -\cos\beta l_3 \\ f_{44} & h_{44} & j_{44} & k_{44} & l_{44} & \Omega\cos\beta l_4+m_{44} & \Omega\sin\beta l_4+n_{44} \\ -\cos\beta l_1 & \cos\beta l_1 & \sin\beta l_1 & \bigcirc & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & -\cos\beta l_2 & -\sin\beta l_2 & \cos\beta l_2 & \sin\beta l_2 & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & -\cos\beta l_3 & -\sin\beta l_3 & \cos\beta l_3 & \sin\beta l_3 \\ f_{43} & h_{43} & j_{43} & k_{43} & l_{43} & \Omega\cos\beta l_3+m_{43} & \Omega\sin\beta l_3+n_{43} \end{vmatrix}$$

$$=-\Omega^2\cos\beta l_2\cdot\sin\beta(l_4-l_3)$$
$$-\Omega\Big\{\cos\beta l_2\cdot[\cos\beta l_3\cdot(l_{44}+n_{44})-\sin\beta l_3\cdot(k_{44}+m_{44})$$
$$+\sin\beta l_4\cdot(k_{43}+m_{43})-\cos\beta l_4\cdot(l_{43}+n_{43})]$$
$$+\sin\beta(l_4-l_2)\cdot(f_{43}+h_{43})-\sin\beta(l_3-l_2)\cdot(f_{44}+h_{44})\Big\}$$