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Permittivity of Solid State Electron Plasma including the Effect of Diffusion

논 문
20～6～2

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Abstract

Permittivities are closely related to lattice vibrations and dispersions relations, and this paper deals with the tensor permittivities which include the effect of diffusion. It is a great convenience in the consideration of plasma waves to treat the plasma as a dielectric medium with its circumference. And, on the assumption that the motion of the ionized donors be neglected the general expression from which the tensor permittivity can be derived is derived from the view point that the plasma can be treated as a hydrodynamical fluid.

The effect of diffusion appears as perturbation terms in the tensor permittivities of the non-streaming solid state electron magnetoplasmas and affects no influence on the anisotropic terms in the specific configuration.

I. INTRODUCTION

On analyzing the characteristics of wave propagations in solid state electron plasmas, permittivities play a major role as in dielectrics, because permittivities are, especially, closely related to lattice vibrations and dispersion relations, and, furthermore, it is a principal target of the current researches to seek the plasmas which possess proper permittivities suitable for the intended specific applications. Therefore, it is of fundamental importance to investigate the influence of diffusion on the expression of the permittivity.

It is a great convience in the consideration of plasma waves to treat the plasma as a dielectric medium with its circumference. The presence of electrons in the plasma gives rise to a convection current, and this current must be accounted for in the derivation of an equivalent permittivity. When a static magnetic field is present, the plasma is anisotropic and the plasma permittivity is a tensor. Anisotropy in a plasma medium is due to the fact that electrons orbit magnetic field lines in one direction only and the plasma properties enter the wave equation only through the tensor dielectric constant.

In solids, because of the thermal vibrations of the lattice, scattering is a very important part of any problem. Aside from phonon scattering of the carriers, there are Rutherford scattering and scattering effect from neutral impurities as well as from faults. For dense semiconductor electron-hole plasmas we also have to take into account electron-hole scattering. For solids, the value of the scattering frequency ν ranges about 10^{10} to 10^{13}sec^{-1} as the temperature increases from liquid helium to room temperature. And it is usually necessary that to observe plasma-wave phenomena in solids the relaxation time of the charge carriers due to the above scatterings must be made sufficiently low so that the wave phenomena are not damped out. This requires using samples of high purity at liquid-helium temperatures and, in this paper, it is assumed that the time is a constant, because, under the ordinary conditions, the devices which utilize the characteristics of the plasmas operate in the constant environment and, also, the constituents of the devices remain unchanged.

With both finite temperature and scattering in the solid, there are diffusion effects whenever the carriers are bunched. Since most of the interesting interactions involve such bunching, diffusion can play a critical role. An important aspect of diffusion concerns the upper frequency limit that imposes on any wave instability involving bunching.

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In the analysis we will concentrate only on the response of the electron gas assuming that the motion of the ionized donors in semiconductors can be neglected and it is, also, assumed that the medium is an infinite one, the E.M. wave which propagates through the medium has the form of exp $j(\omega t-\vec{k}\cdot\vec{r})$, and it will be not considered, here, band-to-band transitions.

Ⅱ. THE GENERAL EXPRESSION

From the Helmholtz equation, the linearized equation of motion for the electron plasma and the magnetization vector and from the assumption of a perturbation in the form of exp $j(\omega t-\vec{k}\cdot\vec{r})$, the following two equations are obtained:

$$\vec{k}\times\vec{k}\times\vec{E}+\frac{\omega^2}{c^2}\underline{\underline{\epsilon}}\cdot\vec{E}=0 \qquad (1)$$

$$j\,\Omega\,\vec{v}=\eta\,\vec{E}+\vec{v}\times\vec{\omega}_c+\eta\,\vec{v}_0\times\vec{B}+j\frac{v_T{}^2\rho}{3\rho_0}\vec{k}, \qquad (2)$$

where $\underline{\underline{\epsilon}}$ and c are the tensor relative permittivity and the velocity of light, respectively, and all other symbols have the same meanings as those in the author's previous paper (JKIEE, September 1971).

From Eq. (1) and its associated equation for $\vec{J}$, we will get

$$\underline{\underline{\epsilon}}\cdot\vec{E}=\epsilon_l\,\vec{E}+\frac{1}{j\omega\epsilon_0}(\rho_0\,\vec{v}+\rho\,\vec{v}_0), \qquad (3)$$

and using Eq. (2) with its related expression for ρ, the above equation becomes

$$\underline{\underline{\epsilon}}\cdot\vec{E}=\epsilon_l\,\vec{E}-\frac{1}{\omega\epsilon_0 Q}\Big\{\frac{\rho_0}{\omega}\Big[W\eta\,\vec{R}+k\,\eta\,\vec{S}(\vec{v}_0\cdot\vec{E}) + \frac{k^2\,v_T{}^2\,\omega_p{}^2\,\epsilon_0}{3\rho_0\,Q}\vec{S}\Big(\hat{k}\cdot\Big[\vec{R}+\frac{k}{W}\,\vec{T}(\vec{v}_0\cdot\vec{E})\Big]\Big)\Big/ \Big[1-\frac{k^2\,v_T{}^2}{3\,WQ}(\hat{k}\cdot\vec{T})\Big]\Big]+\vec{v}_0\,k\,\omega_p^2\epsilon_0\,\hat{k}\cdot\Big[\vec{R} + \frac{k}{W}\vec{T}(\vec{v}_0\cdot\vec{E})\Big]\Big/\Big[1-\frac{k^2 v_T{}^2}{3\,WQ}(\hat{k}\cdot\vec{T})\Big]\Big\} \qquad (4)$$

where ϵ_l is the relative scalar permittivity of the lattice, and

$$Q=\Omega\,(\Omega^2-\omega_c{}^2)$$
$$\vec{R}=\Omega^2\,\vec{E}+j\,\Omega(\vec{\omega}_c\times\vec{E})-\vec{\omega}_c(\vec{\omega}_c\cdot\vec{E})$$
$$\vec{S}=\Omega^2\hat{k}-\vec{\omega}_c(\vec{\omega}_c\cdot\hat{k})+j\,\Omega(\vec{\omega}_c\times\hat{k})$$
$$\vec{T}=\Omega^2\hat{k}-\vec{\omega}_c(\vec{\omega}_c\cdot\hat{k})$$

Eq. (4) is the general expression from which one can define the tensor relative permittivity including the diffusion effect of the plasma, and the equation is derived from the view point that the plasma can be treated as a hydrodynamical fluid.

Ⅲ. DISCUSSION

For the case where the plasma is in the state of non-streaming and the wave vector and the static magnetic field are in the z and y directions, respectively, the permittivity tensor is, from Eq (4), given by

$$\underline{\underline{\epsilon}}=\begin{pmatrix} \epsilon_l-BC & 0 & -jBD \\ 0 & \epsilon_l-BF & 0 \\ jBD & 0 & \epsilon_l-BE \end{pmatrix}, \qquad (5)$$

where

$$B=\frac{1}{\omega\epsilon}\frac{\rho_0}{\Omega(\Omega^2-\omega_c{}^2)}$$
$$C=\eta\,\Omega^2+\frac{k^2\epsilon_0 v_T{}^2\omega_p{}^2\omega_c{}^2\Omega^2}{\rho_0[3\Omega(\Omega^2-\omega_c{}^2)\omega-k^2\,v_T{}^2\Omega^2]}$$
$$D=\eta\omega_c\Omega+\frac{k^2\epsilon_0 v_T{}^2\omega_p{}^2\omega_c{}^2\Omega^3}{\rho_0[3\Omega(\Omega^2-\omega_c{}^2)\omega-k^2\,v_T{}^2\Omega^2]}$$
$$E=\eta\Omega^2+\frac{k^2\epsilon_0 v_T{}^2\omega_p{}^2\Omega^4}{\rho_0[3\Omega(\Omega^2-\omega_c{}^2)\omega-k^2\,v_T{}^2\Omega^2]}$$
$$F=\eta(\Omega^2-\omega_c{}^2)$$

For the case where the wave vector and the static magnetic field are in z and x directions, respectively, the tensor becomes

$$\underline{\underline{\epsilon}}=\begin{pmatrix} \epsilon_l-BF & 0 & 0 \\ 0 & \epsilon_l-BC & jBD \\ 0 & -jBD & \epsilon_l-BE \end{pmatrix} \qquad (6)$$

For the case where both of the wave vector and the static magnetic field are in z direction, the tensor becomes

$$\underline{\underline{\epsilon}}=\begin{pmatrix} \epsilon_l-BC' & jBD' & 0 \\ -jBD' & \epsilon_l-BC' & 0 \\ 0 & 0 & \epsilon_l-BF' \end{pmatrix} \qquad (7)$$

where

$$C'=\eta\Omega^2$$
$$D'=\eta\omega_c\Omega$$
$$F'=\eta(\Omega^2-\omega_c{}^2)+\frac{k^2\epsilon_0 v_T{}^2\omega_p{}^2(\Omega^2-\omega_c{}^2)^2}{\rho_0[3\Omega(\Omega^2-\omega_c{}^2)\omega-k^2\,v_T{}^2\Omega^2]}$$

Observing the expressions of (5), (6) and (7) we can see that each expression, which is for the extreme case in its physical configuration, has four null elements and the expression (5) and (6) are equivalent each other in their physical meanings as one can recognize by comparing their eigenvectors each other. The expression (7) is quite different from the other two ones in its physical reality, even though in its mathematical form it is equivalent to the other two expressions. This arises from the fact that the expression (7) is for the case of the longitudinally applied field as can be expected from its physical configuration.

The permittivity tensor has connection with the conductivity tensor as

$$\underline{\underline{\epsilon}}=\epsilon_l\,\underline{\underline{1}}+\frac{\underline{\underline{\sigma}}}{j\omega\epsilon_0} \tag{8}$$

where $\underline{\underline{\sigma}}$ is the conductivity tensor, and the tensor (7) can be splitted as

$$\underline{\underline{\epsilon}}=\epsilon_l\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}-B\begin{pmatrix}C'&0&0\\0&C'&0\\0&0&F'\end{pmatrix}+jBD'\begin{pmatrix}0&1&0\\-1&0&0\\0&0&0\end{pmatrix}. \tag{9}$$

In the absence of magnetic fields the transport equation in the relaxation time approximation for a longitudinal phonon in a free-electron gas is

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\nabla f+\vec{F}\cdot\nabla_p f=-\frac{f-f_0}{\tau}, \tag{10}$$

where f_0 is the equilibrium distribution function and τ the relaxation time. From Eq. (10) with the definition of the electric current density $\vec{J}$, $\vec{J}$ is given by

$$\vec{J}=\underline{\underline{\sigma}}\cdot\left(\vec{E}+\frac{m\vec{u}}{e\tau}\right)+nec_s\vec{R} \tag{11}$$

with $\vec{u}$ for the local lattice velocity, n the perturbed electron density, c_s the sound velocity and $\vec{R}$ the diffusion vector, and the conductivity tensor is

$$\sigma_{\mu\nu}=-\frac{e^2\tau}{4\pi^3}\int d^3k\frac{v_\mu v_\nu}{1-i\omega\tau+i\vec{q}\cdot\vec{v}\tau}\cdot\frac{\partial f_0}{\partial\varepsilon} \tag{12}$$

with $\vec{q}$ for the wavevector of the phonon. Solving Eq. (12) with the appropriate equilibrium distribution function we will get, for $\vec{q}$ parallel to the z axis,

$$\left.\begin{aligned}\sigma_{xx}&=\sigma_{yy}\neq0\\ \sigma_{zz}&\neq0\end{aligned}\right\} \tag{13}$$

and the nondiagonal terms are zero. Therefore, we can infer that, on the analogy of the above results, the 3rd term on the right-hand side of Expression (9), that is, the off-diagonal elements in the permittivity tensor, denotes the anisotropic properties of the plasma arising as a result of the existence of the applied magnetic field.

The diffusion term is included only in the 2nd term of the right-hand side of Expression (9) and, in this sense, the permittivity tensor (7) is quite different from those of (5) and (6) in its characteristics and in the latter the diffusion terms are in both of the 2nd and the 3rd terms as perturbation. Therfore, when the directions of the wave vector and the magnetic field are same, the effect of diffusion appears only in ϵ_{zz} element.

Substituting the tensor (9) into the constitutive relation $\vec{D}=\underline{\underline{\epsilon}}\cdot\vec{E}$, we obtain

$$\vec{D}=\epsilon_l\vec{E}-(BC'(\hat{x}\hat{x}\cdot+\hat{y}\hat{y}\cdot)+BF'\hat{z}\hat{z}\cdot)\vec{E}-jBD'\hat{z}\times\vec{E} \tag{14}$$

and Expression (14) is an alternative statement of the constitutive relation for the tensor (9) and one can understand the states and the degrees of perturbations in another representation from the expression.

Ⅳ. CONCLUIONS

In the case of non-streaming semiconductor electron magnetoplasmas, the tensor permittivity has four null elements in the specific configuration for the case of diffusion-neglect and the same result holds for the case where the diffusion effect is included in analysis, and in the case of the longitudinal magnetic field the effect affects only ϵ_{zz} term, whereas in the other cases the effect does the anisotropic terms, too, as perturbation.

Ⅴ. REFERENCES

1. P.K. Dubey, *Helicon Waves in an Electron-Hole Collisional Plasma*, Proc. of the IEEE, pp. 719～720, Vol. 59, No. 4, April 1971.
2. J.P. Mckelvey, *Solid-State and Semiconductor Physics*, Harper & Row, 1966, Chap. 7.
3. B. Vural, *Interaction of Spin Waves with Drifted Carriers in Solids*, J. Appl. Phys., Vol. 37, No. 3, pp. 1030～1031, March 1966.
4. A. Yariv, *Quantum Electronics*, John Wiley and Sons, 1967, Chap. 18.
5. M. Born and K. Huang, *Dynamical Theory of Crystal Lattices*, Oxford Univ. Press, Chap. Ⅱ, 1968.
6. F.W. Crawford, *Microwave Plasma Devices-Promise and Progress*, Proc. of the IEEE, January, 1971.
7. C.Kittel, *Quantum Theory of Solids*, John Wiley & Sons, 1967. Chaps 12 and 17.
8. C. Cho, *Dispersion Relation including the Effect of Diffusion for E.M. Wave in Solid State Plasma*, JKIEE, pp 15～18, Vol. 20, No. 5, September 1971.