固狀 프라즈마內에서의 電磁波分散則과 擴散効果

논 문
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Dispersion Relation including the Effect of Diffusion for E.M. Wave in Solid-State Plasma

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ABSTRACT

Up to now, there have been numerous investigations about the effect of diffusion on the wave propagation in gaseous plasmas, but not so much in semiconductor magnetoplasmas. However, currently, it becomes the centor of interest to work with the latter problem, and this paper deals with the dispersion equation including diffusion effect in the latter case to see how diffusion affects the equation in which diffusion term is neglected in the first place, and the analysis is based on the assumption that the plasma can be treated as a hydrodynamical fluid, since, from a macroscopic view point, the plasma interacting with a magnetic field can be considered as a magneto-hydrodynamical fluid, an electrically conducting fluid subjected to electromagnetic force, and the system is linear.

The results of the relation and computation show that in the non-streaming case the diffusion terms appear in the equation as perturbation terms and the amplitude of the wave vector changes parabolically with the variation of the angular frequency and the longitudinal modes are observed.

1. Introduction

It is the intention of this paper to get the expression of dispersion relation including the effect of diffusion and compare the result with the one obtained in the case of diffusion-neglect, and apply the result to the specific cases to see what kinds of waves can exist in those physical configurations, and, also, whether the system exhibits any instability.

In gaseous state many workers have engaged in this field and done numerous distinguished accomplishments, but not so much in semiconductor magnetoplasmas, and it becomes one of current topics in reserach to investigate the nature of wave propagations in semiconductor magnetoplasmas.

The propagation of the wave in a plasma can be described phenomenologically by solving Maxwell's equations with the Boltzmann equation, together. To obtain a dispersion relation the equations will be applied to the case of a monochromatic plane wave which has the form of exp $[j(\omega t-\vec{k}\cdot\vec{r})]$, and in this paper it is assumed that general features of wave do not depend on the details of the band structure and the medium is an infinite one.

2. The General Equation

Maxwell's equations are

$$\nabla\times\vec{E}=-\frac{\partial}{\partial t}\vec{B} \quad (1)$$

$$\nabla\times\vec{H}=\vec{J}+\frac{\partial}{\partial t}\vec{D} \quad (2)$$

$$\nabla\cdot\vec{B}=0 \quad (3)$$

$$\nabla\cdot\vec{D}=\rho \quad (4)$$

and the first moment of the Boltzmann equation, using with the Lorentz equation, leads to the equation of motion, with neglecting the gravitational effect because of its smallness in magnitude,

$$\frac{d\vec{v}}{dt}=\frac{e}{m}(\vec{E}+\vec{v}\times\vec{B})-\nu\vec{v}-\frac{1}{mn}\cdot\nabla\underline{\underline{P}}, \quad (5)$$

where ν is the effective collision frequency leading to

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momentum change and n and $\underline{\underline{P}}$ are the particle density and the pressure tensor, respectively, and the latter is derived, assuming the pressure is isotropic because the plasma is considered as a conducting fluid, from the random walk theory of particles cycloiding between collisions in random phases around magnetic field lines as

$$\underline{\underline{P}}=\frac{m^* n\, v^2{}_T}{3}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}, \tag{6}$$

where $v_T{}^2=\frac{3k_BT}{m^*}$ is the kinetic velocity of the particle of effective temperature T and k_B is Boltzmann's constant.

Because, in this problem, Maxwell's equations and the Boltzmann equation have constant coefficients and construct a linear system, the problem may be solved by taking Fourier transforms in t and r, and we get the following basic equations:

$$j\vec{k}\times\vec{E}=j\omega\mu_0\vec{H} \tag{7}$$

$$-j\vec{k}\times\vec{H}=\sum_i\vec{J}_i+j\omega\epsilon\vec{E} \tag{8}$$

$$j\vec{k}\cdot\vec{B}=0 \tag{9}$$

$$-j\vec{k}\cdot\vec{D}=\sum_i\rho_i \tag{10}$$

$$j\Omega_i\vec{v}_i=\eta_i\vec{E}+\vec{v}_i\times\vec{\omega}_{ci}+\eta_i\vec{v}_{oi}\times\vec{B}+\frac{v^2{}_{Ti}}{3\rho_{oi}}j\vec{k}\rho_i \tag{11}$$

$$\vec{J}_i=\rho_{oi}\,\vec{v}_i+\rho_i\,\vec{v}_{oi} \tag{12}$$

and

$$\vec{k}\cdot\vec{J}_i=\omega\rho_i, \tag{13}$$

where η_i is charge to mass ratio of the ith carrier and $\vec{\omega}_{ci}=\eta_i\vec{B}_0$ and Eq.(13) follows from Eqs.(8) and (10). In deriving Eqs.(7) to (13) we used the following operators:

$$\frac{d}{dt}=\frac{\partial}{\partial t}-j\vec{k}\cdot\vec{v}_{oi} \tag{14}$$

$$j\Omega_i=\frac{d}{dt}+\nu_i \tag{15}$$

and in notation the subscript i denotes the quantities pertaining to various classes of carriers and takes on the symbol e or h for electrons and holes, respectively, and the additional subscript o refers to the $d.c.$ value; the higher order terms was neglected because of its small magnitude.

Substitution of the relation of Eq.(7) into the third term of the right hand side of Eq.(11) leads, after some manipulation, to the following result:

$$j\Omega_i\,\vec{\omega}_{ci}\times\vec{v}_i=\eta_i\frac{W_i}{\omega}\,\vec{\omega}_{ci}\times\vec{E}+\vec{\omega}_{ci}\times(\vec{v}_i\times\vec{\omega}_{ci})$$
$$+\frac{\eta_i}{\omega}(\vec{v}_{oi}\cdot\vec{E})(\vec{\omega}_{ci}\times\vec{k})+\frac{v^2{}_{Ti}}{3\rho_{oi}}\,\vec{\omega}_{ci}\times j\vec{k}\rho_i \tag{16}$$

where $W_i\equiv\omega-\vec{k}\cdot\vec{v}_{oi}$. Similarly,

$$j\Omega_i\cdot\vec{\omega}_i\cdot\vec{v}_i=\eta_i\frac{W_i}{\omega}\,\vec{\omega}_{ci}\cdot\vec{E}+\frac{\eta_i}{\omega}(\vec{v}_{oi}\cdot\vec{E})(\vec{\omega}_{ci}\cdot\vec{k})+\frac{v^2{}_{Ti}}{3\rho_{oi}}\,\vec{\omega}_{ci}\cdot j\vec{k}\rho_i \tag{17}$$

By recombining Eqs.(11), (16) and (17), we get

$$j\Omega_i(\Omega_i{}^2-\omega_{ci}{}^2)\vec{v}_i=\frac{W_i}{\omega}\eta_i\Big\{\Omega_i{}^2\vec{E}+j\Omega_i(\vec{\omega}_{ci}\times\vec{E})-(\vec{\omega}_{ci}\cdot\vec{E})\vec{\omega}_{ci}\Big\}+\frac{\eta_i}{\omega}(\vec{v}_{oi}\cdot\vec{E})\Big\{\Omega_i{}^2\vec{k}-(\vec{\omega}_{ci}\times\vec{k})\vec{\omega}_{ci}+j\Omega_i(\vec{\omega}_{ci}\times\vec{k})\Big\}+\frac{jv^2{}_{Ti}\rho_i}{3\rho_{oi}}\Big\{\vec{k}\Omega_i{}^2-(\vec{\omega}_{ci}\cdot\vec{k})\vec{\omega}_{ci}+j\Omega_i(\vec{\omega}_{ci}\times\vec{k})\Big\} \tag{18}$$

From Eqs.(12) and (13)

$$\rho_i=\frac{1}{W_i}\vec{k}\cdot(\rho_{oi}\,\vec{v}_i) \tag{19}$$

The general equation for wave propagation through the prasma is, from Eqs. (7), (8), (18) and (19),

$$\vec{E}+\frac{k^2}{\omega^2\epsilon\mu_o}\Big[\hat{k}(\hat{k}\cdot\vec{E})-\vec{E}\Big]-\frac{1}{\omega\epsilon}\sum_i\frac{1}{Q}\Big\{\frac{\rho_{oi}}{\omega}\Big[W_i\eta_i\vec{R}+k\eta_i\vec{S}(\vec{v}_{oi}\cdot\vec{E})+\frac{k^2v^2{}_{Ti}\omega_{pi}{}^2\epsilon}{3\rho_{oi}Q}\vec{S}\Big(\hat{k}\cdot\Big[\vec{R}+\frac{k}{W_i}\vec{T}(\vec{v}_{oi}\cdot\vec{E})\Big]\Big)\Big/\Big[1-\frac{k^2v^2{}_{Ti}}{3W_iQ}\Big(\hat{k}\cdot\vec{T}\Big)\Big]\Big]+\vec{v}_{oi}\,k\omega_{pi}{}^2\epsilon\Big(\hat{k}\cdot\Big[\vec{R}+\frac{k}{W_i}\vec{T}(\vec{v}_{oi}\cdot\vec{E})\Big]\Big)\Big/\Big[1-\frac{k^2v^2{}_{Ti}}{3W_iQ}(\hat{k}\cdot\vec{T})\Big]\Big\}=0, \tag{20}$$

where

$$Q=\Omega_i(\Omega_i{}^2-\omega_{ci}{}^2)$$
$$\vec{R}=\Omega_i{}^2\vec{E}+j\Omega_i(\vec{\omega}_{ci}\times\vec{E})-\vec{\omega}_{ci}(\vec{\omega}_{ci}\cdot\vec{E})$$
$$\vec{S}=\Omega_i{}^2\hat{k}-\vec{\omega}_{ci}(\vec{\omega}_{ci}\cdot\hat{k})+j\Omega_i(\vec{\omega}_{ci}\times\hat{k})$$
$$\vec{T}=\Omega_i{}^2\hat{k}-\vec{\omega}_{ci}(\vec{\omega}_{ci}-\hat{k})$$

3. Discussion

In the case of non-streaming plasma, with the configuration of $k/\!/\hat{\eta}$ and $B_o/\!/\hat{y}$, we get, from Eq.(20), the following dispersion equation:

$$\begin{vmatrix}1+A-BC & 0 & -jBD\\ 0 & 1+A-BF & 0\\ jBD & 0 & 1-BE\end{vmatrix}=0, \tag{21}$$

where

$$A=-\frac{k^2}{\omega^2\epsilon\mu_0}$$

$$B=\frac{1}{\omega\epsilon}\sum_i\frac{\rho_{oi}}{\Omega_i(\Omega_i{}^2-\omega_{ci}{}^2)}$$

$$C=\eta_i\Omega_i{}^2+\frac{k^2\epsilon v^2{}_{Ti}\omega_{pi}{}^2\omega_{ci}{}^2\Omega_i{}^2}{\rho_{oi}[3\Omega_i(\Omega_i{}^2-\omega_{ci}{}^2)\omega-k^2v^2{}_{Ti}\Omega_i{}^2]}$$

$$D=\eta_i\omega_{ci}\Omega_i+\frac{k^2\epsilon v^2{}_{Ti}\omega_{pi}{}^2\omega_{ci}{}^2\Omega_i{}^3}{\rho_{oi}[3\Omega_i(\Omega_i{}^2-\omega_{ci}{}^2)\omega-k^2v^2{}_{Ti}\Omega_i{}^2]}$$

$$E=\eta_i\Omega_i{}^2+\frac{k^2\epsilon v^2{}_{Ti}\omega_{pi}{}^2\Omega_i{}^4}{\rho_{oi}[3\Omega_i(\Omega_i{}^2-\omega_{ci}{}^2)\omega-k^2v^2{}_{Ti}\Omega_i{}^2}$$

$$F=\eta_i(\Omega_i{}^2-\omega_{ci}{}^2)$$

and there exist four null elements in the equation, that is, the 2nd row and 1st column, the 1st row and 2nd column, the 3rd row and 2nd column, and the 2nd row and 3rd column.

Comparing Eq.(21) with the result obtained from Eqs.(18) and (19), in which the diffusion term is neglected at the first place, the 2nd terms of C, D and E in Eq.(21) can be regarded as perturbation terms and the degree of its perturbation on the dispersion equation depedds upon the relative magnitudes of variables and parameters and, also, it might be expected that these terms have influence on the type of wave, since the terms are, in general, complex quantities and on instabilities.

ω	k		
	$1+A-BC=0$	$1-BE=0$	$1+A-BF=0$
10^5	$\pm 3.5466\cdot10^6\mp j1.4690\cdot10^6$	$\pm 7.9266\cdot10^3\mp j7.9266\cdot10^3$	
10^6	$\pm 1.1215\cdot10^7\mp 4.6458\cdot10^6$	$\pm 2.5066\cdot10^4\mp j2.5065\cdot10^4$	
10^7	$\pm 3.5470\cdot10^7\mp j1.4699\cdot10^7$	$\pm 7.9286\cdot10^4\mp j7.9246\cdot10^4$	
10^8	$\pm 1.1206\cdot10^8\mp j4.6737\cdot10^7$	$\pm 2.5129\cdot10^5\mp j2.5003\cdot10^5$	
$5\cdot10^8$	$\pm 2.4944\cdot10^8\mp j1.0701\cdot10^8$	$\pm 5.6754\cdot10^5\mp j5.5345\cdot10^5$	No Solutions
10^9	$\pm 3.5075\cdot10^8\mp j1.5572\cdot10^8$	$\pm 8.1257\cdot10^5\mp j7.7273\cdot10^5$	
$5\cdot10^{10}$	$\pm 2.3679\cdot10^9\mp j2.0753\cdot10^8$	No Data Available	
10^{11}	$\pm 2.4363\cdot10^9\mp j2.6550\cdot10^8$		
10^{12}	$\pm 9.9621\cdot10^7\mp j2.4782\cdot10^7$	$\pm 9.9607\cdot10^7\mp j2.4800\cdot10^7$	
10^{13}	$\pm 1.0873\cdot10^9\mp j2.5968\cdot10^7$	$\pm 1.0873\cdot10^9\pm j2.5968\cdot10^7$	

The computer results of the solution of Eq. (21) in the case of $T\approx77°K$, with the typical range of the other parameters, are shown in the table.

As shown in the table, there is a parabolic inclination to the variation of the values of the wave vector (for transverse mode) as the angular frequency changes, and this tendency is quite similar to the case of non-diffusion, so it might be possible to regard the effect of the diffusion on the dispersion relation as a perturbation. Furthermore, with both finite temperatures and scattering in the solid, there are diffusion effects whenever the carriers are bunched and, in this case, diffusion can play a critical role. Therefore, one may, perhaps, find out some instabilities by taking narrower intervals of ω variation than those in the table, even though there is not observed any instability from the data in the table.

In the table, there is no solution of 1+A−BF=0. This means the e.m.wave whose electric field is polarized to the applied magnetic field can not propagate through the medium, and it is, also, observed that, from the data of 1−BE=0, there exist longitudinal modes which propagate through the medium. In the view point of Maxwell field theory, there can not exist any mode whose wave vector and electric field have same direction. Therefore, the solution indicates that acoustic mode can exist in and propagate through the medium.

4. Conclusions

In the case of non-streaming semiconductor magnetoplasma, the determinant equation for the dispersion relation including the effect of diffusion has the form that the elements of the 2nd row and 1st column, the 1st row and 2nd column, the 3rd row and 2nd column, and the 2nd row and 3rd column have the value of zero, and, as far as the data show, the diffusion effect can be regarded as a perturbation and, also, the variation of k value depends on ω in parabolic fasion. Furthemore, the results of solution show that there can exist acoustic mode, too.

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6. References

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