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Added Mass of Two Dimensional Cylinders with the Sections of Straight Frames Oscillating Vertically in a Free Surface

by

J. H. Hwang*

直線肋骨型船斷面을 가지는 柱狀體의 自由水面에서의 上下動에 隨伴되는 附加質量

黃 宗 屹*

直線肋骨 및 單一 背骨을 가지는 배의 斷面과 同一한 斷面의 柱狀體가 高振動數로 理想流體의 自由水面에서 上下動을 할 때 流體의 附加質量을 Schwarz-Christoffel 變換을 利用하여 解析하고 系統的인 計算을 하였으며 그 結果를 Lewis forms에 對한 結果와 比較하였다.

이기서 提示한 解析法은 Lewis 以來의 等角寫像法에 依한 側面 및 船底에서의 接線條件에 關係없이 適用될 수 있으며, 複背骨의 直線肋骨型船斷面의 柱狀體에 對해서 擴張될 수 있다.

This work is a general treatment of added mass calculation of two-dimensional cylinders with straight-framed sections and chines oscillating in the free surface of an ideal fluid with high frequencies. Two and three parameter families in vertical oscillations are treated by employing Schwarz-Christoffel transformation. The results are presented with regards to geometrical parameters such as chine angles, sectional area coefficient and beam draft ratio.

Introduction

The hydrodynamic added mass of two dimensional cylinders oscillating vertically at high frequencies in the free surface is of interest to ship vibration, and sometimes to ship motion problems. The general class of problems dealing with cylinders of straight frames and chines has not been treated previously.

After F.M. Lewis' initial work [1]** in 1929, Prohaska [2] has evaluated the inertia coefficient for a number

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* Member; Associate Professor, Department of Naval Architecture, College of Engineering, Seoul National University, Seoul, Korea.

** Numbers in brackets refer to References at the end of the paper.

of shapes by means of conformal transformations. Landweber and Macagno [3] have treated more general cases including horizontal oscillations and derived the relations between the added mass and the sectional area coefficient as well as beam-draft ratio. They [4] have also extended the range of available shapes by a more general class of transformations involving a third parameter based on the moment of inertia of the section about the transverse axis. Above works, strictly speaking, have dealt with oscillations at high frequencies, $\omega \to \infty$. Actually, the frequency of oscillations in the range of interest in ship motions has an effect on the added mass. Thus recently, Grim [5] and Tasai [6] have separately considered such effects, and Porter [7] and Paulling [8] have contributed by experimental confirmation.

For straight line sections, Lewis [1] has evaluated the inertia coefficient for rectangle and rhombus sections vibrating vertically by employing the Schwarz-Christoffel tranformation. And, Wendel [9] has analyzed the added masses for rectangular sections with bilge keels.

Recently Vugts [10] has performed theoretical computations of hydrodynamic coefficient of cylinders with rectangle, triangle and other typical sections using the same conformal tranformation as was used for Lewis form. His work includes checking computation with experiments at finite frequencies for all three modes of oscillation: heave, sway, and roll.

Hwang and Kim [11] have investigated cylinders with the typical straight frames in vertical oscillation at high frequencies employing the Schwarz-Christoffel transformation, which includes added mass coefficient calculated for twelve typical sections with vertical side and flat bottom using manual integration in the analysis. By comparing the results with those of the Lewis forms having same sectional area coefficient and beam-draft ratio, it was concluded that the magnitude of chine angle seems to be a dominant factor over the sectional area coefficient on added mass coefficient for straight-framed sections. Since then, a number of computations has now been performed and a systematic relationship between the added mass coefficient and each variable concerned can be presented for various straight frame and chine sections.

General Formulation

Formulation of the Problem

The added mass calculation for a two dimensional cylinder with straight-framed section, when it oscillates vertically at high frequencies in the free surface of an infinite invicid fluid, may be accomplished by the Schwarz-Christoffel transformation as stated previously [11]. Take the y-axis in the free surface and the x-axis normal to it as shown in Fig. 1.

The transformation of the interior of a polygon PQR-VWX in the z-plane into a half plane above the real axis in the ζ-plane may be expressed in the form

Nomenclature

A = sectional area
b = half beam of section
B = full beam of section
g = acceleration of gravity
G, G' = designation of section and its image
H = draft of section
k, ϕ_i = corresponding value of φ at vertices of section
k_2 = added mass coefficient for vertical vibration
K = wave number = ω^2/g
m' = added mass for vertical vibration
n = distance measured along normal to side of section
r = modulus of complex variables in z plane
r_k = length of side of section near free surface
r_1 = length of side of section near bottom
s = arc length measured along side of section
T = kinetic energy of fluid
U = vertical component of velocity of section
w = complex potential = $\phi + i\psi$
x, y = horizontal and vertical co-ordinates in plane of section

$$\frac{dz}{d\zeta} = C(\zeta - \xi_1)^{\frac{\alpha_1}{\pi}-1} (\zeta - \xi_2)^{\frac{\alpha_2}{\pi}-1} (\zeta - \xi_3)^{\frac{\alpha_3}{\pi}-1}\cdots\cdots, \tag{1}$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \cdots\cdots = (n-2)\pi,$$

where z and ζ are the complex variables with $z=x+iy$ and $\zeta=\xi+i\eta$, C may be complex or real, $\xi_1, \xi_2, \xi_3, \cdots\cdots$ are the particular values of ξ at the vertices of the polygon and $\alpha_1,\alpha_2,\alpha_3,\cdots\cdots$ are the angles at corresponding vertices as shown. Integrating (1) we get

$$z = C\int\left[(\zeta - \xi_1)^{\frac{\alpha_1}{\pi}-1} (\zeta - \xi_2)^{\frac{\alpha_2}{\pi}-1} (\zeta - \xi_3)^{\frac{\alpha_3}{\pi}-1}\cdots\cdots\right] d\zeta + L, \tag{2}$$

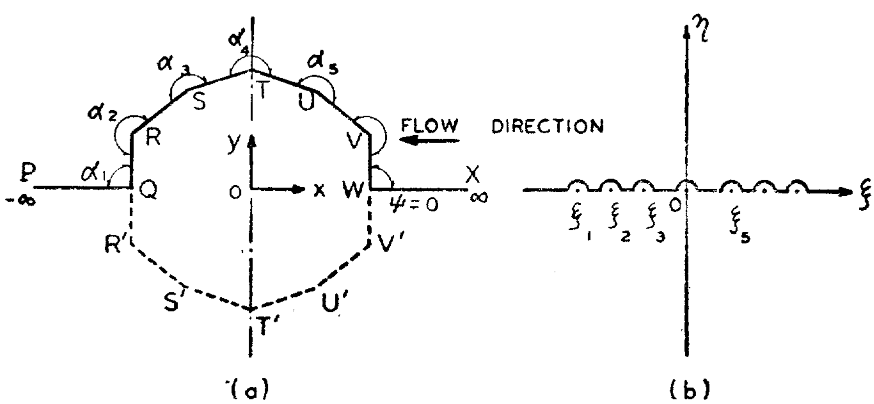

Fig. 1 Mapping of polygonal boundary into a real axis

where L is an arbitrary constant which can be removed by a proper selection of origin in the z-plane. The proper choice of C will fix the scale and orientation.

In applications such as this we are concerned with simple polygons with two of its sides extending to the infinities as in the above figure. The factors corresponding to $\xi = -\infty$ and ∞ are omitted from the equation of transformation, and the angle α does not appear. The complex potential for the rectilinear flow in ζ-plane is expressed by

$$w = \zeta \tag{3}$$

for the unit velocity, where

$$w(\zeta) = \varphi + i\psi,$$

Nomenclature

z = complex variables = $x + iy$

α_n = interior angle at vertices of polygon

β,γ = parameters controlling chine angles

ζ = complex variables = $\xi + i\eta$

ξ,η = horizontal and vertical co-ordinates in ζ plane

ξ_n = points on real axis of ζ-plane

θ = angle of polar co-ordinate system in z-plane

λ = reciprocal of beam-draft ratio = H/B

ρ = mass density of fluid

ϕ = velocity potential refered to the flow around the stationary polygon : $-\nabla\varphi=(u,v)$ velocity components

ϕ' = velocity potential refered to the flow around the moving polygon

ψ = stream function refered to the flow around the stationary polygon

ψ' = stream function refered to the flow around the moving polygon

ω = circular frequency of oscillation of cylinder

and φ and ψ are the velocity potential and the stream function respectively.
Hence we get

$$\varphi_i = \xi_i, \quad i = 1, 2, 3, \cdots\cdots. \tag{4}$$

Then, a rectilinear flow in ζ-plane may be transformed into a flow in z-plane with a polygonal boundary which is at rest in the fluid by the transformation

$$\frac{dz}{dw} = (w - \varphi_1)^{\frac{\alpha_1}{\pi}-1} (w - \varphi_2)^{\frac{\alpha_2}{\pi}-1} (w - \varphi_3)^{\frac{\alpha_3}{\pi}-1}, \cdots\cdots\cdots. \tag{5}$$

Now we can consider the complex potential for the flow around a stationary polygon in z-plane, say

$$w(z) = \varphi + i\psi. \tag{6}$$

In Fig.1 (a), polygon QRSTU∼U′T′S′R′Q is symmetrical about both y-axis and x-axis, and the part of the figure TUVWV′U′T′ (say G) may be considered as a hull section with straight-frames. The figure T′S′R′QRST is denoted as G' hereafter.

The rectilinear flow around the closed polygon QRSTU∼S′R′Q is symmetrical about both x-axis and y-axis. Therefore it is sufficient to consider the flow around the polygon PQRSTUVWX whose boundaries consist of stream line corresponding to $\psi=0$.

Boundary Conditions

Equation (5) represents the flow past a stationary polygon. We wish to obtain the energy of the flow of a moving polygon in a fluid stationary at infinity, we must accordingly add to the values of φ and ψ the terms

$$\varphi_1 = -x. \qquad \psi_1 = -y \tag{7}$$

giving

$$\varphi' = \varphi - x, \quad \psi' = \psi - y \tag{8}$$

for the moving polygon, where φ' and ψ' are velocity potential and stream function refered to the flow around the moving polygon.

When the body is oscillating with an angular frequency ω, the boundary condition on the free surface is

$$\frac{\partial\varphi'}{\partial x} + K\varphi' = 0 \quad \text{on } x = 0, \; y > \frac{B}{2}, \tag{9}$$

where $K=\frac{\omega^2}{g}$, and g is the acceleration of gravity. When ω is very large, the boundary condition becomes

$$\varphi' = 0 \quad \text{on } x=0, \; y > \frac{B}{2}, \tag{10}$$

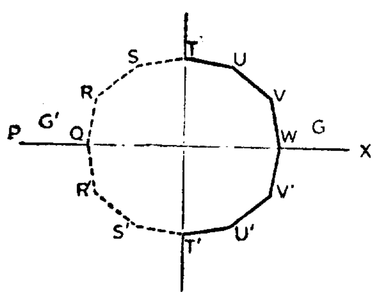

Fig. 2 Characteristics of the section

In the case of vertical oscillation of G the boundary condition (10) is satisfied by supposing that the double section GG' oscillates as a single rigid form, so that the boundary condition on GG' becomes

$$\frac{\partial\varphi'}{\partial n} = \frac{\partial x}{\partial n} \tag{11}$$

where n is the direction of the outward normal to GG'.

Furthermore,

$$\frac{\partial x}{\partial n} = -\frac{\partial y}{\partial s}, \quad \frac{\partial\varphi'}{\partial n} = \frac{\partial\psi'}{\partial s}$$

where s denotes the length along the boundary.
Hence, we get the following relation,

$$\frac{\partial\psi'}{\partial s} = -\frac{\partial y}{\partial s}. \tag{12}$$

We may therefore take the boundary condition,

$$d\psi' = -dy \quad \text{or } \psi' = -y, \tag{13}$$

along the polygonal boundary. And it will be recalled that this same condition is satisfied by imposing a uniform flow along the positive x axis as appearing in Eq. (7). The infinity condition may therefore be taken as the fluid to have a uniform flow at infinity with vanishing of disturbances.

Kinetic Energy of Fluid

The kinetic energy T of a fluid at rest at infinity is given by

$$T = -\frac{1}{2}\rho\int\varphi'\frac{\partial\varphi'}{\partial n}dS$$

or

$$T = -\frac{\rho}{2}\int\varphi'\,d\psi' \tag{14}$$

where dS denotes an elementary area, ρ is the mass density of the fluid, and the integral extends over all the boundaries of the fluid.

Since $\varphi'=0$ on the free surface for the vertical oscillations when ω is very large, the kinetic energy integral vanishes over this boundary. Therefore the kinetic energy of the fluid below the free surface is half of that obtained when a submerged whole polygonal cylinder GG' move vertically.

Added Mass

The added mass of the vibrating polygonal cylinder is then given by

$$m' = \frac{2T}{U^2},$$

where U is the corresponding intantaneous velocity.
If $U=1$,

$$m' = 2T. \tag{15}$$

For a prismatic body, whose cross section is a semi-ellipse, the added mass per unit length is given by

$$m' = \frac{1}{2}\pi\rho b^2, \tag{16}$$

where b designates half-breadth at waterline.
For other forms it may be written in the form

$$m' = k_2\frac{1}{2}\pi\rho b^2, \tag{17}$$

using k_2 as the added mass coefficient.
From (15) and (17), we get

$$k_2 = \frac{2T}{\frac{1}{2}\pi\rho b^2} \tag{18}$$

or

$$k_2 = \frac{16T}{\pi\rho B^2}, \tag{19}$$

where B designates full-breadth at waterline.

Added Mass for Straight Framed Section

Straight Framed Sections with Raked Side

In the case of a two dimensional cylinder with the general straight-framed section and a single chine (Fig. 3), the flow around its section may be obtained from equation (6) with

$$\alpha_1 = \alpha_5 = \beta\pi, \qquad \alpha_2 = \alpha_4 = \gamma\pi$$

$$\alpha_3 = [5 - 2(\beta + \gamma)]\pi$$

and

$$\varphi_1 = -1,\ \varphi_2 = -k,\ \varphi_3 = 0,\ \varphi_4 = k,\ \varphi_5 = 1$$

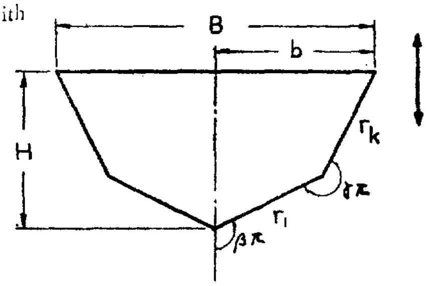

Fig. 3 Straight-framed section with raked side

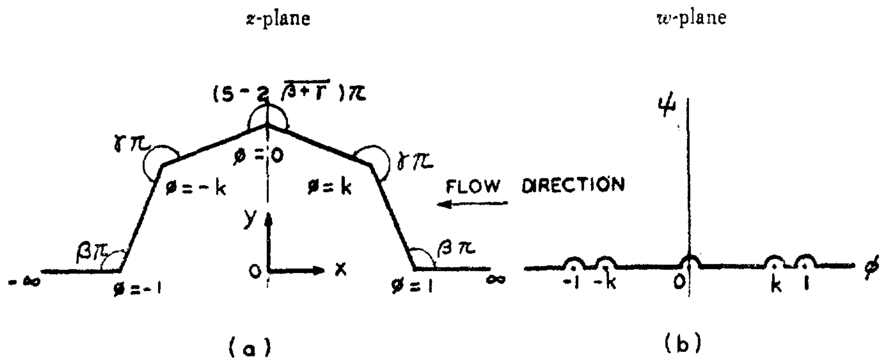

Fig. 4 Mapping of straight-framed section with raked side

Hence the transformation corresponding to the present problem becomes

$$\frac{dz}{dw} = (w^2 - 1)^{\beta-1}(w^2 - k^2)^{\gamma-1} w^{2(2-\overline{\beta+\gamma})} \tag{20}$$

or

$$z = \int (w^2 - 1)^{\beta-1}(w^2 - k^2)^{\gamma-1} w^{2(2-\overline{\beta+\gamma})}\, dw + \text{const.}, \tag{21}$$

where

$$\frac{1}{2} \leqq \beta < 1, \qquad 1 < \gamma < \frac{3}{2}.$$

We are only interested in evaluating the intergral (21) along the boundaries of the section. For this, $\psi=0$ and we have

$$z = re^{i\theta} = \int (\varphi^2 - 1)^{\beta-1}(\varphi^2 - k^2)^{\gamma-1} \varphi^{2(2-\overline{\beta+\gamma})}\, d\varphi + \text{const.}, \tag{22}$$

where r and θ are polar coordinates.

Let r_k and r_1 be the lengths of the sides of the polygon corresponding to $(\varphi=0,\ \varphi=k)$ and $(\varphi=k,\ \varphi=1)$ respectively.

Then, we have

$$r_k = |z_k - z_0| = \int_0^k (1-\varphi^2)^{\beta-1} (k^2-\varphi^2)^{\gamma-1} \varphi^{2(2-\overline{\beta+\gamma})} d\varphi \tag{23}$$

$$(0 < \varphi < k < 1)$$

$$r_1 = |z_1 - z_k| = \int_k^1 (1-\varphi^2)^{\beta-1} (\varphi^2-k^2)^{\gamma-1} \varphi^{2(2-\overline{\beta+\gamma})} d\varphi \tag{24}$$

$$(0 < k < \varphi < 1)$$

Where z_0, z_k and z_1 are the values of z corresponding to the points at $\varphi=0, k$ and 1 respectively.

The integral of (23) is approximated as follows by employing the hypergeometric function (see Appendix A)

$$r_k = \frac{1}{2} k^{3-2\beta} \frac{\Gamma(\gamma)}{\Gamma(1-\beta)} \sum_{n=0}^{\infty} \frac{\Gamma(1-\beta+n)\,\Gamma\left(\frac{5}{2}-\overline{\beta+\gamma}+n\right)}{\Gamma\left(\frac{5}{2}-\beta+n\right)} \frac{k^{2n}}{n!}$$

$$(0 < k < 1) \tag{25}$$

Similarly (24) becomes (see Appendix B)

$$r_1 = \frac{1}{2} (1-k^2)^{\beta+\gamma-1} \frac{\Gamma(\gamma)}{\Gamma\left(\beta+\gamma-\frac{3}{2}\right)} \sum_{n=0}^{\infty} \frac{\Gamma\left(\beta+\gamma-\frac{3}{2}+n\right)\Gamma(\beta+n)}{\Gamma(\beta+\gamma+n)} \frac{(1-k^2)^n}{n!} \tag{26}$$

$$(0 < k < 1)$$

If we denote B and H as the full beam and draft of the section respectively, then we get

$$b = r_k \sin(2-\overline{\beta+\gamma})\pi + r_1 \sin(1-\beta)\pi \tag{27}$$

$$H = r_k \cos(2-\overline{\beta+\gamma})\pi + r_1 \cos(1-\beta)\pi \tag{28}$$

with $b = B/2$.

And the area of the section becomes

$$A = r_k^2 \cos(2-\overline{\gamma+\beta})\pi \sin(2-\overline{\gamma+\beta})\pi$$
$$+ r_1 \sin(1-\beta)\pi\,[2r_k \cos(2-\overline{\gamma+\beta})\pi + r_1 \cos(1-\beta)\pi] \tag{29}$$

The kinetic energy of the entrained water per unit lenth of cylinder is given by

$$2T = 2\rho \int_0^b (\varphi - x)\, dy$$
$$= 2\rho \left[\int_0^b \varphi dy - \int_0^b x dy\right] \tag{30}$$

from (13) and (14).

And y increment of the side r_k and r_1 of the section are expressed by

$$dy = dr_k \sin(2-\overline{\beta+\gamma})\pi$$

and

$$dy = dr_1 \sin(1-\beta)\pi,$$

respectively.

Hence, (30) becomes

$$2T = 2\rho \left[\sin(2-\overline{\beta+\gamma})\pi \int_0^k (1-\varphi^2)^{\beta-1}(k^2-\varphi^2)^{\gamma-1} \varphi^{5-2(\beta+\gamma)} d\varphi\right.$$

$$+\ \sin(1-\beta)\pi\int_k^1(1-\varphi^2)^{\beta-1}(\varphi^2-k^2)^{\gamma-1}\,\varphi^{5-2(\beta+\gamma)}\,d\varphi-\frac{A}{2}\Big] \qquad (31)$$

by virtue of equations (23) and (24).

Integrals involved in the equation (31) may be evaluated by the same approximation as employed in equations (23) and (24).

Then we have

$$2T=\rho\Big[\,k^{2(2-\beta)}\sin\,(2-\overline{\beta+\gamma})\pi\frac{\Gamma(\gamma)}{\Gamma(1-\beta)}\sum_{n=0}^{\infty}\frac{\Gamma(3-\overline{\beta+\gamma}+n)}{(2-\beta+n)(1-\beta+n)}\frac{k^{2n}}{n!}$$

$$+(1-k^2)^{2\beta+\gamma-1}\sin\,(1-\beta)\pi\frac{\Gamma(\gamma)}{\Gamma(\beta+\gamma-2)}\sum_{n=0}^{\infty}\frac{\Gamma(\beta+n)}{(\beta+\gamma+n-1)(\beta+\gamma+n-2)}\frac{(1-k^2)^n}{n!}-A\Big] \qquad (32)$$

Therefore the added mass coefficient k_2 for the vertical oscillation will be reduced to

$$k_2=\frac{8}{\pi B^2}\Big[\,k^{2(2-\beta)}\sin(2-\overline{\beta+\gamma})\,\pi\frac{\Gamma(\gamma)}{\Gamma(1-\beta)}\sum_{n=0}^{\infty}\frac{\Gamma(3-\overline{\beta+\gamma}+n)}{(2-\beta+n)(1-\beta+n)}\frac{k^{2n}}{n!}$$

$$+(1-k^2)^{2\beta+\gamma-1}\sin(1-\beta)\pi\frac{\Gamma(\gamma)}{\Gamma(\beta+\gamma-2)}\sum_{n=0}^{\infty}\frac{\Gamma(\beta+n)}{(\beta+\gamma+n-1)(\beta+\gamma+n-2)}\frac{(1-k^2)^n}{n!}-A\Big] \qquad (33)$$

Straight-framed Section with Vertical Side

When α_3 is π instead of $[5-2(\beta+\gamma)]\pi$ in the preceding section *i.e.* in the case of the wall sided profile, we have

$$\beta+\gamma=2.$$

In this case the denominator of the second term in the bracket of the equation (32) beoomes zero, therefore we must analyze seaprately from the preceding section as a special case.

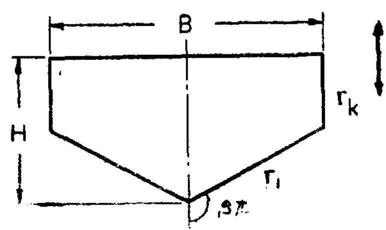

Fig. 5 Straight-framed section with vertical side

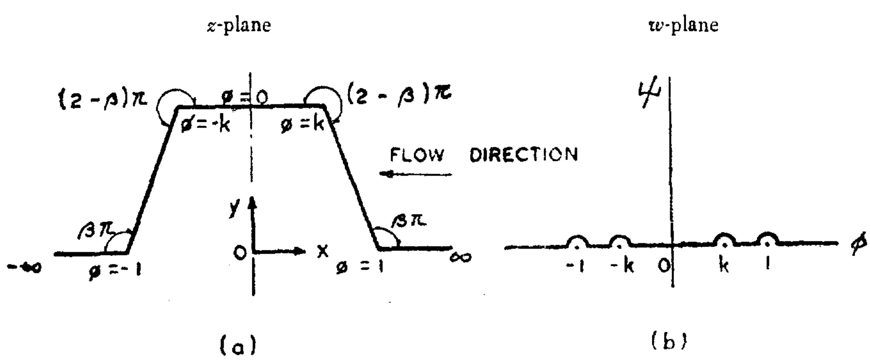

Fig. 6 Mapping of straight-framed section with vertical side

The flow around such a section may be obtained by the following transformation

$$\frac{dz}{dw} = (w^2 - 1)^{\beta-1} (w^2 - k^2)^{1-\beta} \tag{34}$$

where $\frac{1}{2} \leq \beta < 1$.

Therefore we have

$$r_k = \int_0^k (1-\varphi^2)^{\beta-1} \ (k^2 - \varphi^2)^{1-\beta} \ d\varphi \tag{53}$$

$$r_1 = \int_k^1 (1-\varphi^2)^{\beta-1} \ (\varphi^2 - k^2)^{1-\beta} \ d\varphi \tag{36}$$

These integrals are performed by the same approximation as in equations (23) and (24), but they are easily obtained by substituting γ by $2-\beta$ in the equations (25) and (26). Thus it follows that

$$r_k = \frac{1}{2} k^{3-2\beta} (1-\beta) \sum_{n=0}^{\infty} \frac{\Gamma(1-\beta+n)\ \Gamma\left(\frac{1}{2}+n\right)}{\Gamma\left(\frac{5}{2}-\beta+n\right)} \frac{k^{2n}}{n!}, \tag{37}$$

$$r_1 = \frac{1}{2\sqrt{\pi}} (1-k^2)\ \Gamma(2-\beta) \sum_{n=0}^{\infty} \frac{\Gamma\left(\frac{1}{2}+n\right) \Gamma(\beta+n)}{(n+1)\,!} \frac{(1-k^2)^n}{n!} \tag{38}$$

where $0 < k < 1$, and

$$b = r_1 \sin (1 - \beta)\pi,$$

$$H = r_k + r_1 \cos (1 - \beta)\pi,$$

$$A = b(r_k + H).$$

The kinetic energy of the entrained water is given by

$$2T = 2\rho \ [\sin (1-\beta)\pi \int_0^{r_1} \varphi \ dr_1 - \frac{A}{2}]$$

$$= \rho \ [2 \sin (1-\beta)\pi \int_k^1 (1-\varphi^2)^{\beta-1}(\varphi^2 - k^2)^{1-\beta} d\varphi - A], \tag{39}$$

since dy is zero along the parallel side to the flow.

Let

$$I = \int_k^1 \varphi(1 - \varphi^2)^{\beta-1} \ (\varphi^2 - k^2)^{1-\beta} \ d\varphi. \tag{40}$$

This integral may be reduced to

$$I = \frac{1}{2} (1 - k^2) \ B(\beta, 2-\beta)$$

Hence by virtue of the property of Gamma function, we obtain

$$I = \frac{1}{2}(1 - k^2)(1 - \beta)\frac{\pi}{\sin \beta\pi}$$

Then (39) becomes

$$2T = \rho \ [(1 - k^2)(1 - \beta)\pi - A]$$

$$= \rho \, [2\,(1 - k^2)(1 - \beta)\pi - B(r_k + H)] \tag{42}$$

Therefore added mas coefficient k_2 for vertical oscillation in this case is given by

$$k_2 = \frac{8}{B^2}\,(1 - k^2)\,(1 - \beta) - \frac{4}{\pi B}\,(r_k + H) \tag{43}$$

Designate sectional area coefficient by σ. Then

$$\sigma = \frac{1}{2}\left(1 + \frac{r_k}{H}\right) \tag{44}$$

It follows that

$$k_2 = \frac{8}{B^2}\,(1 - k^2)(1 - \beta) - \frac{4}{\pi}\,\sigma\lambda, \tag{45}$$

where $\lambda = \dfrac{H}{B}$.

Numerical Results and Discussion

Straight Framed Section with Vertical Side

This class of problems belongs to two parameter family, containing parameters k and β. For numerical results, values of r_k, r_1, B, H, B/H, A, σ, k_2 were calculated by computer for the following k values, and β values, taken in turn for each k.

k: 0.10, 0.20, 0.30, 0.40, 0.45, 0.50, 0.55, 0.60, 0.70, 0.80, 0.90, 0.93, 0.95, 0.98

β: 0.50, 0.55, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95.

The results of the computation are plotted in Fig. 7 and Fig 8. The values of added mass coefficient k_2 with parameter β on the base of B/H are plotted in Fig. 7 and k_2 on the base of β with parameter B/H are plotted in Fig. 8.

The upper limit of B/H for constant β corresponds to triangular section with same β. Therefore curves of k_2 for the general sections must be within the two limits given by those of rectangular and triangular sections. When β equals to 0.5, the section corresponds to the rectangle. The calculated values of k_2 for $\beta=0.5$ from equation (45) coincide well with those of Lewis formula for rectangular section [1] as can be seen.

If B/H and β are chosen, the sectional area coefficient is predetermined.

Thus

$$\sigma = 1 - \frac{1}{4\lambda}\tan\,(\beta - 0.5)\pi. \tag{46}$$

This equation shows that the relation between σ and $B/H(=1/\lambda)$ is linear with constant parameter β in two parameter family of straight-framed sections.

The curves of k_2 on the base σ with parameter β are plotted in Fig. 10 and the same curve with parametter B/H are plotted in Fig. 11.

From Fig. 7 and Fig. 9 it is clear that the values of the added mass coefficient k_2 increase if the values of β decrease.

Straight-framed Section with Raked Side

This class of problems belongs to three parameter family, containing parameter k, β and γ. For numerical results, values of the same geometrical and physical quantities as in the preceding section were calculated for the following k, β and γ values. For each combination of k and β an appropriate value of γ satisfying

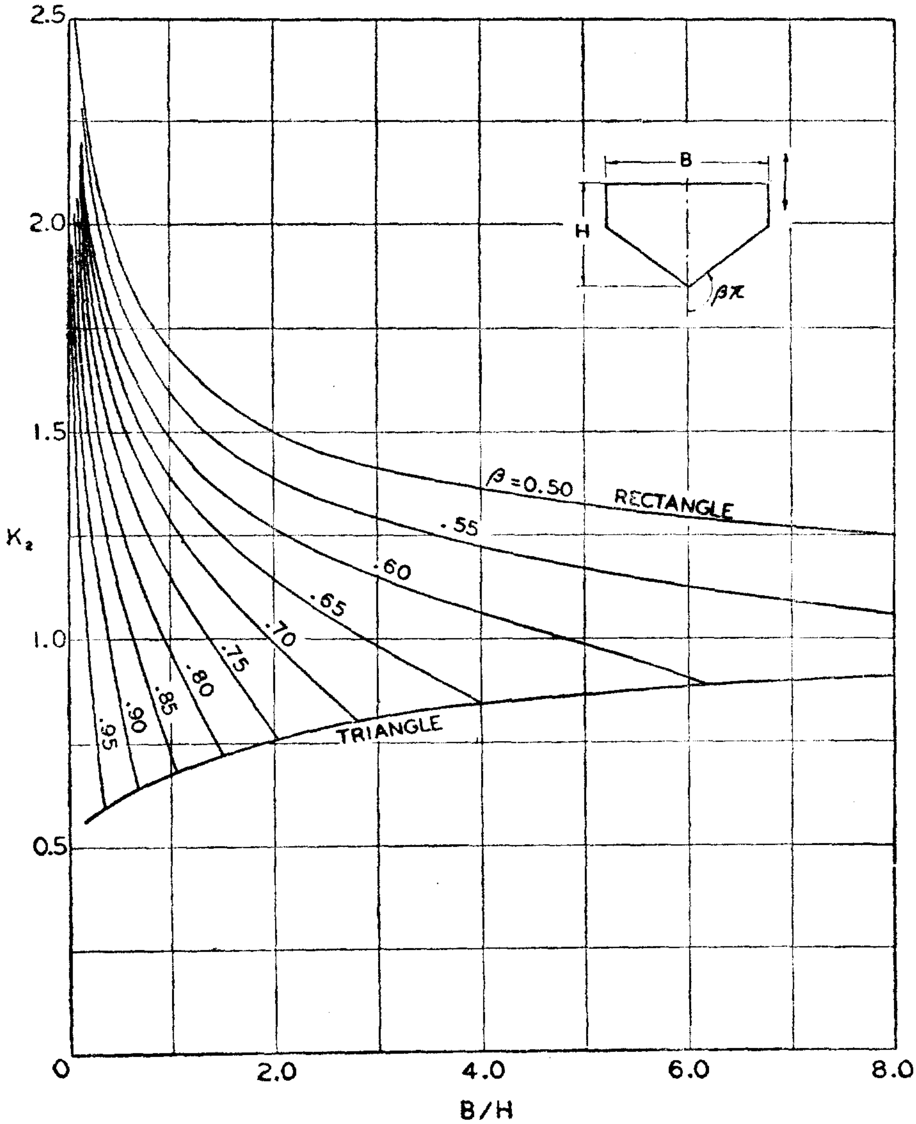

Fig. 7 Added mass coefficients versus beam-draft ratios with parameter β

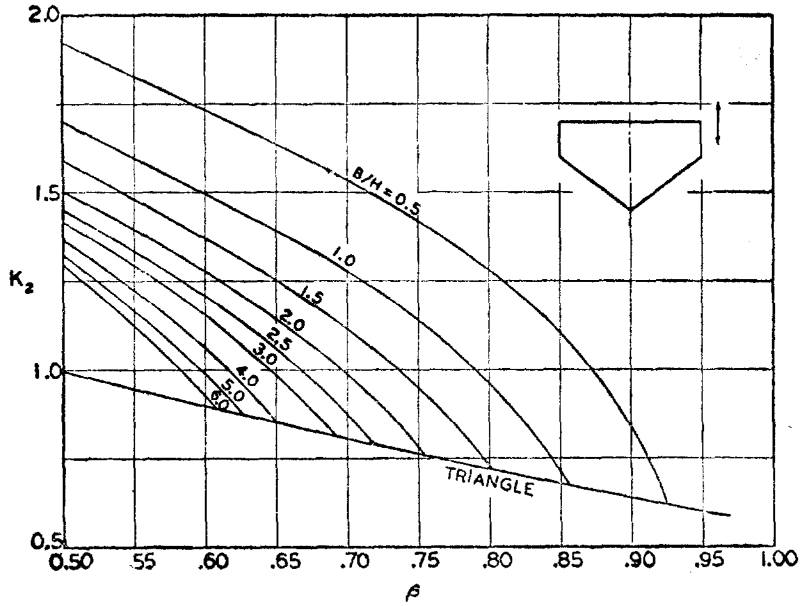

Fig. 8 Added mass coefficients versus β with parameter B/H

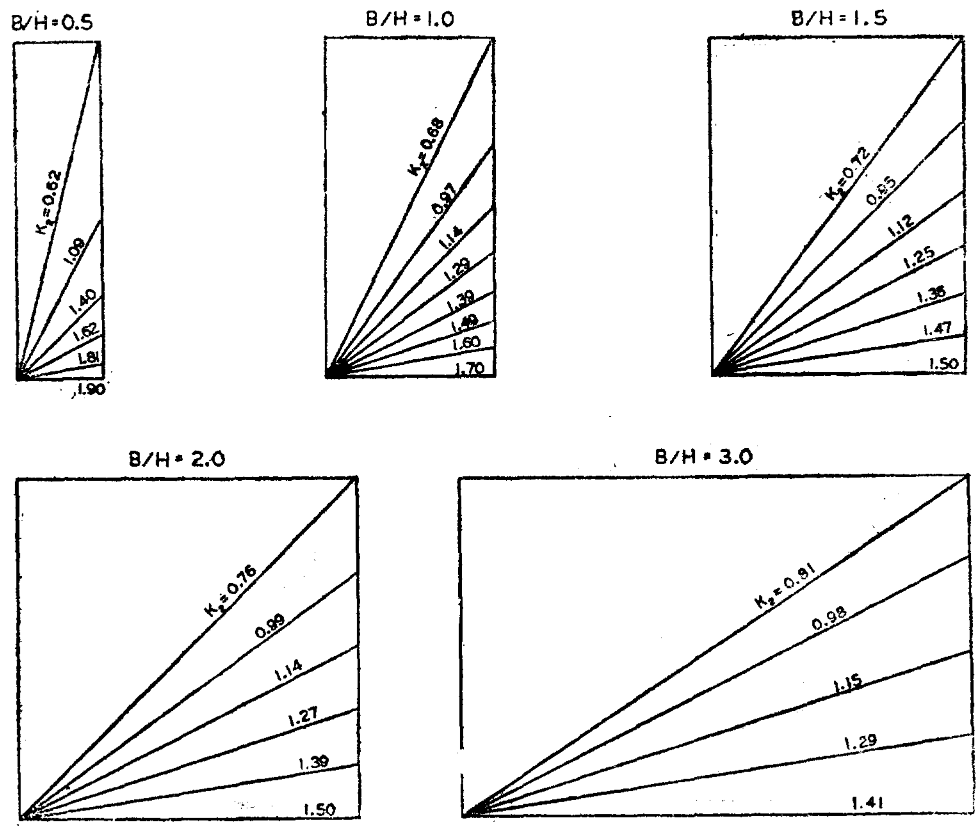

Fig. 9 Section shapes and added mass coefficients

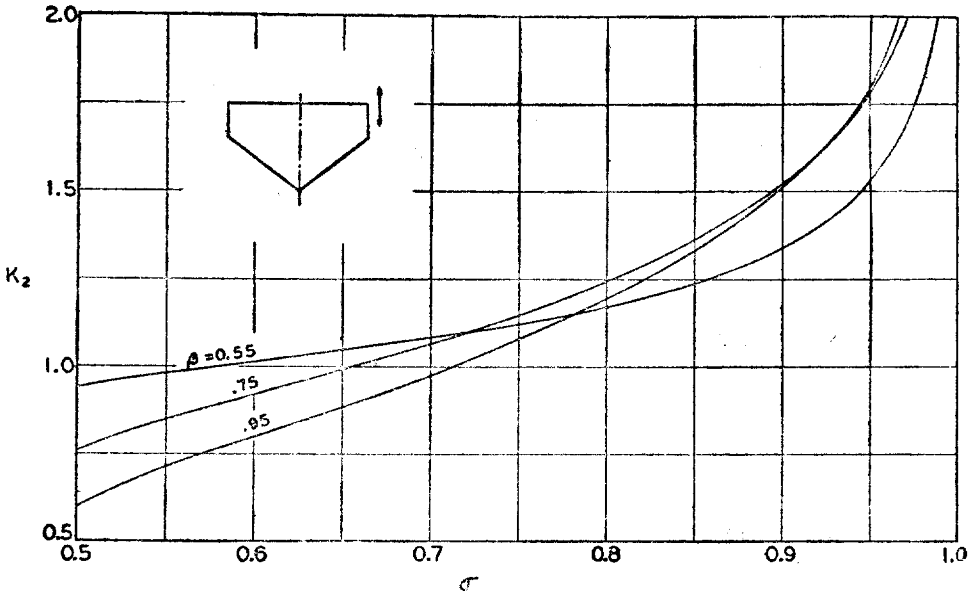

Fig. 10 Added mass coefficients versus sectional area coefficients with parameter β

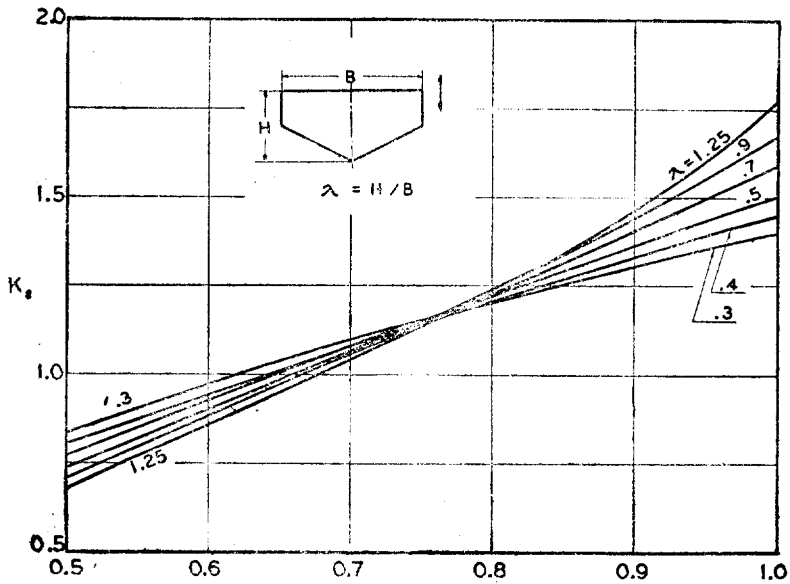

Fig. 11 Added mass coefficients versus sectional area coefficients with parameter λ

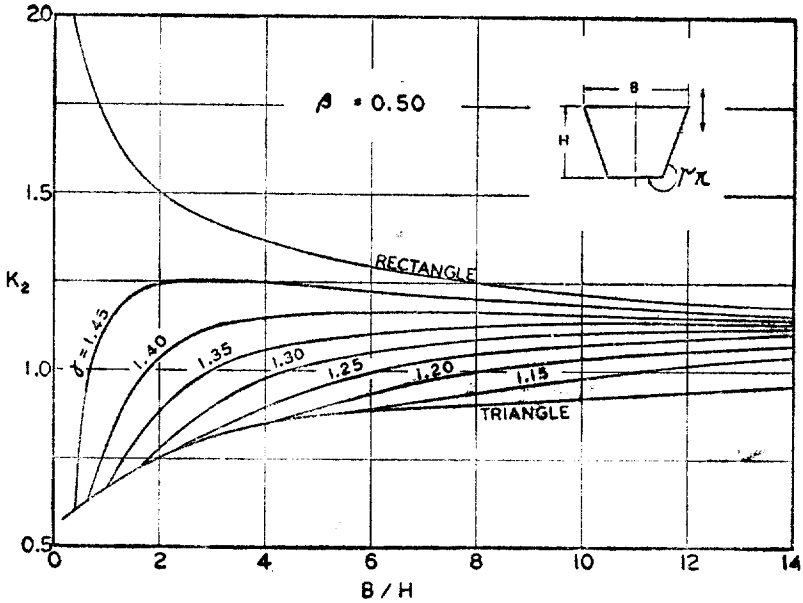

Fig. 12 (a)

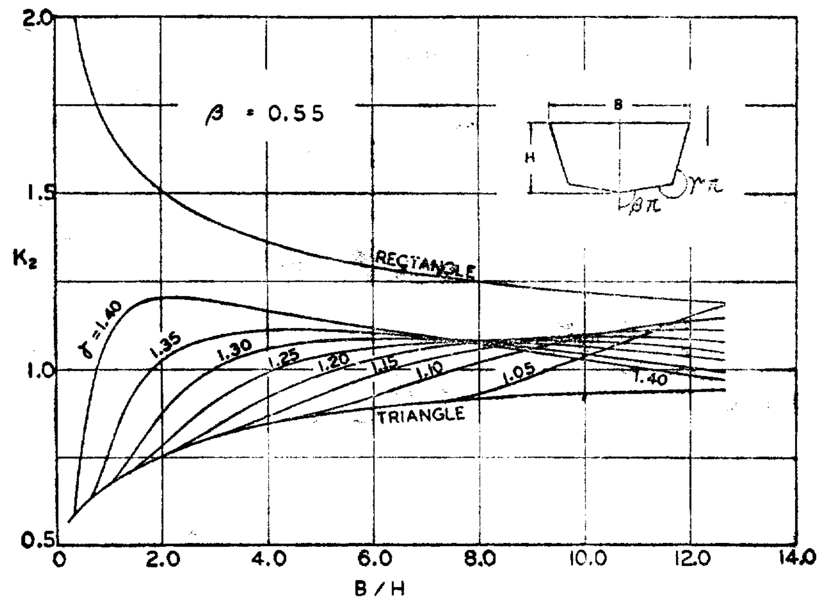

Fig. 12 (b)

Fig. 12 Added mass coefficients versus sectional area coefficients with parameer β

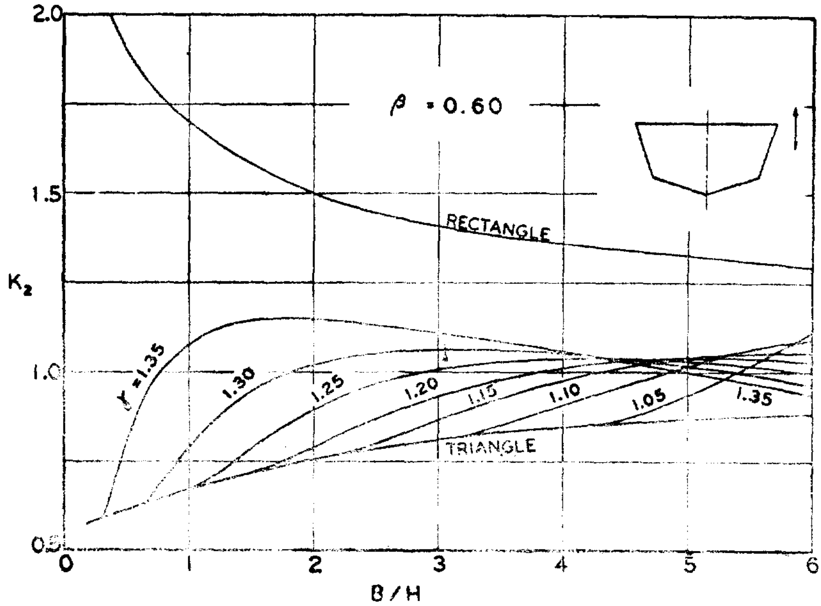

Fig. 12 (c)

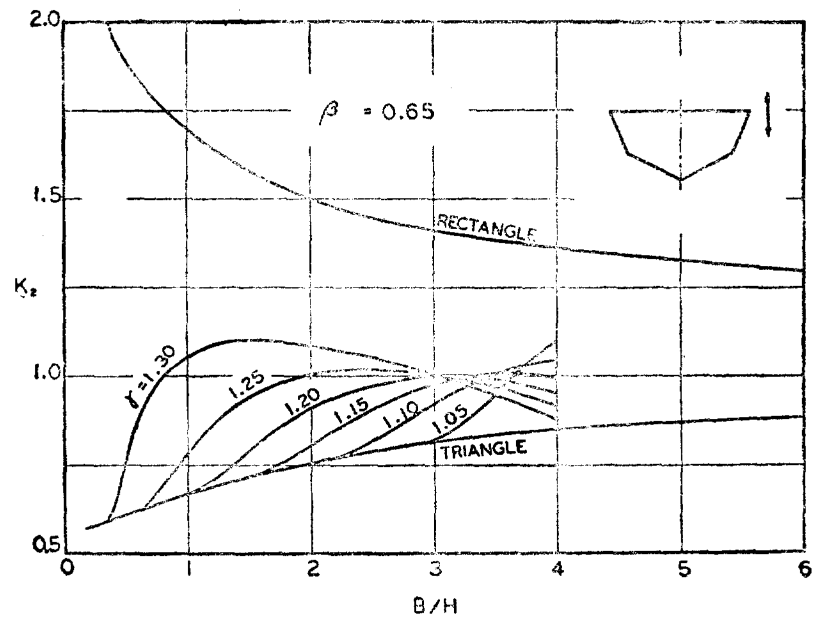

Fig. 12 (d)

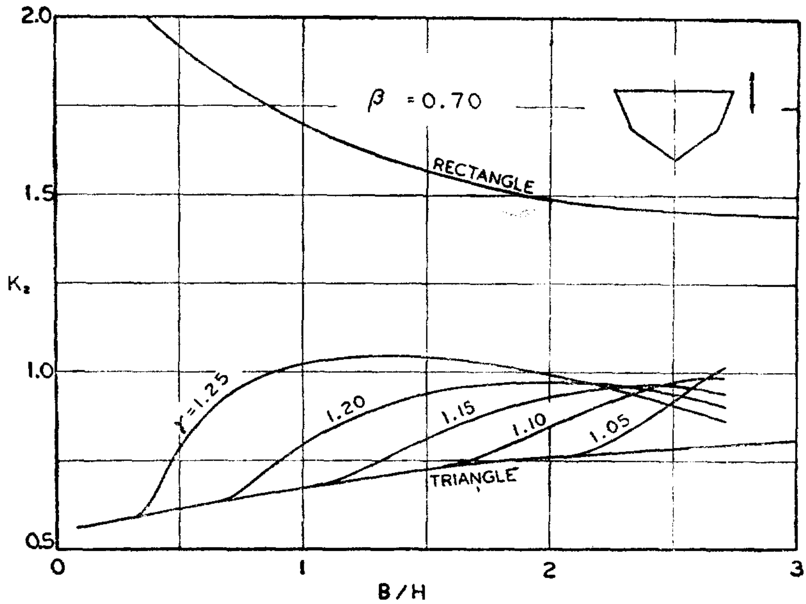

Fig. 12 (e)

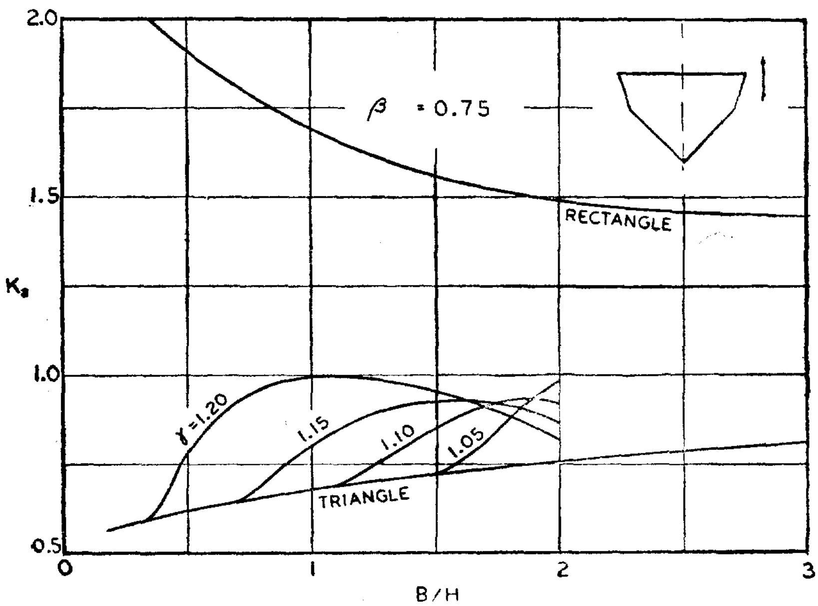

Fig. 12 (f)

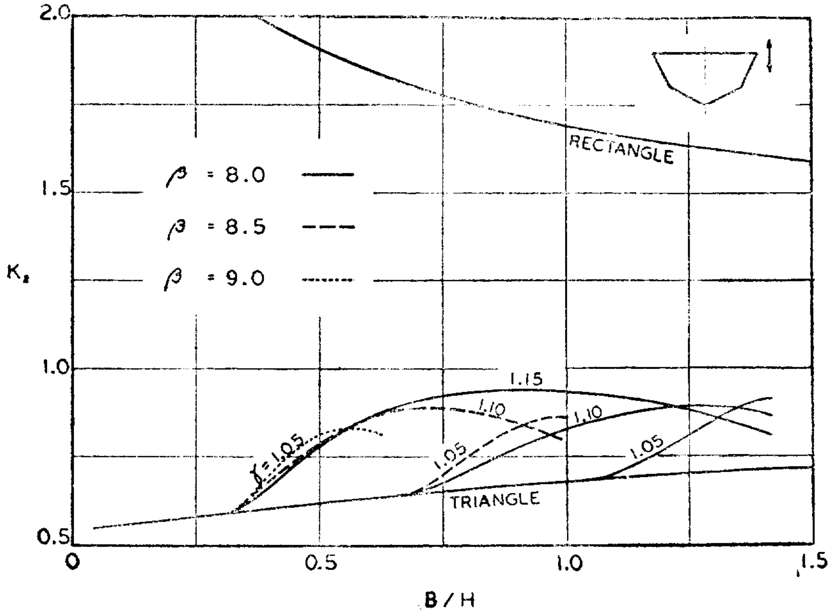

Fig. 12 (g)

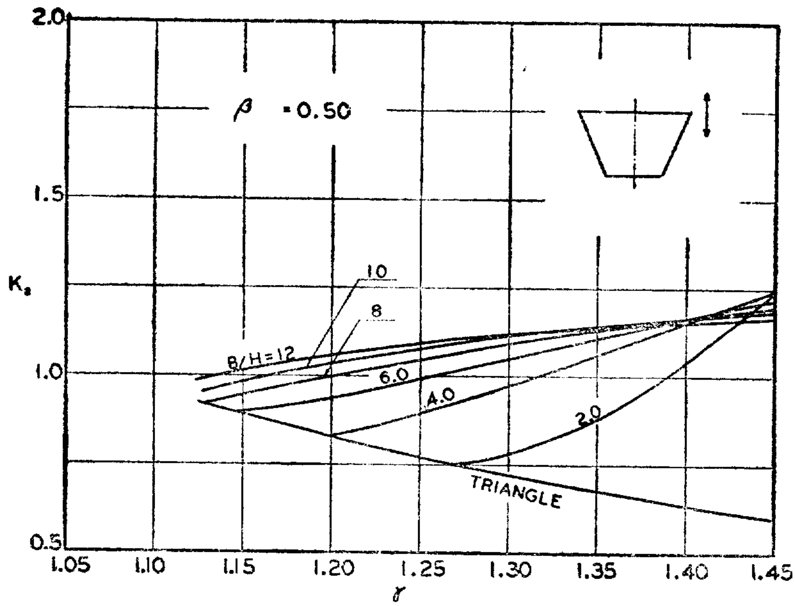

Fig. 13 (a)

Fig. 13 Added mass coefficients versus γ with parameter B/H

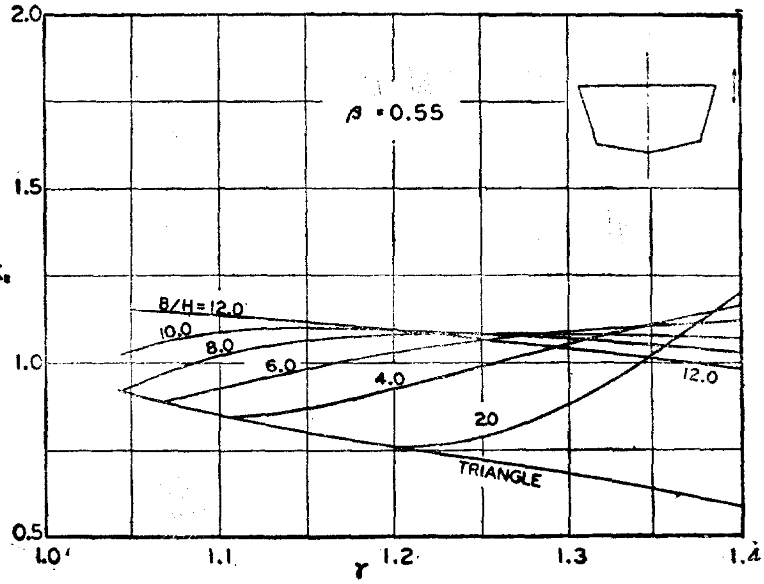

Fig. 13 (b)

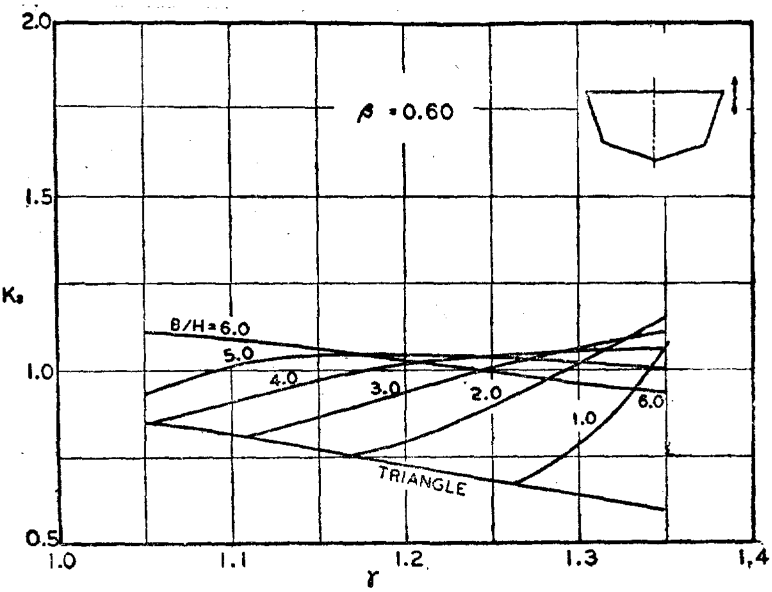

Fig. 13 (c)

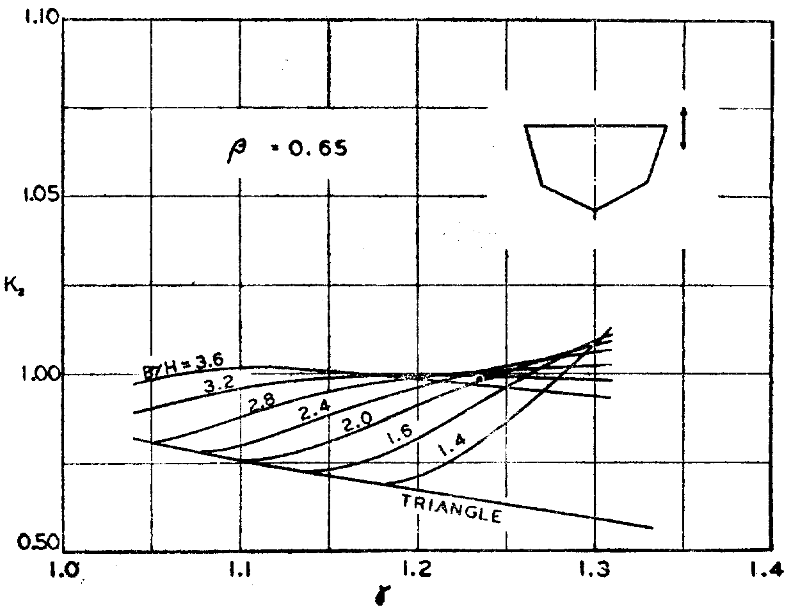

Fig. 13 (d)

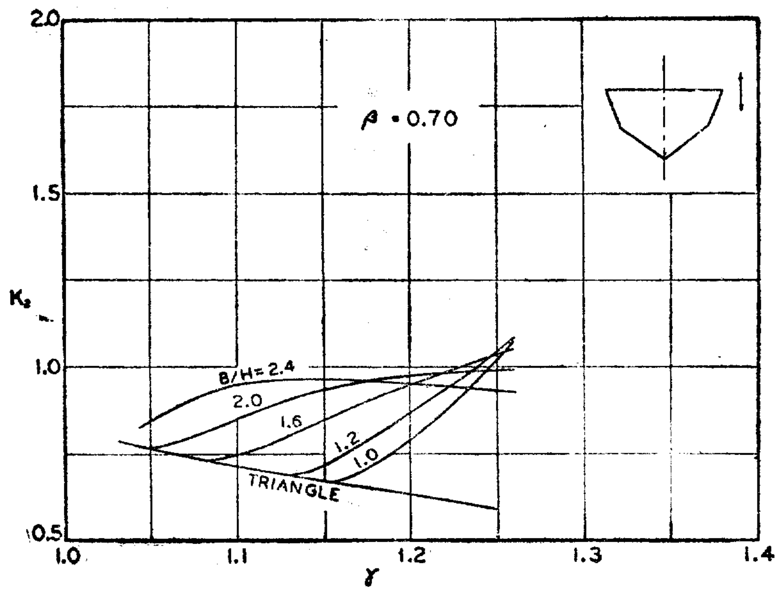

Fig. 13 (e)

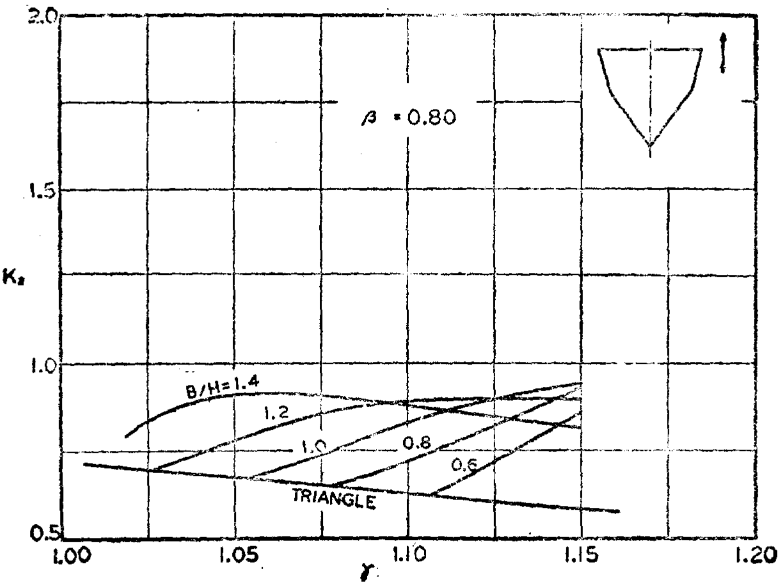

Fig. 13 (f)

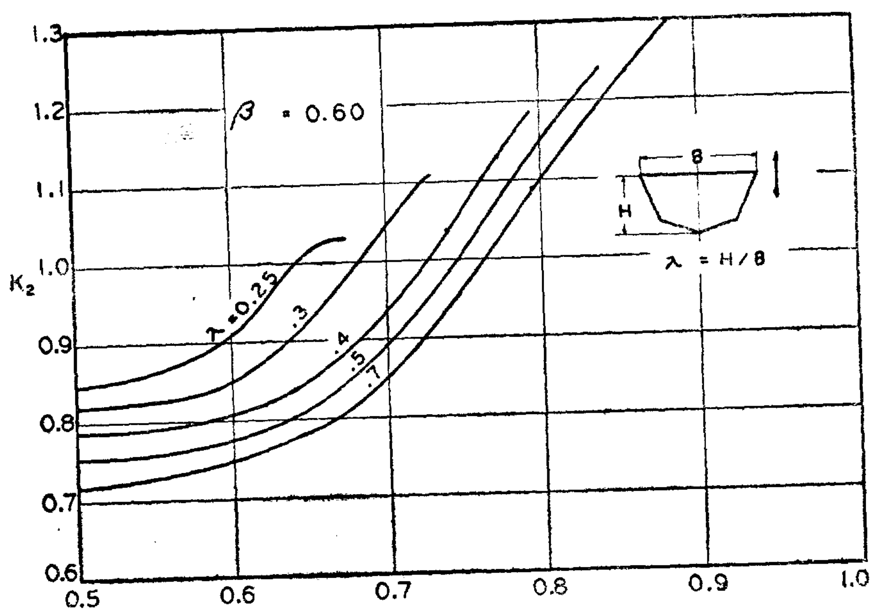

Fig. 14 Added mass coefficients versus sectional area coefficients with parameter λ

the relation $1.5 < \beta + \gamma < 2$ were taken.

k: 0.10, 0.20, 0.30, 0.40, 0.45, 0.50, 0.55, 0.60, 0.70, 0.80, 0.90

β: 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90

γ: appropriate values between 1.05 and 1.45 satisfying the relation $1.5 < \beta + \gamma < 2$ for each β value above.

The results of the computation are plotted in Fig. 12 and Fig. 13. Figure 12 shows the values of k_2 with parameters β and γ on the base of B/H, and Fig. 13 shows the values of R_2 on the base of γ with parameters β and B/H.

The whole points lie between those of rectanglular and triangular section in Fig. 14.

It is obvious that the lower limit of B/H for constant β and γ corresponds to the triangular section with a bottom angle ($\beta\pi$) taking γ value.

If B/H, β and γ are chosen, the sectional area coefficient σ is predetermined.

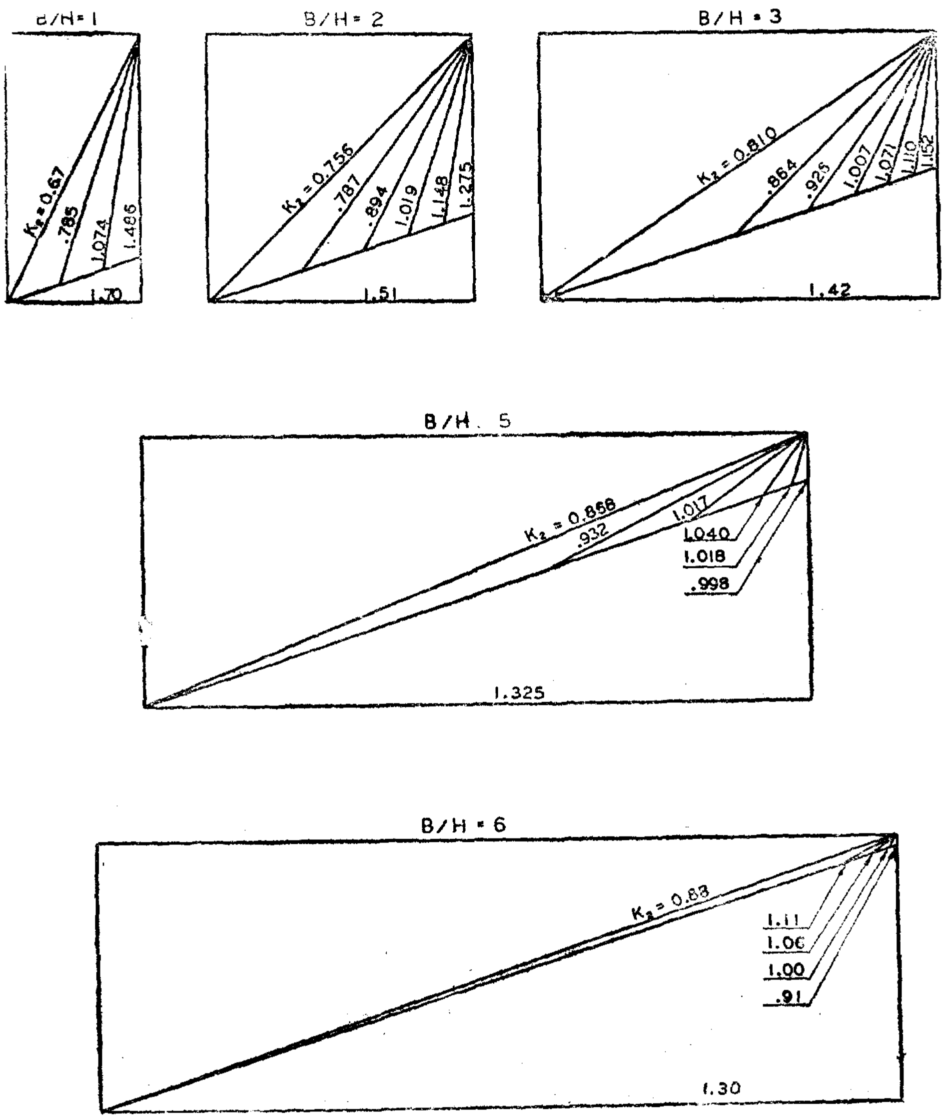

Fig. 15 Section shapes and added mass coefficient (β = 0.60)

Thus

$$\sigma = \frac{1}{2} + \frac{\lambda\left\{\frac{1}{2\lambda} - \tan(2-\beta+\gamma)\pi\right\}\left\{1 - \frac{1}{2\lambda}\tan(\beta-0.5)\pi\right\}}{1-\tan(\beta-0.5)\pi\,\tan(2-\beta+\gamma)\pi} \tag{47}$$

It is seen from above equation that the contours of constant β and γ induce the hyperbolic relation between σ and $B/H(=1/\lambda)$ in three parameter family of straight-framed section. The curves of k_2 on the base of parameter λ at constant β value of 0.60, for example, are plotted in Fig. 14.

Several sections and corresponding added mass coefficients k_2 when $\beta = 0.6$ are shown in Fig. 15 to show the effect of γ values. From Fig. 12 and Fig. 15 it is seen that the values of added mass coefficient k_2, at contant β values, increase with the γ values in considerable range of beam draft ratio and decrease while γ increase comparatively from certain high beam draft ratios. It is interesting to note that the every k_2 curve in Fig. 12 shows that its values are closer to that of trianglular section and are far from that of rectangular section.

Comparison of Straight-framed Section and Lewis Form.

It is seen from the results of computation in the preceding sections that in straight-framed section the chine angles are also basic parameters having marked effects on added mass coefficients as well as beam-draft ratio and sectional area coefficient. Thus the comparison between straight-framed sections and Lewis forms on added mass coefficient is of little merit if it is done only at the same beam-draft ratio and sectional area coefficient, neglecting the chine angles.

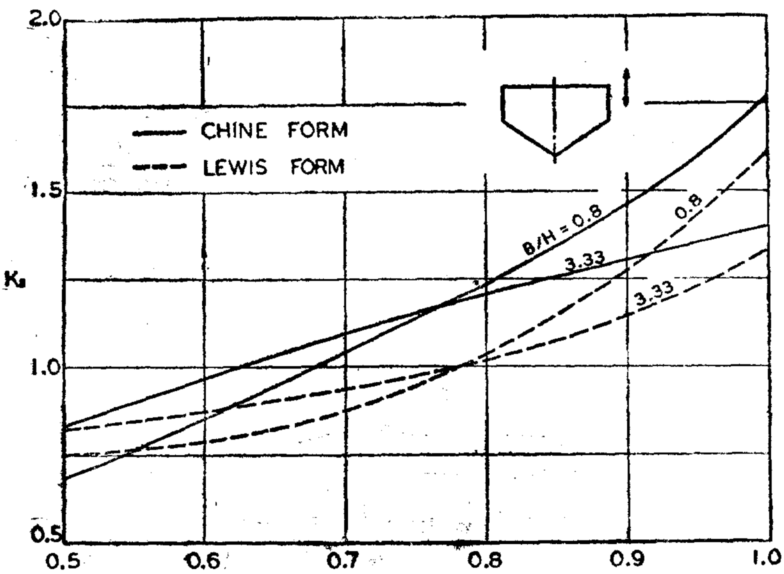

Fig. 16 The comparison of straight-framed sections with vertical side and Lewis forms on added mass coefficients

For the two parameter family of straight-framed section, however, angle β is predetermined if beam-draft ratio and sectional area coefficient are chosen. Therefore, for this class of straight-framed sections it is clear that the comparison of the straight-framed section and Lewis form is possible provided σ and B/H are used as basic parameters.

For B/H = 2 ($\lambda = 0.5$) and 3.33 ($\lambda = 0.3$), for examples, the straight-framed sections of vertical side give greater values of added mass coefficient than those of Lewis form in almost entire region of sectional area coefficients. The maximum difference of about 20 percent over Lewis form are attained near the sectional area coefficient 0.8 and it is maintained in the considerably broad band as shown in Fig. 16.

The differences decrease as σ values approach to the rectangles (σ = 1) or triangles (σ = 0.5).

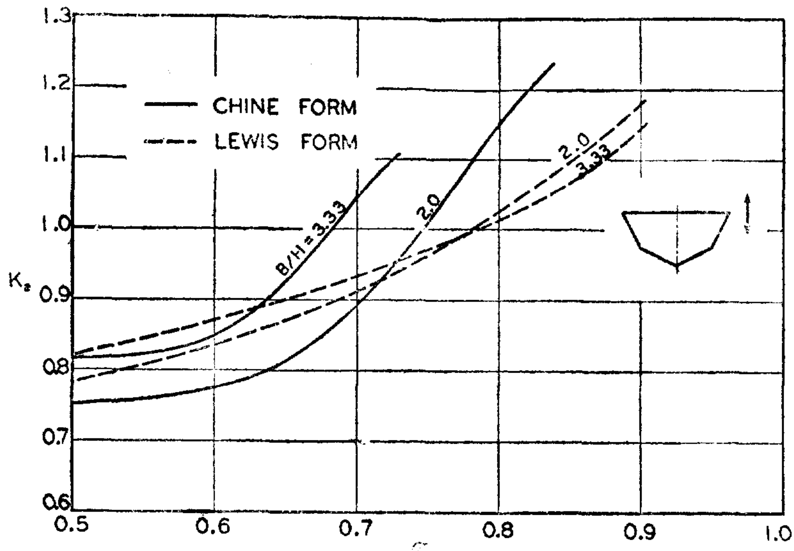

Fig. 17 The comparison of straight-framed sections with raked side (β=0.60) and Lewis froms on added mass coefficients

Hwang and Kim [11] have shown, from their computation on added mass coefficients for twelve typical straight-framed sections having vertical side and flat bottom and covering four groups of chine angles, that the added mass coefficient of those sections had considerably higher values than those of Lewis forms having same sectional area coefficients and beam draft ratios and that the increments ranged from 3% to 30% in whole being significantly controlled by the chine angle as stated at the early part of this paper.

Nevertheless, from the results of systematic computation on the straight-framed section with raked side and bottom, as examplified for the constant angle β of 0.60, added mass coefficients associated with vertical oscillation at high frequency give greater value than those of Lewis forms beyond the range of σ = 0.63 for B/H = 3.33 (λ = 0.3) and beyond the range of over σ = 0.73 for B/H = 2.0(λ = 0.5) as shown in Fig. 17. And it is interesting to note that the straight-framed sections for β = 0.60 give smaller added mass coefficients than those of Lewis forms at those σ values smaller than 0.63 for B/H = 3.33 and σ values smaller than 0.73 for B/H = 2.0. It appears that these results are mainly dependent upon the slope of the sides of the section.

Summary and Conclusion

The foregoing analyses have demonstrated a general technique of employing Schwartz-Christoffel transformation for the added mass calculation of two-dimensional cylinders with straight-framed sections and chines oscillating vertically in the free surface of an ideal fluid at high frequencies.

Specifically, two and three parameter families, including sections found in practical chine ship forms such as with raked sides and deadrise bottoms, were analyzed and the results were found to be well within the expected range of values compared to Lewis forms and other previous works.

The study shows that the parameter controlling chine angle is a significant one as are other parameters such as beam draft ratios and sectional area coefficients. It is significant to note that the method such as employed here raises no difficulty with regard to raked sides and deadrise, and thus such effects may be studied further in detail by employing this technique.

In real flow, due to the sprays on the sides and eddies at the knuckles, if any, certain differences in added mass coefficients are expected. And the effect of frequencies of oscillation is of interest in practical application. An experimental study, therefore, to complement the above would show further interesting results, and such is recommended.

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References

1. F.M. Lewis, "The Inertia of the Water Surrounding a Vibrating Ship," *Trans. SNAME*, vol. 37, 1929.
2. C.W. Prohaska, "Vibrations verticales de navire," *Bulletin de L'Association Technique Maritime et Aeronautique*, 1947.
3. L. Landweber and M. Macagno, "Added mass of Two-Dimensional Forms Oscillating in a Free Surface," *Journal of Ship Research*, vol. 1. No. 3, 1957.
4. L. Landweber and M. Macagno, "Added Mass of a Three-Parameter Family of Two-Dimensional Forms Oscillating in a Free Surface," *Journal of Ship Research*, vol. 2, No. 4, 1959.
5. O. Grim, "Die Schwingungen von Schwimmenden, Zweidimensionalen Körpern," *Hamsburgische Schiffbau-Versuchsanstalt, Gesellschaft* Report 1171, 1959.
6. F. Tasai, "On the Damping Force and Added Mass of Ships Heaving and Pitching," *Journal of Zosen Kiokai*, vol. 105, 1959.
7. W.R. Porter, "Pressure Distributions, Added Mass, and Damping Coefficients for Cylinders Oscillating in a Free Surface," *University of California Report*, Series 82, Issue 16, 1960.
8. J.R. Paulling, "Measurement of Pressures, Forces and Radiating Waves for Cylinders Oscillating in a Free Surface," *University of California Report*, Series 82, Issue 23, 1962.
9. K. Wendel, "Hydrodynamische Massen und hydrodynamische Massenträgheitsmomente," *STG*, vol. 44, 1950.
10. J. H. Vugts, "The Hydrodynamic Coefficients for Swaying, Heaving and Rolling Cylinders in a Free Surface", *ISP*, vol. 15, No. 167, 1968
11. J.H. Hwang and K.C. Kim, "A Study on the Added Mass Associated with the Vertical Oscillation of the Straight-Framed Ships in a Free Surface," *Science and Engineering Report, College of Engineering, Seoul*

National University, vol. 3, No. 1 (written in Korean), 1968.
12. E.T. Whittaker and G.N. Watson, *A Course of Modern Analysis*, Cambridge University, Press, 1927.

APPENDIX A

Evalution of Integral (23)

$$r_k = \int_0^k (1 - \varphi^2)^{\beta-1}(k^2 - \varphi^2)^{\gamma-1}\varphi^{4-2(\beta+\gamma)}\, d\varphi \tag{23}$$

Let $(\varphi/k)^2 = t$, then (23) becomes

$$r_k = \frac{1}{2}k^{3-2\beta}\int_0^1 t^{3/2-\overline{\beta+\gamma}}(1 - t)^{\gamma-1}\,(1 - k^2 t)^{\beta-1}dt \tag{48}$$

In evaluating the above integral we employ the following relation [12]

$${}_2F_1(a,\, b;\, c;\, x) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 t^{b-1}(1 - t)^{c-b-1}(1 - tx)^{-a}dt \tag{49}$$

$$[\, Re(c) > Re(b) > 0,\ |x| < 1\,]$$

Then (23) will be reduced to

$$r_k = \frac{1}{2}k^{3-2\beta}\,\frac{\Gamma\left(\frac{5}{2} - \overline{\beta+\gamma}\right)\Gamma(\gamma)}{\Gamma\left(\frac{5}{2} - \beta\right)}\ {}_2F_1\left(1 - \beta,\ \frac{5}{2} - \overline{\beta+\gamma};\ \frac{5}{2} - \beta;\ k^2\right). \tag{50}$$

since $|k| < 1$ and, $\frac{5}{2} - \beta > \frac{5}{2} - \overline{\beta+\gamma} > 0$.

Hypergeometrical function ${}_2F_1(a,\, b;\, c;\, x)$is also expressed as follows,

$${}_2F_1(a,\, b;\, c;\, x) = \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)\,\Gamma(b+n)}{\Gamma(c+n)}\,\frac{x^n}{n!} \tag{51}$$

$$(|x|<1)$$

Then

$$\begin{aligned}&{}_2F_1\left(1 - \beta,\ \frac{5}{2} - \overline{\beta+\gamma};\ \frac{5}{2} - \beta;\ k^2\right)\\ &= \frac{\Gamma\left(\frac{5}{2} - \beta\right)}{\Gamma(1-\beta)\,\Gamma\left(\frac{5}{2} - \overline{\beta+\gamma}\right)}\sum_{n=0}^{\infty}\frac{\Gamma(1-\beta+n)\,\Gamma\left(\frac{5}{2} - \overline{\beta+\gamma} + n\right)}{\Gamma\left(\frac{5}{2} - \beta + n\right)}\,\frac{k^{2n}}{n!}\end{aligned} \tag{52}$$

$$(|k|<1)$$

From (49) and (51) we have

$$r_k = \frac{1}{2}k^{3-2\beta}\frac{\Gamma(\gamma)}{\Gamma(1-\beta)}\sum_{n=0}^{\infty}\frac{\Gamma(1-\beta+n)\,\Gamma\left(\frac{5}{2} - \overline{\beta+\gamma} + n\right)}{\Gamma\left(\frac{5}{2} - \beta + n\right)}\,\frac{k^{2n}}{n!} \tag{25}$$

Since $|k|<1$ in the present problem, the series appeared in the right side of (25) converges. It is noted that $\frac{1}{2} \leq \beta < 1$, $1 < \gamma < \frac{3}{2}$ and $\frac{3}{2} < \beta + \gamma < 2$.

The integral appeared in the first term of equation (31) may be readily evaluated similarly.

APPENDIX B

Evaluation of Integral (24)

$$r_1 = \int_k^1 (1 - \varphi^2)^{\beta-1} (\varphi^2 - k^2)^{\gamma-1} \varphi^{2(2-\overline{\beta+\gamma})} d\varphi \tag{24}$$

$$(0 < k < \varphi < 1)$$

Let $(\varphi^2 - k^2)/(1 - k^2) = t$, then (24) becomes

$$r_1 = \frac{1}{2}(1 - k^2)^{\beta+\gamma-1} \int_0^1 (1 - t)^{\beta-1} t^{\gamma-1} [k^2 + (1 - k^2)t]^{3/2-\overline{\beta+\gamma}} dt. \tag{53}$$

Since $|k| < 1$ and the above integral does not seem to be approximated to any convergent series as the previous one we condesider a new transformation of variable, say

$$1 - t = u.$$

Then (53) becomes

$$r_1 = \frac{1}{2}(1 - k^2)^{\beta+\gamma-1} \int_0^1 u^{\beta-1} (1 - u)^{\gamma-1} [1 - (1 - k^2)u]^{3/2-\overline{\beta+\gamma}} du.$$

By the same technique applied in equation (23) we have

$$r_1 = \frac{1}{2}(1 - k^2)^{\beta+\gamma-1} \frac{\Gamma(\gamma)}{\Gamma(\beta + \gamma - \frac{3}{2})} \sum_{n=0}^{\infty} \frac{\Gamma(\beta + \gamma - \frac{3}{2} + n)\Gamma(\beta+n)}{\Gamma(\beta + \gamma + n)} \frac{(1-k^2)^n}{n!} \tag{26}$$

Since $|k| < 1$, there follows $0 < 1 - k^2 < 1$ and $\beta + \gamma > \beta > 0$, the series in (26) converges. The integral appearing in the second term of equation (31) may be evaluated by the same method.