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Slope design optimization framework for road cross section using genetic algorithm based on BIM

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Abstract: This paper presents the development of an optimization framework for road slope design. Recognizing the limitations of current manual stability analysis methods, which are time-consuming, are error-prone, and suffer from data mismatches, this study proposes a systematic approach to improve efficiency, reduce costs, and ensure the safety of infrastructure projects. The framework addresses the subjectivity inherent in engineers' decision-making process by formalizing decision variables, constraints, and objective functions to minimize costs while ensuring safety and environmental considerations. The necessity of this framework is embodied by the review of existing literature, which reveals a trend toward specialization within sub-disciplines of road design; however, a gap remains in addressing the complexities of road slope design through an integrated optimization approach. A genetic algorithm (GA) is employed as a fundamental optimization tool due to its well-established mechanisms of selection, crossover, and mutation, which are suitable for evolving road slope designs toward optimal solutions. An automated batch analysis process supports the GA, demonstrating the potential of the proposed framework. Although the framework focuses on the design optimization of single cross-section road slopes, the implications extend to broader applications in civil engineering practices. Future research directions include refining the GA, expanding the decision variables, and empirically validating the framework in real-world scenarios. Ultimately, this research lays the groundwork for more comprehensive optimization models that could consider multiple cross-sections and contribute to safer and more cost-effective road slope designs.

Key words: Slope design, optimization, road cross-section, building information modeling

1. INTRODUCTION

Rapid urbanization in densely populated regions, with the scarcity of available land, has resulted in a large number of both natural and man-made slopes, especially in cities like Hong Kong, where land is extremely valuable. The territory's hilly geography has led to a landscape with numerous slopes, posing potential safety risks to the built environment and population. According to the Hong Kong Government's Highway Slope Manual [1], an average of 166 landslides annually reportedly affected roads between 1984 and 1998. Most of these incidents were related to man-made slopes, and some resulted in fatalities.

To mitigate such risks, the road design process includes iterative stability analyses to evaluate different design options, ensuring slope safety. The road design process usually commences with the representation of a site's original landscape from geotechnical investigation, which is essential for laying out the initial plan outlining the horizontal alignment. The elevation details are provided later through vertical alignment. Subsequently, cross-sectional views at each station describe the specifics of the road structure, including the pavement and barriers, as well as the guidelines for modifying the landscape through cut and fill volumes. This step is critical in determining how the road will interact with the existing topography. A preliminary design of the road comes with the above-mentioned design criteria, thereby authentically leading to the slope stability analysis alongside whole cut and fill slope sections of a road to check whether the slopes satisfy the safety requirements. After slope stability analyses, any sections deemed unstable are redesigned, making the road design process both iterative and repetitive. Revisions to initial designs are common, as they are frequently updated based on the outcomes of stability analyses.

Challenges arise in the manual process of stability analysis, which is required along the full length of the road, especially in varied terrains. This manual approach is not only time-consuming and prone to errors, but also often results in data mismatches when mapping the additional information necessary for a comprehensive analysis. These additional data, which include material properties not covered in the initial design, must be manually inputted, creating a task prone to inconsistencies and inefficiencies. A previous work by the author [2] partially addressed this problem by providing a formal representation of data exchange for slope stability analysis to improve interoperability.

Yet the decision-making process underlying these technical tasks is often reliant on engineers' subjective judgment, leading to subjective interpretations and potentially inconsistent results. This subjectivity extends to the final road design, where engineers may default to conservative measures across the project if instability is detected in any section. The absence of a structured framework for making decisions, particularly in slope design, poses significant risks. Engineers' subjective decisions may not align with the most efficient or cost-effective solutions, highlighting the need for a systematic approach to enhance efficiency, reduce costs, and ensure the long-term sustainability of infrastructure projects.

Addressing these issues, the establishment of a formalized optimization framework for slope design decision-making in road projects is imperative. Such a framework would enable a road slope design with assured safety, controlled cost, and a sufficient search within the feasible domain defined by the constraints, thereby enhancing the efficiency and reliability of the design process.

This study adopts single cross-section road slope design optimization as an example. Section 2 introduces the existing literature on road design optimization topics, followed by Section 3's introduction of the optimization framework, including the objective function, decision variables, and optimization method. Section 4 concludes this study and proposes future research directions.

2. POINTS OF DEPARTURE

2.1. Diversity of optimization objectives in road projects

Plenty of optimization objectives exist in the realm of road design, ranging from reducing travel time and lowering construction expenses to enhancing safety measures and mitigating environmental consequences. Although individual studies may concentrate on a singular goal, the practice of multi-objective optimization (MOO) is extensively recognized in the literature. The primary objectives, although diverse, are identified as cost, construction timing, and environmental impact.

Construction projects are often dominated by the dual concerns of cost and project duration. The primary optimization focus in some literature is cost reduction, as in road projects discussed by Vázquez-Méndez et al. [3], whereas other studies integrate cost optimization with secondary aims, such as minimizing emissions [4] and augmenting road safety [5]. This includes designing roadways to enhance traffic flow and diminish congestion, thereby reducing emissions from stationary vehicles [4]. Moreover, the adoption of sustainable materials like permeable pavements contributes to these environmental goals [6]. Noise pollution reduction is also being addressed through the reconfiguration of road networks to reduce traffic noise [7] and the exploration of sound-absorbing materials [8]. Safety is a pivotal aspect of road project optimization, and researchers have devised various methods to optimize road safety, taking into account factors such as geometric design [9] and the use of protective barriers [10]. Addressing and mitigating risks in road projects are equally critical, which involve identification, impact assessment, and the creation of resilient designs [5]. Accounting for uncertainties, as explored by [11], further strengthens the resilience of these optimization strategies.

2.2. Optimization methodologies in road design research

The application of various optimization techniques has been documented across road design studies., prevalent methodologies such as genetic algorithms, particle swarm optimization, and bilevel programming are categorized alongside machine learning due to its rising prevalence in recent research. A considerable number of papers have employed mathematical programming methods to reach optimal solutions, which include an array of techniques from bilevel to dynamic programming. Metaheuristic algorithms, notably genetic algorithms and particle swarm optimization, are commonly selected in the literature for road design challenges. Yet, other metaheuristic approaches such as ant colony optimization [12] and simulated annealing [11] are gaining attention. The integration of multiple algorithms or the combination of algorithms with mathematical programming techniques is sometimes employed to enhance optimization outcomes [12]. Machine learning methods present novel approaches to road design optimization problems, as demonstrated by [13], who harnessed a synergy of reinforcement learning and deep learning techniques.

2.3. Sub-disciplines and themes considering road design optimization

Disciplinary keywords such as "transportation" and "traffic assignment" have been categorized under two main topics: traffic and network as well as geometric design. Optimization for road material design is also a critical theme. The complexity of network design poses significant challenges within the transportation sector. Consistent academic pursuits have sought to enhance traffic performance, including vehicular queue length reductions [13]. At the same time, signal control optimization has been researched in conjunction with network design [13]. Geometric road design, which encompasses the physical layout of roadways, has seen a progression from earlier research focusing on traditional problems [14] to more contemporary studies that tackle current challenges such as safety and emissions [15]. The quest for optimal road performance continues with a focus on the materials used in construction. Research into the composition and properties of asphalt mixtures aims to optimize durability and sustainability [16]. Vaitkus et al. [8] delve into the potential of high modulus asphalt concrete to extend the lifespan of road surfaces whereas Bellara et al. [6] investigate the environmental advantages of warm mix asphalt.

2.4. Research on road slope design optimization

Regarding the topic of road slope design optimization, researchers have mostly conducted retrospective case study analyses with existing landslide cases or specific locations of road slopes [17,18,19]. In addition, optimization studies have been carried out on the road structure slope along alignment [20,21]. Road side slope design optimization studies are limited. Momo et al. [22] proposed an optimization model for resource road-based vertical alignment and side slopes in 3D model using convex optimization. However, the optimization based on 3D model is usually time and resource consuming, which leads to difficulties in implementation in design practice. Therefore, there is a need to propose an optimization framework for road slope design that considers general design parameters of side slopes, ensures safety, controls cost, and is also efficient to apply during practice.

3. OPTIMIZATION FRAMEWORK

3.1. Objective function

The objective of this framework is to reduce costs while still ensuring the safety of slope design, within the defined constraints. Therefore, the objective function of the optimization framework represents the total cost to be minimized. The scope of this study starts from a single cross-section slope design; cut and fill balancing throughout the project along with hauling costs is not considered at this stage. The hypothetical scenario is shown in Figure 1, which is a section of road design with an identical cross-section design. The depth along the alignment only allows for volume calculations.

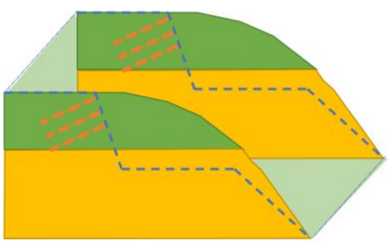

Figure 1. An illustrative view of hypothetical scenario of the optimization framework

The objective function is a quantitative measure used to evaluate the performance of a given cross-section design. This study uses this function to minimize the overall cost of the project while adhering to engineering standards and safety requirements. The objective function is composed of three main cost components, each of which reflects significant aspects of the construction process.

Cut and Fill Cost: This component of the objective function accounts for the expenses associated with the excavation (cut) and embarkment (fill) of earth materials to achieve the desired road cross-section slope. It includes the expenses associated with the excavation of material from cut areas and the use of this material in fill areas to establish the desired road profile. Costs here are determined by the volume of material handled and the complexity of the excavation or fill process. This cost also encompasses the operational costs of machinery and labor for the cut and fill work. This aspect of the objective function aims to optimize the balance between cut and fill volumes to reduce the need for additional material and the costs of its disposal or acquisition. A well-designed cut and fill plan can minimize waste and optimize the use of onsite materials, thereby reducing the overall project cost.

Reinforcement Cost: This part of the function reflects the costs associated with stabilizing the created slopes. It includes the expenses for materials and the installation of any necessary reinforcement structures, such as retaining walls, anchors, or geosynthetic layers, that are required to ensure the structural integrity and safety of the slope. Effective reinforcement ensures the long-term stability and safety of the slopes, reducing the risk of landslides or slope failures. The costs can vary widely depending on the complexity of the stabilization measures needed and the factor of safety deemed acceptable for the project, usually specified by specifications in different regions.

Land Used Cost: This component considers the economic implications of the land area required for the road project. In urban or high-value areas, the cost of land acquisition or the opportunity cost of using the land for the road (as opposed to other potential uses) can be substantial. The objective function thus includes these costs to encourage designs that minimize the footprint of the road, thereby conserving land and reducing the overall impact on the environment and surrounding communities.

Each of these components must be carefully balanced to achieve a cost-effective and safe road design. These three criteria constitute the following objective function form:

$$C = V_c*P_c+V_f*P_f+N_r*P_r++A*P_l \quad (1)$$

where V_c represents the cut volume and V_f represents the fill volume; Nr represents the number of reinforcement items; A represents the area of land use; and P_c, P_f, P_r, and P_l represent the unit price for cut, fill, reinforcement item, and land use, respectively. The objective function is used to compare different design alternatives, with the goal of finding the one that provides the best overall performance.

3.2. Decision variables and constraints

With the objective function specified, Table 1 introduces the decision variables and constraints in the optimization framework.

Table 1. Decision variables and constraints

Decision Variables	Symbol	Type	Unit	Constraints
Number of material types of cut slope	N_c	integer	N/A	N/A
Number of material types of fill slope	N_f	integer	N/A	N/A
Steepness of cut slope (material 1)	θ_{C1}	real	degree	upper limit and lower limit
Steepness of cut slope (material 2)	θ_{C2}	real	degree	upper limit and lower limit
Steepness of cut slope (material 3)	θ_{C3}	real	degree	upper limit and lower limit
…				
Steepness of cut slope (material N_c)	θ_{CNc}	real	degree	upper limit and lower limit
Steepness of fill slope (material 1)	θ_{F1}	real	degree	upper limit and lower limit
Steepness of fill slope (material 2)	θ_{F2}	real	degree	upper limit and lower limit
Steepness of fillslope (material 3)	θ_{F3}	real	degree	upper limit and lower limit
…				
Steepness of fillslope (material N_f)	θ_{FNf}	real	degree	upper limit and lower limit
Width of berm	W_b	real	m	upper limit and lower limit

Steepness of berm	θ_b	real	degree	upper limit and lower limit
Number of slope face	N_s	integer	N/A	N/A
Height of each slope face	H	real	m	upper limit and lower limit
Reinforcement (Yes/No)	R	boolean	N/A	N/A
Length	L	real	m	upper limit and lower limit
Size	R_r	real	m	upper limit and lower limit
Spacing	S	real	m	upper limit and lower limit
Location start	X_r,Y_r	real	m,m	upper limit and lower limit
Number	N_r	integer		upper limit and lower limit
Direction/inclination	θ_d	real	degree	upper limit and lower limit
Factor Of Safety (FOS)	FOS	real		upper limit and lower limit
Distance from center line to the slope boundary (Left)	B_l	real	m	upper limit and lower limit
Distance from center line to the slope boundary (Right)	B_r	real	m	upper limit and lower limit

N_c (number of material types of cut slope): This integer variable represents the number of different geological material types present in the cut slope area. The design must account for each material type due to the materials' unique mechanical properties.

N_f (number of material types of fill slope): Similar to Nc, this integer variable indicates the number of different material types in the fill slope area, which affects the stability and design of the fill slope.

θ_{Cx} (steepness of cut slope for material x): These real variables (θ_{C1}, θ_{C2}, θ_{C3}, ..., θ_{CNc}) denote the angles of the cut slopes for each material type. They are measured in degrees and are bounded by upper and lower limits based on engineering standards and safety requirements.

θ_{Fx} (steepness of fill slope for material x): Analogous to the θ_{Cx} variables, these real variables (θ_{F1}, θ_{F2}, θ_{F3}, ..., θ_{FNf}) represent the steepness of the fill slopes for each material type, also constrained by upper and lower limits for design and safety considerations.

W_b (width of berm): This real variable specifies the horizontal width of the berm, which is a flat or gently sloping area that breaks the continuity of a steep slope. The width is measured in meters and has design constraints to ensure functionality and safety.

θ_b (steepness of berm): This real variable indicates the angle of the berm in degrees. It must also adhere to upper and lower limits for effective operation and stability.

N_s (number of slope face): An integer variable that denotes the number of distinct faces or segments of the slope. Different faces may have different characteristics or design requirements.

H (height of each slope face): A real variable measured in meters representing the vertical height of each slope face. This variable is important for calculating the overall stability of the slope and is subject to design constraints.

R (reinforcement, Yes/No): A boolean variable indicating whether reinforcement design is to be used in the slope design.

L (length): This real variable specifies the length of elements such as soil nails or piles, measured in meters, critical for the stabilization process and subject to design limits.

R_r (size): A real variable indicating the size (typically diameter) of reinforcement elements such as soil nails, measured in meters, with constraints for effective soil reinforcement.

S (spacing): This real variable represents the spacing between reinforcement elements, measured in meters, which affects the strength and stability of the reinforced soil mass.

X_r,Y_r (location start): These real variables give the starting coordinates for the placement of reinforcement elements, measured in meters, with design constraints to ensure proper positioning.

N_r (number): An integer variable representing the quantity of reinforcement elements such as soil nails, which needs to be optimized within certain limits.

θ_d (direction/inclination): A real variable that specifies the inclination angle of reinforcement elements, measured in degrees, with design constraints for proper installation and effectiveness.

FOS (factor of safety): A real variable representing the factor of safety, which is a measure of the reliability of the slope stability. It is dimensionless and must meet specified upper and lower limits for safety assurance.

B_l (distance from center line to the slope boundary—left): This real variable measures the horizontal distance from the road center line to the left slope boundary, measured in meters, and is subject to design constraints for road alignment and land use.

B_r (distance from center line to the slope boundary—right): Similar to B_l, this real variable measures the distance from the road center line to the right slope boundary, also measured in meters and within certain design limits.

3.3. Optimization method

This study employs a genetic algorithm (GA) as the core optimization method to address the slope design in road construction projects. The GA is a bio-inspired heuristic that simulates the process of natural evolution, making it adept at navigating large, multi-dimensional search spaces to identify optimal solutions.

The application of a GA to slope design is particularly appropriate as the first trial, due to the nature of the problem, which requires balancing various factors such as safety, cost, and environmental impact. The GA's ability to evolve solutions over generations by emulating natural selection mechanisms enables it to improve the design alternatives. As the algorithm iterates, it converges on the most effective design that satisfies the design requirements and constraints.

The suitability of GA in optimizing slope design is further embodied by its efficiency and practicality. First, the GA significantly reduces the time and computing resources required, especially when compared to an exhaustive search approach. Unlike an exhaustive search, which evaluates every possible solution in the search space, the GA intelligently navigates toward areas of potential optimum by simulating the process of natural selection. This targeted search not only conserves computational resources, but also expedites the optimization process, delivering timely solutions that are essential in dynamic project environments. Second, the GA serves as a starting point for the optimization process. As a classic and well-established method, it provides a robust framework for tackling optimization problems. Its widespread application across various domains demonstrates its capability as a preliminary method.

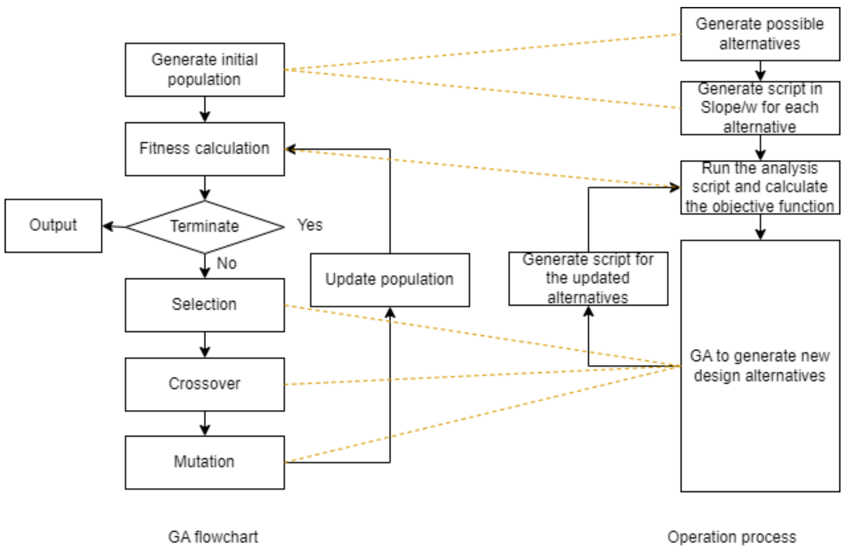

Figure 2. A general GA flowchart and the operation process of the optimization framework in this study

Figure 2 depicts a classic process of GA together with the possible operation process of this study. The genetic algorithm initiates with the generation of an initial population, representing a diverse range of slope design solutions. Each member of this population is evaluated for its fitness, a quantifiable measure derived from a predefined objective function specific to slope stability considerations. The algorithm progresses iteratively, provided the termination criteria (e.g., a maximum number of iterations and a threshold level of fitness) are not met. During each iteration, the GA conducts a selection process, where superior designs are chosen based on their fitness scores. These selected designs undergo crossover, a procedure where pairs of solutions are combined to produce new designs that inherit characteristics from both parents. To maintain diversity within the population and prevent premature convergence, mutation introduces random variations to the offspring. Subsequent to the introduction of these new designs, the population is updated, and the process repeats. Upon meeting the termination criteria, the algorithm yields the optimal design, which is the output of the GA.

Parallel to the GA workflow, an operation process is introduced. Initially, a set of potential design alternatives is established. For each alternative, an analysis script is generated within the Slope/W module of GeoStudio, which is a specialized tool for conducting slope stability analysis. The execution of these scripts facilitates the calculation of the objective function, assessing the viability of each design. The calculated fitness values from the Slope/W analysis are then fed back into the GA. This feedback loop allows the GA to generate new, more effective design alternatives based on the performance metrics obtained from the previous solutions. In turn, updated scripts for these refined alternatives are generated, ensuring a consistent and informed evolution of the design options.

To facilitate the optimization process, an automated batch slope analysis workflow has been developed based on GeoStudio 2023.1.2. This workflow allows for the rapid evaluation of numerous slope design alternatives, providing critical feedback (i.e., FOS results) to the genetic algorithm. The automation process streamlines the iterative assessment of potential solutions.

4. CONCLUSION AND DISCUSSION

This paper addresses the critical and complex issue of road slope design by proposing an optimization framework for a single cross-section. The need for such a framework stems from identified deficiencies in subjective decisions and rule of thumb during current practices, which may not lead to an optimal design, balancing costs, safety, and environmental impacts, in a holistic manner. A review of the existing literature indicates a trend toward many sub-disciplines in road design, yet there remains a gap in specifically addressing the complexity of road slope design through an integrated optimization approach.

To fill this gap, an optimization framework is introduced, with a defined objective function to minimize costs, relevant decision variables, and necessary constraints tailored to the single cross-section slope design of a road. The use of GA is an approach for evolving road slope designs toward optimal solutions. The GA's capability in managing optimization problems is demonstrated through mechanisms of selection, crossover, and mutation, and the operation process is also introduced, illustrated, and explained in this scenario. An automated batch analysis process is developed to facilitate this process. As it is a classic and well-established method, the GA functions as a first and fundamental optimization tool in this study.

In conclusion, the proposed optimization framework contributes to the field of road slope design. It enriches the existing body of knowledge theoretically and provides a practical tool for achieving safer, more cost-effective, and environmentally conscious road slopes. Future endeavors will include the further refinement of the GA, the expansion of the decision variables considered, and the empirical validation of the framework in real-world settings. This research is expected to build the fundamentals for future optimization models, with more optimization methods and extended optimization scope, such as by considering multiple cross-sections of road slope design.

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