시비율 제어된 벅 LED 구동기의 모델링 및 해석

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Modeling and Analysis of Duty-Cycle-Controlled Buck LED Driver

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ABSTRACT

A discrete time domain modeling for the duty-cycle-controlled buck LED driver is presented in this paper. Based on the modeling result, a root locus analysis for the buck LED driver is done to derive the stability boundaries of feedback gains.

1. Introduction

Over the past few years, light-emitting diode (LED) technology has emerged as a promising technology for residential, automotive, decorative and medical applications. This is mainly caused by the enhanced efficiency, energy savings and flexibility, and the long lifetime. Today, LEDs are available for various colors and they are suitable for white illumination. Up to now, numerous attempts have been made to characterize the current-mode control system[1]-[4]. However, all mentioned modeling approaches are related to voltage regulated converters. Very little work has been done in the area of dynamic modeling for the current regulated LED driver[5].

In this paper, the systematic discrete time domain approach[6]-[7] is adapted to modeling and analysis for the duty-cycle-controlled buck LED driver shown in Fig. 1. Root locus analysis is employed to derive the stability boundaries.

2. Discrete time domain modeling of duty-cycle-controlled buck LED driver

$$\delta X_{k+1} = A \cdot \delta X_k + B \cdot \delta v_r \qquad (1)$$

where

$$\delta X_{k+1} = [\ \delta i_{k+1}\ \ \delta v_{k+1}\ ]^T,\ \delta X_k = [\ \delta i_k\ \ \delta v_k\ ]^T,$$

$$\mathrm{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},\ \mathrm{B} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix},$$

$$a_{11} = 1 - \frac{1}{(1-D)}\frac{k_p+k_{ni}D}{(k_p+k_{ni}D/2+S_r)},\ a_{12} = \frac{1}{R_s}\frac{1}{(1-D)}\frac{1}{(k_p+k_{ni}D/2+S_r)},$$

$$a_{21} = R_s\frac{k_{ni}(k_{ni}D/2-S_r)}{(k_p+k_{ni}D/2+S_r)},\ a_{22} = 1 - \frac{k_{ni}}{(k_p+k_{ni}D/2+S_r)},$$

$$b_1 = \frac{1}{R_s}\frac{1}{(1-D)}\frac{1+k_p+k_{ni}D}{(k_p+k_{ni}D/2+S_r)},\ \ b_2 = -\frac{k_{ni}(1+k_{ni}D/2-S_r)}{(k_p+k_{ni}D/2+S_r)},$$

$$\mathrm{D} = V_o/V_i,\ k_p = \frac{R_1}{R_2},\ k_{ni} = k_i T_s = \frac{T_s}{R_2 C_1},\ S_r = \frac{M_e}{(V_o R_s/L)}\frac{D}{(1-D)}.$$

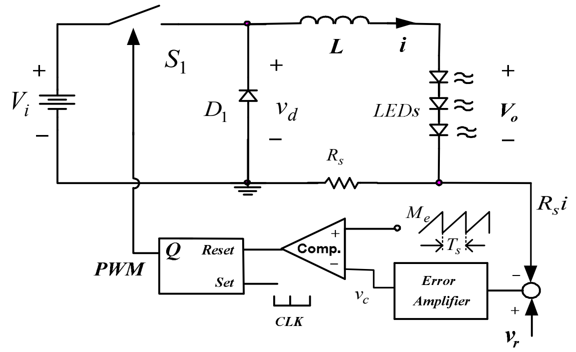

Fig. 1. Duty-cycle-controlled buck LED driver with constant-frequency controller

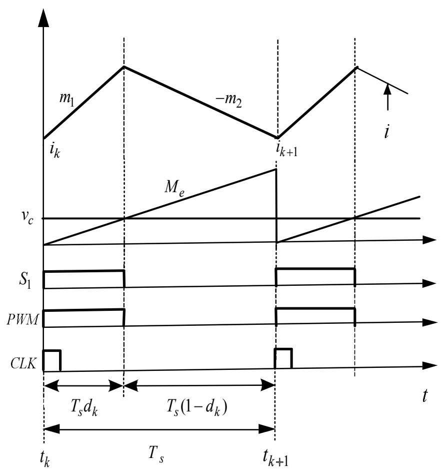

Fig. 2. Key theoretical waveforms of Fig. 1

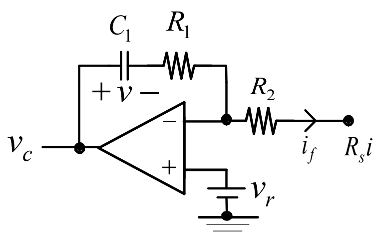

Fig. 3. Proportional-Integral error amplifier circuit

3. Analysis

Bode plots have been commonly used to assess the stability of the closed-loop system by finding the phase margin, but these plots cannot give information on the dynamic behavior of the individual state variables. On the other hand, root locus analysis can provide the engineer with the stability and the transient performance of the individual state variables related to the location of the roots of the characteristic equation.

To analyze the stability and dynamic characteristics of the closed-loop system, the eigenvalues of the system matrix is evaluated. The eigenvalues of A is the solutions of

$$|A - zI| = 0 \qquad (2)$$

where I is the identity matrix. The following root locus analysis is performed for $R_s = 1$.

The root locus as a function of the P gain k_p for $k_{ni} = 0.2$, D=0.45, and $S_r = 0.82$ is shown in Fig. 4. The eigenvalues λ_1 and λ_2 in the z-plane are plotted. Unlike the peak-current-controlled buck LED driver reported in [8], this duty-cycle-controlled buck LED driver is unstable for $k_p = 0$. The eigenvalue λ_1 is dominated by the inductor current state. The transient response of λ_1 after a disturbance is underdamped when k_p is between 0 and 0.6. At $k_p = 0.6$, the system response is critically damped. And then λ_1 moves towards the origin of the unit circle with increasing k_p, which means the inductor current becomes faster. When k_p is greater than 0.6, the transient response of the inductor current is overdamped. Increasing k_p much further, the current response is underdamped with a natural resonant frequency equal to $f_s/2$ due to the negative real value of λ_1. On the other hand, the eigenvalue λ_2 is dominated by the capacitor voltage state of the error amplifier. The transient response of λ_2 is underdamped when k_p is between 0 and 0.6 due to the two complex roots. Then, λ_2 moves towards the unit circle with increasing k_p, which means the capacitor voltage becomes slower. The capacitor voltage is overdamped when k_p is greater than 0.6.

Fig. 5 shows stability boundaries of k_{ni} as a function of D. When k_{ni} is between zero and the stability boundary, the system is stable. While the peak- current-controlled buck LED driver can be stable by selecting a proper I gain for $k_p = 0$ [8], this duty-cycle-controlled buck LED driver is always unstable for $k_p = 0$. The stability boundary of k_{ni} is increasing with increasing k_p for D<0.5. But, this stability boundary of k_{ni} is decreasing with increasing k_p for D> 0.5. The stable range of k_{ni} is very wide for a fixed k_p and D. However, the design engineer need to select the optimum k_p and k_{ni} for a good transient response instead of the simple stable gains.

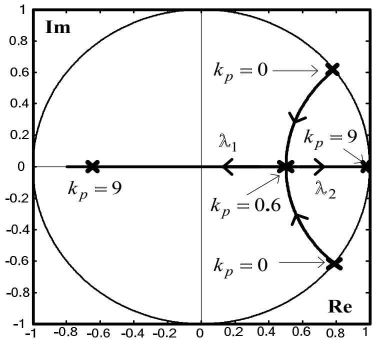

Fig. 4. Root locus as a function of the P gain k_p(k_{ni}=0.2, D=0.45, S_r=0.82)

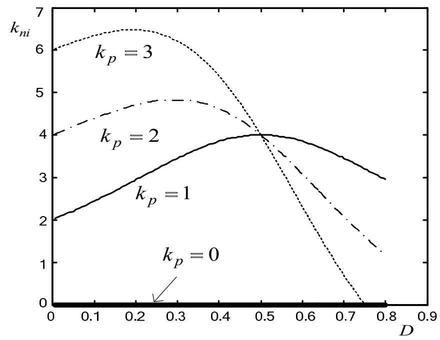

Fig. 5. Theoretical stability boundaries of k_{ni} as a function of D ($S_r = \frac{1.0D}{1-D}$)

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