Basic Study of the Optimization of the Gain Parameters $\alpha$, $\beta$ and $\gamma$ of a Tracking Module for ARPA system on Board High Dynamic Warships

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Abstract: The purpose of this paper is to determine the optimal values of the gain parameters used in the tracking module for a highly dynamic warship. The algorithm of the tracking module uses the $\alpha$-$\beta$-$\gamma$ filter to compute accurate estimates and update the state variables, that is, positions, velocity and acceleration. The filtering coefficients $\alpha$, $\beta$ and $\gamma$ are determined from set values of the damping parameter, $\xi$. Optimization is achieved by plotting a range of the damping parameter $\xi$ against the corresponding residual error and then selecting the best value of $\xi$ with the minimum residual error. Optimal values of the smoothing coefficients are subsequently computed from the selected damping parameter, $\xi$.

Key word : Tracking module, $\alpha$-$\beta$-$\gamma$ filter, optimization, smoothing, prediction, state variables, residual error

1. Introduction

Different methods are used for tracking. However, the most commonly used method is the Kalman filter and its modifications. The $\alpha$-$\beta$-$\gamma$ filter is one of the special cases of the general solution provided by the Kalman filter. It is a third order filter that computes the smoothed estimates of position, velocity and acceleration for the $n^{th}$ observation, and also predicts the next position and velocity.

The $\alpha$-$\beta$-$\gamma$ filter is a simple target tracking filter. It has less computational load since it has only three design parameters hence it is quite simple to implement and control as compared to the Kalman filter which involves a large number of computations.

This study aims to analyze the performance of the $\alpha$-$\beta$-$\gamma$ tracking module and suggests optimal values of the $\alpha$, $\beta$ and $\gamma$ smoothing parameters.

2. $\alpha$-$\beta$-$\gamma$ filter

- The equations for the smoothed estimations are as follows:
  \[ x_k = x_{k-1} + \xi (x_k - x_{k-1}) \]  
  \[ \dot{x}_k = \dot{x}_{k-1} + \xi (\dot{x}_k - \dot{x}_{k-1}) \]  
  \[ a_k = a_{k-1} + \xi (a_k - a_{k-1}) \]  
- The predicted position and velocity are expressed as shown below:
  \[ x_{k+1} = x_k + \frac{T}{2} \gamma (a_k + a_{k+1}) \]  
  \[ \dot{x}_{k+1} = \dot{x}_k + \frac{T}{2} \gamma (a_k + a_{k+1}) \]  
- The smoothing parameter for position is determined as follows [Mafailza et al., 2004]:
  \[ \alpha = 1 - \xi \]  
  \[ \beta = \frac{1}{2} \xi (\xi + 1) \]  
  \[ \gamma = \frac{1}{2} \xi (\xi + 1) \]
3. Simulation and Evaluation by Position

3.6 Figures of Target position when \( \xi \) is varying

Fig. 13-14: Target position when \( \xi \) is varying

3. Simulation and Evaluation by Position

3.7 Summation of Residual Error T-P, T-S

Fig. 17-20: Summation of Residual Error T-P, T-S

3. Simulation and Evaluation by Position

4. Evaluation by Velocity and Acceleration

4.2 Equations to Get Theoretical Velocity Acceleration

1. Equations of Theoretical Position
\[ x(t) = x_0 + v_t t + \frac{1}{2} a t^2, \] \[ y(t) = y_0 + v_y t + \frac{1}{2} a t^2, \]

2. Equations of Theoretical Velocity
\[ \dot{x}(t) = v_t + a t, \] \[ \dot{y}(t) = v_y + a t, \]

3. Equations of Theoretical Acceleration
\[ \ddot{x}(t) = a, \] \[ \ddot{y}(t) = a, \]

\[ \sum_{i=1}^{N} x_{i}\] = 20670.60 m \[ \sum_{i=1}^{N} y_{i}\] = 112220.20 m

\[ x(t) = x_0 + v_t t + \frac{1}{2} a t^2, \] \[ y(t) = y_0 + v_y t + \frac{1}{2} a t^2, \]

\[ \dot{x}(t) = v_t + a t, \] \[ \dot{y}(t) = v_y + a t, \]

\[ \ddot{x}(t) = a, \] \[ \ddot{y}(t) = a, \]
4. Evaluation by Velocity and Acceleration

4.3 Comparison of Theoretical Velocity and Acceleration with Smoothed Velocity and Acceleration

Fig. 23-28 Comparison of Theoretical Velocity and Acceleration with Smoothed Velocity and Acceleration

5. Conclusion and Prospects

5.2 Optimal $\zeta$ of Varying Velocity

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>Initial Velocity</th>
<th>Average Velocity</th>
<th>OKE$\zeta$</th>
<th>OKE$\zeta$</th>
<th>OKE$\zeta$</th>
<th>OKE$\zeta$</th>
<th>Recommended $\zeta$</th>
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<td>12.9</td>
<td>10.34</td>
<td>0.77</td>
<td>0.78</td>
<td>0.75</td>
<td>0.78</td>
<td>0.5–0.7</td>
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<tr>
<td>15</td>
<td>30</td>
<td>31.3</td>
<td>19.24</td>
<td>0.65</td>
<td>0.68</td>
<td>0.62</td>
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<tr>
<td>30</td>
<td>50</td>
<td>50.36</td>
<td>30.38</td>
<td>0.60</td>
<td>0.64</td>
<td>0.56</td>
<td>0.62</td>
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<td>90</td>
<td>88.43</td>
<td>54.02</td>
<td>0.53</td>
<td>0.57</td>
<td>0.49</td>
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<tr>
<td>70</td>
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<td>111.4</td>
<td>69.28</td>
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<td>0.45</td>
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<tr>
<td>90</td>
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<td>145.4</td>
<td>91.20</td>
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<td>0.54</td>
<td>0.43</td>
<td>0.51</td>
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</tbody>
</table>

(OK$\zeta$PEP Optimized $\zeta$ evaluated by summation of residual error of true position and predicted position; OK$\zeta$PEP Optimized $\zeta$ evaluated by summation of residual error of true position and smoothed position; OK$\zeta$PEP Optimized $\zeta$ evaluated by summation of residual error of theoretical acceleration and smoothed acceleration; OK$\zeta$PEP Optimized $\zeta$ evaluated by summation of residual error of theoretical acceleration and smoothed acceleration)

Fig. 38 Optimal $\zeta$ of Varying Velocity

5.1 Initial Velocity=50m/s, Average Velocity=30m/s

<table>
<thead>
<tr>
<th>Evaluation Items</th>
<th>Optimal $\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summation of Residual Error of True Position and Predicted Position</td>
<td>0.60</td>
</tr>
<tr>
<td>Summation of Residual Error of True Position and Smoothed Position</td>
<td>0.64</td>
</tr>
<tr>
<td>Summation of Residual Error of Theoretical Velocity and Smoothed Velocity</td>
<td>0.56</td>
</tr>
<tr>
<td>Summation of Residual Error of Theoretical Acceleration and Smoothed Acceleration</td>
<td>0.62</td>
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Conclusion

1. Under the conditions of initial speed of 50m/s, average speed of 30m/s, the optimal $\zeta$ for prediction is 0.60 while the smoothed optimal $\zeta$ is 0.54.
2. Optimal $\zeta$ is inversely proportional to the velocity.
3. Based on the simulation results, the best results in all cases of varying initial speed are obtained when $\zeta$ lies in the interval [0.5,0.7].

Prospects

- $\alpha - \tilde{\beta} - \gamma$ filter will be used instead of $\alpha - \tilde{\gamma}$ filter to improve the precision of tracking.
- The filter will be employed to track the target while both own ship and target are in motion.