트리제거 기법을 이용한 희소신호 복원

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Sparse Signal Recovery via a Pruning-based Tree Search

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ABSTRACT

In this paper, we propose a sparse signal reconstruction method referred to as the matching pursuit with a pruning-based tree search (PTS-MP). Two key ingredients of PTS-MP are the pre-selection to put a restriction on columns of the sensing matrix to be investigated and the tree pruning to eliminate unpromising paths from the search tree. In our simulations, we confirm that PTS-MP is effective in recovering sparse signals and outperforms conventional sparse recovery algorithms.

1. Introduction

In recent years, compressive sensing (CS) has received much attention as a means to recover sparse signals in underdetermined system [1]-[4]. In CS paradigm, the key finding is that as long as the signal to be recovered is sparse, one can recover the signal with far less number of measurements than traditional approaches. The goal of this paper is to introduce an effective sparse recovery algorithm based on the tree search with pruning, referred to as the matching pursuit with a pruning-based tree pruning (PTS-MP). Two key features of PTS-MP are a restriction on columns of the sensing matrix to be investigated (pre-selection) and the pruning strategy to remove the unpromising paths (pruning-based tree search). From the simulations, we show that our approach significantly reduces the computational burden of exhaustive tree search yet achieves excellent reconstruction performance.

2. Matching Pursuit with a Pruning-based Tree Search

The pre-selection is performed to 'roughly' select the column indices which are highly likely to be the support T (index set of nonzero entries). If the pre-selection is performed, then the search set is reduced from $\Omega = \{1, 2, \dots, \mathrm{N}\}$ to Θ, where Θ is a small subset of Ω and $|\Theta| << N$. In the pre-selection, one can use any conventional algorithm to construct Θ, such as the OMP algorithm running more than K-iterations [3] or the generalized OMP algorithm [4].

In the pruning-based tree search, we only use the elements in Θ as the branches so that we can restrict the number of paths and thus lessen the search complexity. When Θ is constructed, PTS-MP performs tree search for sparse signal reconstruction. The tree has K layers, and the goal is to find a path with cardinality K that minimizes the cost function $\mathrm{J}(\Lambda) = \left\| y - \Phi_{\Lambda} x_{\Lambda} \right\|_2$ ($|\Lambda| = K$). In each layer, new branch is added using the new element to the existing path. If we denote the path at layer (iteration) i as $\hat{s}_1^i$, then $\hat{s}_1^i$ is the causal set chosen in first i iterations.. Since visiting all possible paths is exhaustive, we introduce a pruning strategy to remove unpromising paths from the tree. This pruning decision is performed by comparing the cost function of the path and the pruning threshold (smallest cost function among all paths investigated).

To make a proper decision, therefore, we have no way but to consider the cost function of full-blown path
and hence need a noncausal set $\tilde{s}_{i+1}^K$ in the pruning process. This noncausal set $\tilde{s}_{i+1}^K$ is temporarily needed for the pruning operation and can be easily obtained by choosing $\mathrm{K} - \mathrm{i}$ indices of columns in $\Omega \backslash \hat{s}_1^i$ whose magnitude of the correlation with the residual $r_{\hat{s}_1^i}$ is maximal, where

$$r_{\hat{s}_1^i} = \mathrm{y} - \Phi_{\hat{s}_1^i} \hat{x}_{\hat{s}_1^i}. \tag{1}$$

And

$$\hat{x}_{\hat{s}_1^i} = \Phi^{+}_{\hat{s}_1^i}\mathrm{y}. \qquad (2)$$

For example, if $\mathrm{K}-\mathrm{i}=2$, $\Omega \backslash \hat{s}_1^i = \{2,5,6,10,11,\ldots\}$ and

$$\left|\phi_5' r_{\hat{s}_1^i}\right| > \left|\phi_2' r_{\hat{s}_1^i}\right| > \left|\phi_{11}' r_{\hat{s}_1^i}\right| > \left|\phi_{10}' r_{\hat{s}_1^i}\right| > \left|\phi_6' r_{\hat{s}_1^i}\right| > \cdots,$$

then the noncausal set is $\tilde{s}_{i+1}^K = \{2,5\}$.

Once the noncausal set is obtained, it is combined with the causal set as $\tilde{s}_1^K = \hat{s}_1^i \cup \tilde{s}_{i+1}^K$ and then its residual $r_{\tilde{s}_1^K} = \mathrm{y} - \Phi_{\tilde{s}_1^K}\hat{x}_{\tilde{s}_1^K}$ ($\hat{x}_{\tilde{s}_1^K} = \Phi^{+}_{\tilde{s}_1^K} y$) is computed. Using the l_2 −norm of the residual $r_{\tilde{s}_1^K}$, PTS−MP decides whether to prune the path $\hat{s}_1^i$ or not. To be specific, if the magnitude

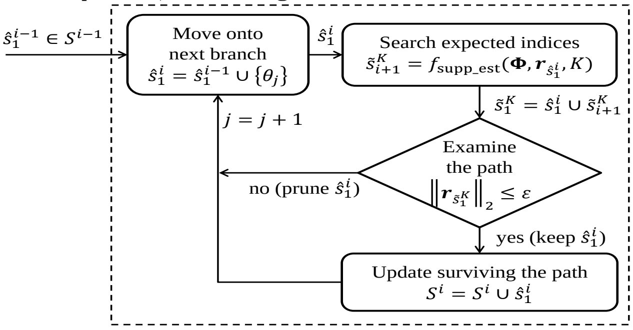

Figure 1 Pruning operation of PTS−MP

of $r_{\tilde{s}_1^K}$ is greater than the pruning threshold ε, then $\hat{s}_1^i$ is regarded to be hopeless and hence is removed from the tree. After the path examination in i−th layer is finished, the pruning threshold is updated to the minimum l_2 −norm of the residual among all surviving paths.

3. Empirical Results and Discussions

In this section, we provide the empirical performance of PTS−MP with existing sparse signal recovery algorithms. The sensing matrix of size 100×256 where each entry is from independent Gaussian random variable is used for the simulation. In order to measure the performance, the exact recovery ratio (ERR) and the mean squared error (MSE) are used for the noiseless and the noisy settings, respectively.

Fig. 2 shows the ERR performance of sparse signal recovery algorithms. Overall, we observe that the addition of tree search process provides substantial gain in performance. In particular, when K is large, performance gain caused by the tree search stage is noticeable. For example, the ERR of PTS−MP at $\mathrm{K}=35$ is 0.94 while that of OMP and CoSaMP are 0.24 and 0.1, respectively.

In Fig. 3, we plot the MSE performance of the sparse recovery algorithms as a function of signal−to−noise ratio (SNR) in the noisy setting. In this test, we set the sparsity level to K = 30 so that 8% of entries of x are nonzero. Overall, we observe that the performance gain of PTS−MP improves with SNR.

In summary, we proposed an effective sparse signal recovery algorithm referred to as matching pursuit with a pruning−based tree search (PTS−MP). In order to overcome the shortcoming of greedy algorithm, PTS−MP performs the tree search and investigates multiple promising candidates. In our empirical simulation, we observed that PTS−MP provides excellent recovery performance in both noiseless and noisy scenarios.

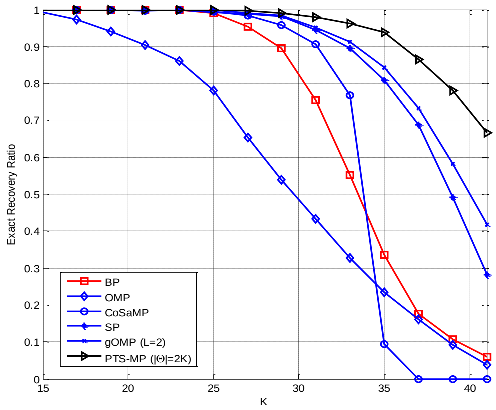

Figure 2 ERR performance

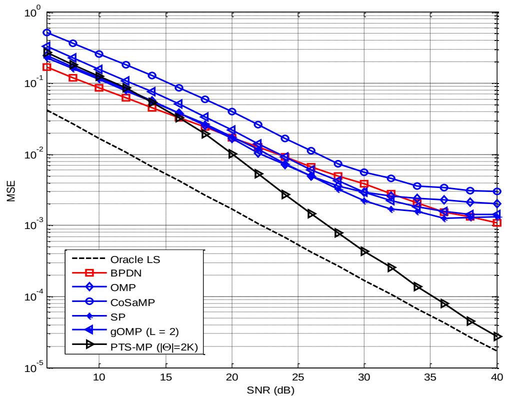

Figure 3 MSE performance

ACKNOWLEDGEMENT

This work was partly supported by the Brain Korea 21 Plus Project in 2015, the ICT R&D program of MSIP/IITP [B0126-15-1017, Spectrum Sensing and Future Radio Communication Platforms] and the National Research Foundation of Korea (NRF) grant funded by the Korean government(MSIP)(2014R1A5A1011478)

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