Quantum Spin Hall Effect And Topological Insulator

Ilyoung Lee[1] Hwan Joo Yu[2] Won Tae Lee[3]
Seoul National Univercity, Department of Physics (1)
Sunkyunkwan University, Department of Physics (2,3)
1 Gwanak-ro, Gwanak-gu, Seoul 151-742

Fractional quantum Hall Effect (FQSH) is one of most fundamental issues in condensed matter physics, and the Topological insulator becomes its prominent applications. This article reviews the general frameworks of these development and the physical properties. FQSH states and topological insulators are supposed to be topologically invariant under the minor change of geometrical shape or internal impurities. The phase transitions involved in this phenomena are known not to be explained in terms of symmetry breaking or Landau-Ginsburg theory. The new type of phase transitions related to topological invariants has acquired new name – topological phase transition. The intuitive concepts and the other area having same type of phase transitions are discussed.

Keywords: Fractional Quantum Hall Effect, Topological Insulator, Topological Phase Transition, Topological Orders
PACS: 73.43.-f, 71.20.-b, 03.65.Vf

I. INTRODUCTION

The study of two-dimensional (2D) electrons in a strong perpendicular magnetic field has become an extremely important research subject during the last two and a half decades. The basic experimental observation for a two-dimensional electron gas subjected to a strong magnetic field is nearly no loss

$$\sigma_{xx} -> 1 \quad (1)$$

On the other hand, Hall conductance is given by

$$\sigma_{xy} -> n\frac{e^2}{h} \quad (2)$$

which is quantized with integer n times (e^2/h). This quantization is universal and independent of all microscopic details such as the type of semiconductor, the purity of the sample, the precise value of the magnetic field, and so forth.

The term (e^2/h) is actually a fine structure constant

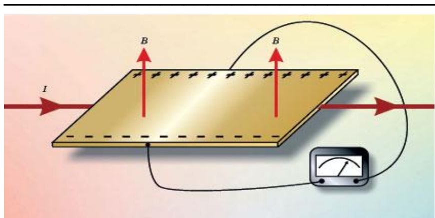

Figure 1. Edwin Hall's 1878 experiment was the first demonstration of the Hall effect. A magnetic field *B* normal to a gold leaf exerts a Lorentz force on a current *I* flowing longitudinally along the leaf. That force separates charges and builds up a transverse "Hall voltage" between the conductor's lateral edges. Hall detected this transverse voltage with a voltmeter that spanned the conductor's two edges

experiments could give an appealingly straightforward precision measurement of the fine-structure constant, yielding a value of 1/137.0360 0300(270). It is surprising to find such a remarkable precision despite of many lousy conditions and different materials being used in real experiments. The theoretical study on this question has arrived the recognition that the Hall conductance at the plateaus has topological significance. It can be understood in terms of topological invariants.

Figure 1 and 2 below show its experimental apparatus and data. The first paper of integer quantum Hall effect was reported in 1980 by Klitzing[1]. This experimental discovery of integer quantization rivals superconductivity in its fundamental significance as a manifestation of quantum mechanics on macroscopic scales, but it was just beginning. In 1982 shortly after the discovery of the IQHE, Horst Stormer, Daniel Tsui, and coworkers at AT&T Bell Laboratories discovered that, in 2D electron gases confined at very high-quality semiconductor interfaces, the Hall conductance develops precisely quantised plateaus at fractional values of (e^2/h).[2] That unexpected discovery required a revision of the quantum Hall paradigm

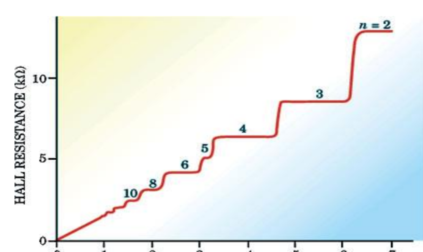

Figure 2. The integer quantum Hall effect. Plotting the Hall resistance of a low-temperature two-dimensional electron gas against the strength of the imposed magnetic field normal to the gas plane, one finds a stairlike quantized sequence of Hall conductances very precisely equal to ne^2/h, where *n* is the integer that characterizes each plateau. The natural unit of resistance defined by this effect is about 26 kΩ. (Adapted from M. Paalanen, D. Tsui, A. Gossard, *Phys. Rev. B.* **25**, 5566 [1982].)

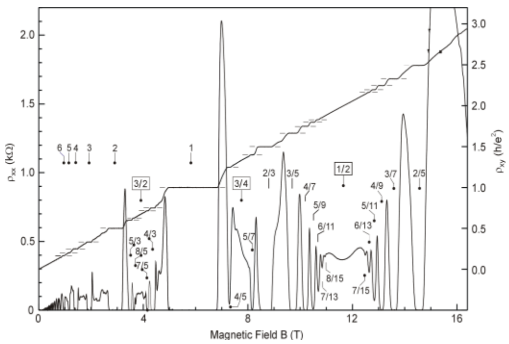

Figure 3: FQHE: In the lowest Landau level (LLL), a number of plateaus at fractional values of h/fe2 appear in the Hall resistance, accompanied by vanishing longitudinal resistivity. These fractional Quantum Hall states are caused by Coulomb interactions between the electrons in a strongly correlated system. These measurements were performed by J.H. Smet and the sample grown by W. Wegscheider

to a new theory in which electron–electron interaction plays a central role. We call this new state the state of Fractional quantum Hall Effect (FQHE). FQHE is another new area of material physics, and it has many different features from simple QHE. FQHE is a collective behavior in a two-dimensional system of electrons. In particular magnetic fields, the electron gascondenses into a remarkable liquid state, which is very delicate, requiring high quality material with a low carrier concentration, and extremely low temperatures. As in the integer quantum Hall effect, a series of plateaus form in the Hall resistance. Each particular value of the magnetic field corresponds to a filling factor .

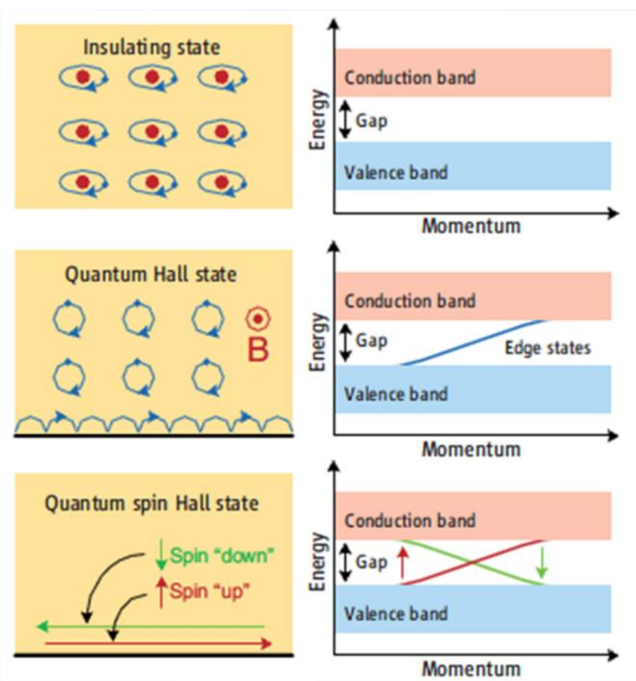

Figure 4: Graphical view of differences among insulator, QHE, and QSHE

The Spin Hall Effect [3] consists in the spin accumulation on the lateral boundaries of a current-carrying sample. No magnetic field is needed. It was predicted by M. I. Dyakonov and V. I. Perel in 1971 and observed experimentally more than 30 years later. Soon after, the existence of a Quantum Spin Hall Effect (QSHE) was developed by Kane and Mele.[4]

The first difference when comparing with QHE is that QSHE doesn't need external Magnetic field. This difference is quite important in application of making small nano scale structure, considering that magnetic field can't be made in small scale. More important fact is that QSHE has a very useful edge properties to have spin filtered current. A spin up carrier will see an effective B-field going into the page due to the spin-orbit coupling, on the while spin down carrier will see an effective magnetic field going out from the page. This generates two opposing edge currents that are spin filtered. The currents are suffer no dissipation. The lossless device is more than academic interest. One goal of spintronics is to circumvent the resistive heating that occurs when electrons are pushed through ever-smaller transistors, capacitors, and other components. Band-based spin flow is dissipation less by nature.

The edge spin current of QSHE is known to have time reversal symmetry. If the symmetry is broken somehow, the QSHE state vanishes. For example, if we put external magnetic field on it, the FQSE and the spin current on the edge will disappear.

Another important fact is that there exist Dirac points in the energy band. Even though velocity of electron is far less than light velocity, electron plays like massless or zero mass. Those phenomena has been confirmed by ARPES experiment by showing linear energy-momentum dispersion relationship that satisfies the Dirac equation and edge electrons are therefore called Dirac fermions in QSHE or topological insulators.

The topological insulator is the 3-dimension QSH bulk system, but the band structure is made up of an odd number of two dimensional conical surfaces, much like the electronic structure of graphene. The term topological insulator has been used to denote this new phase of matter, whereas the QSHE is a two dimensional example.

topological insulatorThe topological insulator becomes a rich repository of peculiar physics not seen in the ordinary physics. The first one is existence of Majorana fermion. It is also the solution of Dirac equation, but its solution, wave function is real, which is different from complex Dirac's solution. Since the anti-particle is defined to be conjugate of its wave function, anti-particle of Majorana particle must be same to itself. Originally this idea aroused in particle

physics long time ago 1930s. However, recently there is a report that Majorana-like quasi particle has been detected from the research group of topological insulator.[5]
They are guaranteed by particle-hole symmetry in spinless superconductors, and expected as egde states in superconductors with topological order.

Physics seems to never stop to surprise us. Another subtle theoretical topic associated topological insulator is anyon.
The early idea of anyon is a type of particle that occurs only in 2d systems, with properties much less restricted than fermions and bosons. The operation of exchanging two identical particles may cause a global phase shift but cannot affect observables. Fermion and boson satisfies.

$$|\Psi_1\Psi_2> = \pm|\Psi_2\Psi_1> \qquad (3)$$

Howeve, in 2d system quasiparticles can be observed which obey statistics

$$|\Psi_1\Psi_2> = e^{i\theta}|\Psi_2\Psi_1> \qquad (4)$$

When such quasi-particles are interchanged the many-body wave function representing them is multiplied with a phase factor which can differ from ± 1. Anyons are crucial for the understanding of the fractional quantum Hall effect.
They are actually neither fermion, nor boson.

Historical overview and short introduction of new physics associated in QSHE and topological insulator are given in this section. The topological insulator as a 3d realization of QSHE are spotlighted due to expected nano-application in spintronics. In section 2, the current direction of material search and way of theoretical calculation is sketched. And the related topological orders will be mentioned.

II. MATERIAL SEARCH AND CALCULATION

TI has energy gaps, and electric current cannot flow inside the material, but there is a topologically protected conducting surface to make possible current flow. Generally, it is possible for insulator to have a current flowing surface. However it is very vulnerable to even small perturbation and easy to lose conductivity. It is a characteristics that the surface of topological insulator is strongly protected by time reversal symmetry and able to maintain conductivity.

The an-initio calculation of electronic structure by using Density functional theory is an effective way to search of material candidate and to predict its properties. The first material as strong topological insulator was $Bi_{1-x}sb_x$ alloy. However, the energy gap of this material is only several tens of meV, and it is found to be difficult to use the surface conductivity effectively because of mixture of thermal excitations. The new material candidates Bi_2Te_3, Bi_2Se_3, Sb_2Te_3 are found to have bigger energy gap and their researches are going on experimentally and theoretically.

Bi_2Se_3 as a most representative TI has rhombohedral lattice structure. Change of the topological order by band inversion is a key ingredient of a topologically nontrivial material.[6] The energy band inversion of above material by spin-orbit coupling is observed and confirmed by theoretical calculation.
Looking at k-space of surface state, one find the linear relationship between energy and momentum at the vicinity of Dirac point. This linear dispersion relation is out of ordinary expectation. Any massive particle can't have linear relation between energy and momentum. This can happen only in massless case in Dirac's relativistic equation. However, the velocity of particle is 0.2% of the light velocity according to calculation.[H. Zhang et al., Nature Physics 5, 438 (2009)] The same linear relationship is also observed in grapheme. So, they are Dirac particles.

The surface state of thick TI is topologically protected, but if the thickness become thin, surfaces of up and bottom would interact and could make a energy gap. This thin TI is very sensitive to its thickness, and physical variables changes also sensitively, when perpendicular electric field is applied on it. It is a very good sample, which we can control physical properties by setting the external condition, for example, the electric field.

The search for new TI material has been launched competitively in many institutes, and first-principle calculation of band inversion and electronic structure receives attention, since it can show many physical properties with cheap cost, before confirming experimentally. Pb-based chalcogenide series having similar lattice structure to Bi_2Se_3 is reported to have conducting surface [7]. Another trial is to use ternary Heusler compound, whose band inversion is possible, but energy gap is almost zeo. In such a case, one can try to derive TI structure by transforming lattice structure or making quantum dots like HgTe.[7]

Up to this time, the surface study of Bi_2Se_3 or similar TI are limited on the plane (1,1,1), but another direction of surfaces are also considered as nanoribbon or step edge. One of the recent calculation of plane (2,2,1) shows the typical Dirac cone in which there are found 2 linear energy bands are crossed. The result is supposed to agree with the study of Bi_2Se_3 claiming strong TI has odd number of topological energy band. So, the ab-initio calculation by DFT can be a effective and reliable method of studying material design and search.

The Landau quantization and Laughlin's model to explain fractional quantum hall effect are not repeated here.[9] For the theoretical calculation, the knowledge for symmetries is absolute necessary to understand. It is like boundary conditions of partial differential equations.

Following is a list up considered in QSH and TI.

1.Berry phase: phase acquired over the course of a cycle, when the system is subjected to cyclic adiabatic processes, which results from the geometrical properties of the parameter space of the Hamiltonian. In another word, when the eigenfunction of the Hamiltonian moves along the closed curve in parametric space, it gets phase – angle between before and after circular motion.

The phenomenon was first discovered in 1956, and rediscovered in 1984. It can be seen in the Aharonov–Bohm effect and in the conical intersection of potential energy surfaces.

2.TKNN invariant: In eq.(2) the number indicating quantization "n" is called TKNN invariant. It is related to the concept of Berry phase in Brilloiun zone.

From the mathematical point of the view, the following three objects are the same thing : the magnetic field *B*, the Berry curvature F, and the Gaussian curvature of *K* (geometry). All of them are described by the same mathematical structure: fiber bundles, showing that they are all quantized due to topological reasons, which is known as topological quantization.

$$\oiint B \cdot dS = \frac{ch}{2q_e} \cdot n \quad \text{for magnetic field}$$

known as the magnetic charge, which measures the number of magnetic monopole inside M.

$$\oiint K \cdot dS = 2\pi \, \chi_M$$

known as the Euler characteristic, which measures the topological nature of the manifold *M.*

$$\oiint F \, d\vec{k} = 2\pi C .$$

known as the TKNN invariant or the Chern number[9], which measures the quantized Hall conductivity for *a* topological insulator.

3. Time Reversal Symmetry:

Accoding to the thermodynamic Law, Nature does not repect Time Reversal Symmetry. On the other hand, Maxwell's equations without material absorption or Newtonian mechanics without friction are time-reversal invariant at the macroscopic level. In QSH state, there are two factors, directions of time and spin are inverted, Therefore, the system is not changed. On the contrary, IQH state needs external magnetic field and it changes the sign.

4. Inversion symmetry: $\Pi \,|\, x,\sigma >= |-x,\sigma >$
where x is the spatial coordinate and sigma signifies the spin. $\Pi^2 = 1$ is self-obvious and the eigenvalue of the operator is $\pm$.

III. TOPOLOGICAL ORDERS

The states of matter are distinguished by their internal structures, called the orders. Physicists discovered many other states of matter in last century, such as superfluids, ferro and antiferromagnets, and liquid crystals that appear in every electronic devices. All those states of matter have different internal structures or orders.

Meanwhile, the term topology is not new term in mathematics, which studies invariant quantity under the continuous deformation. Keeping this in mind, the topological phase transition can't be hard topic. For example, area of a certain closed figure changes under the continuous mapping even without cutting or ripping. Therefore, area is not the object of topological study. However, the fact that the sum of angles of triangles are always 180 degree under the transformation of stretching or shrinking the sides of triangles in the Euclid plane. So the sum of angles of triangles (or generally polygons) is topologically invariant under the transformation.

Suppose 2 physical states which have topological invariants respectively, a1 and a2, whatever their name is. Now, assume the state is transformed from state of a1 to state of a2, then, it is a topological phase transition or topological transform. Topological order should be the name of the structure, which has its topological invariant. IQH is quantized by an integer (Chern number or TKNN), and each state belonging to that number could be interpreted as topological state. If one IQH state with n1 changes to other state with n2, the change of state would be topological phase transition. Topological insulator has many wave function in Brillouin zone with its own topological charge, or topological invariant quantity. Therefore, physics of topological insulator can be described with the topological variables. Topological space, in which physics appears, is not limited to the real space. As shown above, Brillouin zone is the momentum space. Any space can be allowed as a space of topological phase transition.

The development of a crystal order reduces the continuous translation symmetry of a gas to a discrete translation symmetry of a crystal. A general theory is needed to gain a deeper understanding of states of matter and the associated internal orders. Based on the relation between orders and symmetries, Landau developed a general theory of orders and their transitions. Landau's theory is so successful and it seems to explain all transitional problems, at least in principle.

However, since late 1980s, it has become gradually apparent that Landau symmetry-breaking theory may not describe all possible orders. In an attempt to explain high temperature superconductivity the chiral spin state was introduced. At first, physicists still wanted to use Landau symmetry-breaking theory to describe the chiral spin state. They identified the chiral spin state as a state that breaks the time reversal and parity symmetries, but not the spin rotation symmetry. This should be the end of story according to Landau's symmetry breaking description of orders. However, it was quickly realized that there are many different chiral spin states that have exactly the same symmetry, so symmetry alone was not enough to characterize different chiral spin states. This means that the chiral spin states contain a new kind of order that is beyond the usual symmetry description.

Although topologically ordered states usually appear in strongly interacting boson/fermion systems, a simple kind of topological order can also appear in free fermion systems. This kind of topological order correspond in integral quantum Hall state, which can be characterized by the Chern number of the filled energy band if we consider integral quantum Hall state on a lattice. Theoretical calculations have proposed that such Chern number can be measured for a free fermion system experimentally. It is also well known that such a Chern number can be measured by edge states.

Knowing that FQH liquids exist only at certain filling factors, such as 1/3, 2/5, 4/7, one cannot help to guess that FQH liquids should have some internal orders. Different filling factors should be due to those different internal patterns. The hypothesis of internal patterns can also help to explain the rigidness of FQH liquids. Somehow a compression of a FQH liquid can break its internal pattern and cost a finite energy. However, the hypothesis of internal patterns appears to have one difficulty
Theoretical studies indeed reveal that it is possible to construct many different FQH states presumably with different internal orders. It was realized that, however, these internal orders are different from any other known orders and cannot be observed in any conventional ways. What is really new about the orders in FQH liquids is that they are not associated with any symmetries, and cannot be described by Landau's theory.[7, 8] This new kind of order is called *topological order*. Topological order is a new concept of physics.

The topological phase transition as a research area is in very early stage, and topological insulator can be a nice topic for the future theoretical research.

IV. CONCLUSION AND DISCUSSION

The development of QSHE and Topological insulators are reviewed schematically. The topological insulator is the 3d version of QSHE. Its future research attract attention due to the possible application of nano electronics. Last few years since the initial explorations into topological insulators, the level of interest has grown exponentially. There are now dozens of experimental groups around the world, along with countless theorists, studying all aspects of these materials. The material searching for TI and inventing structures for engineering nano devices could be most popular research area. Nobody can guess what kind of product come out from what places. Theorists are expected to make an effort for increasing calculation power. It is actually vital ingredient for the design of new material and new structures. However, looking TI in another side, it is the treasure place of new physics. Realization of the Dirac particle, fractional charge, Majorana particle, anyon, and even magnetic monopole. TI is also a fascinating place of studying topological phase transition and topology itself.

With all those activity there is great hope that some of the ambitious proposals based on topological insulators can be realized, along with others that have not yet even been conceived.

REFERENCES

[1] K. v. Klitzing, G. Dorda, and M. Pepper, *Phys. Rev. Lett.* 45 494 (1980).
[2] D.C. Tsui, H.L. Stormer, A.C. Gossard (1982). PRL **48** (22): 1559.
[3]S. Murakami, N. Nagaosa and S.C. Zhang, Phys. Rev. Lett. 93, 156804 (2004).
[4] C.L. Kane and E.J. Mele, Physical Review Letters 95, 226801 (2005).
[5] Mourik V, Zuo K, Frolov S M, Plissard S R, Bakkers E P A M, and Kouwenhoven L P. 2012 Science 336, 1003,
[6] Fu, Liang; C. L. Kane (2007-07-02). *Physical Review B* **76** (4)
[7] H. Jin et al., arXiv:1007.5480 (2010)
[8] S. Chadov et al., Nature Materials 9, 541 (2010)
[9] Laughlin, R. B. *Phys. Rev. B.* **23** (10) (1981)