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외부 유체의 영향을 고려한 원통형 튜브 내 파동 전파 특성 연구

Nonaxisymmetric propagation of waves in a circular pipes surrounded by liquid

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1. Introduction

Dispersion curves of the propagating modes in a circular pipes surrounded by liquid were calculated, which includes nonaxisymmetric modes as well as axisymmetric mode. Exact analytic solutions were obtained by using the theory of elasticity that can be applied to any tube thickness or frequency. Thus, pressure radiated from the structure toward surrounding liquid was calculated. Results of the current study can be applied to the estimation of the radiated noise from underwater structures such as pile driving or similar shapes.

2. Theory

2.1 Displacement vectors and stress

According to Gauge invariance, displacement vector, $\vec{u}$, can be described in terms of scalar potential, Φ, and vector potential, $\vec{\psi}$, as:

$$\vec{u} = \nabla\Phi + \nabla \times \vec{\psi}, \qquad (1)$$

where each potential satisfies wave equation of:

$$\nabla^2\Phi = \frac{1}{c_c^2}\frac{\partial^2\Phi}{\partial t^2}, \quad \nabla^2\vec{\psi} = \frac{1}{c_s^2}\frac{\partial^2\vec{\psi}}{\partial t^2}, \qquad (2)$$

where c_c and c_s are longitudinal and shear sound speeds in the material respectively. Suppose the coordinate system under investigation as shown in Fig. 1. Assuming dependence of $\exp[i(k_z z - \omega t)]$, where k_z is axial wavenumber, solutions to Eq. (2) are obtained by the product of trigonometric functions, $\cos n\theta$ or $\sin n\theta$, where n is non-negative integer, and the Bessel functions with the order of n and the arguments of $q_c r$ and $q_s r$ where $q_c^2 = k_c^2 - k_z^2$, $q_s^2 = k_s^2 - k_z^2$, and $r^2 = x^2 + y^2$. Once displacement vectors along r, θ, and z are obtained, stress elements in the cylindrical coordinates are obtained from following equations:

$$\tau_{rr} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}\right) + 2\mu\frac{\partial u_r}{\partial r}, \quad (3)$$

$$\tau_{r\theta} = \mu\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right), \qquad (4)$$

$$\tau_{rz} = \mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right), \qquad (5)$$

where λ and μ are Lamé constants.

In the coordinates system as shown in Fig. 1, displacement vectors and stress elements in the elastic solid are described by the infinite series of the linear combination of $J_n(q_c r)$, $J_n(q_s r)$, $Y_n(q_c r)$, and $Y_n(q_s r)$ where n is counted from zero to infinity.

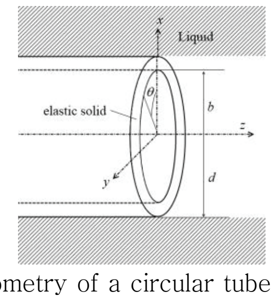

Fig. 1. Geometry of a circular tube surrounded by liquid.

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Since surrounding liquid is assumed as inviscid, in the liquid, $\mu = 0$ and only longitudinal component of wave motion is allowed, which describes the displacement vectors and stress elements in the surrounding liquid by the linear combination of $H_n^{(1)}(q_w r)$ where $q_w^2 = k_w^2 - k_z^2$, $k_w = \omega / c_w$, and c_w is intrinsic sound speed in the liquid.

2.2 Boundary Conditions

Boundary conditions imposed between liquid-elastic solid interfaces are: i) continuity of normal displacement (gives 1 eq.), ii) continuity of normal stress (gives 1 eq.), and iii) vanish of shear stresses (gives 2 eqs.). Since inside of the tube is considered as vacuum (or air), boundary conditions imposed on the inner surface of the tube are: i) vanish of normal stress (gives 1 eq.), ii) vanish of shear stresses (gives 2 eq.). From the boundary conditions, seven equations associated with unknown coefficients are obtained and, thus, the eigenmodes propagating along the *z*-axis are calculated by from the condition of the non-trivial solutions to the associated unknown coefficients.

3. Results

Figure 2 shows the dispersion curves of the several eigenmodes calculated for a stainless steel 304 tube with 8 mm of thickness and 1.35 m of outer diameter. Outside of the tube surrounding the tube is water. Material properties used in the calculation are displayed in Table 1. Horizontal axis is the wavenumber-inner radius product defined in water, and the vertical axis is phase speed of the mode normalized by the sound speed in water. Each mode is identified by modal indices *n* and *m*.

Table 1 Material properties used in the calculation.

Material	ρ (kg/m^3)	c_c (m/s)	c_s (m/s)
ss304	7900	5,675	3,141
water	998	1,486	

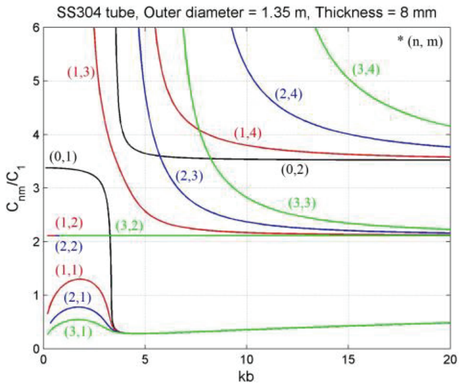

Fig. 2. Dispersion curves for several modes.

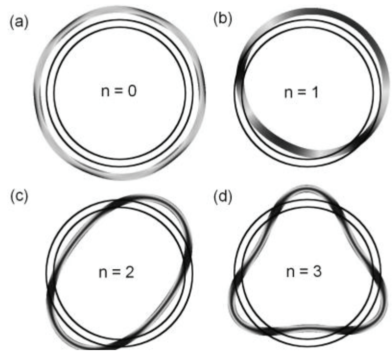

Fig. 3. Mode shapes of the eigenmodes.

Those indices are related to the angular and radial motions of the tube respectively. For each value of *n*, mode shape of the tube is plotted in Fig. 3. Each mode has eigenmodes that exist to the zero-frequency limit while the others diverge at their cut-off frequencies. Once eigemmodes are obtained within the frequency range interested, pressure radiating from this structure toward surrounding liquid can be calculated by the infinite series of the Hankel function.

4. Conclusion

Dispersion curves of the eigenmodes in a circular tube surrounded by the liquid are calculated by using the exact theory of elasticity and the elastic boundary conditions. Using the dispersion relation of the mode, radiating pressure from the structure toward the surrounding liquid can be calculated.