Reclaiming Multifaceted Financial Risk Information from Correlated Cash Flows under Uncertainty

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ABSTRACT: Financial risks associated with capital investments are often measured with different feasibility indicators such as the net present value (NPV), the internal rate of return (IRR), the payback period (PBP), and the benefit-cost ratio (BCR). This paper aims at demonstrating practical applications of probabilistic feasibility analysis techniques for an integrated feasibility evaluation of the IRR and PBP. The IRR and PBP are concurrently analyzed in order to measure the profitability and liquidity, respectively, of a cash flow. The cash flow data of a real wind turbine project is used in the study. The presented approach consists of two phases. First, two newly reported analysis techniques are used to carry out a series of what-if analyses for the IRR and PBP. Second, the relationship between the IRR and PBP is identified using Monte Carlo simulation. The results demonstrate that the integrated feasibility evaluation of stochastic cash flows becomes a more viable option with the aide of newly developed probabilistic analysis techniques. It is also shown that the relationship between the IRR and PBP for the wind turbine project can be used as a predictive model for the actual IRR at the end of the service life based on the actual PBP of the project early in the service life.

Keywords: Feasibility study; Risk analysis; net present value; internal rate of return; payback period

1. INTRODUCTION

Financial risks associated with capital investments are multifaceted. In industry, different feasibility indicators are commonly used together to address various aspects of capital investments [2,9]. For example, the net present value (NPV) measures the amount of capital gains; the benefit-cost ratio (BCR) and the internal rate of return (IRR) measure the efficiency of capital investment; and the payback period (PBP) measures the liquidity of cash flows from the investment.

These feasibility indicators are mostly applied to deterministic discounted cash flow (DCF) analysis under the assumption that the costs and revenues in the cash flow layout can be predicted with certainty. Once specific values of individual feasibility indicator are determined, decision criteria (or hurdle rates) are used to determine whether the proposed investment is economically justified or not. For example, an investment can be economically justified when its IRR is greater than the minimum attractive rate of return (MARR) of the organization, or when its payback period is less than the maximum attractive payback period (MAPP).

The usefulness of financial information from the deterministic feasibility analysis, however, is questioned when it is perceived that most, if not all, cash flow estimates in real world are subject to inherent uncertainty due to the lack of information and the unforeseeable risk factors in the future.

This paper demonstrates an integrative implementation of a probabilistic feasibility analysis procedure. The framework utilizes computational merits of two recently proposed analysis tools for probabilistic analysis of the IRR and PBP to carry out various what-if analyses for stochastic cash flows. These methods are formulated based on basic statistics and probability theories, which make them robust, intuitive, and easily implementable. In addition, Monte Carlo simulation is used to identify statistical relationships between the IRR and PBP.

2. METHODOLOGIES

2.1 Taylor-series approximation for probabilistic IRR analysis

The cash flows of conventional capital investments in the construction industry can be represented with a sequence of cost flows, $\{c_0,\ldots,c_m\}$, followed by a sequence of revenue flows, $\{b_{m+1},\ldots,b_N\}$. Suppose that $\mathbf{X}=\{c_0,\ldots,c_m,b_{m+1},\ldots,b_N\}=\{x_0,x_1,\ldots,x_N\}$ represents such a cash flow. Then, the IRR of a cash flow over N years is determined from the value of the interest rate, r, that makes the NPV zero. That is,

$$NPV(N,r)=\sum_{i=0}^{m}\frac{c_i}{(1+r)^i}+\sum_{j=m+1}^{N}\frac{b_j}{(1+r)^j}=\sum_{i=0}^{N}\frac{x_i}{(1+r)^i}=0 \quad (1)$$

When the cash flow variables are considered random variables, the internal rate of return r in Equation (1) is also a random variable.

In spite of the general acceptance of IRR in industry, analytical methods for dealing with the uncertainty associated with the IRR analysis are limited. Other than Monte Carlo simulation, the Hillier method [3,4,6] has been recognized as the most practical approach to the probabilistic IRR problem. The Hillier method is based on the observation that the probability that IRR is less than an arbitrary value of interest rate r is equal to the probability that the present value is less than zero for the chosen interest rate r. That is,

$$\Pr\{IRR < r\} = \Pr\{NPV < 0 \mid r\} \tag{2}$$

The calculation of the right-hand side of Equation (2) is straightforward from Equation (1) because Equation (1) is statistically a linear combination of random variables. Therefore, the cumulative distribution function of the IRR can be generated from repetitive calculations of Equation (2) at a number of discrete points over the range of potential IRRs.

Recently, a pragmatic solution to the probabilistic IRR problem was proposed by Kim and Reinschmidt [6]. The method was formulated by applying second order Taylor-series approximation to discounted cash flows. Suppose that Y is a function of M random variables, $X = (x_0, x_1,\ldots,x_M)$. Then the mean and variance of Y can be estimated by a Taylor series expansion of Y with respect to the means $\bar{X}$ of the input variables [1,5].

$$E\left[Y\left(x_0, x_1, ..., x_M\right)\right] \cong Y\left(\bar{x}_0, \bar{x}_1, ..., \bar{x}_M\right) + \frac{1}{2}\sum_{j=0}^{M}\sum_{k=0}^{M}\left.\frac{\partial^2 Y}{\partial x_j \partial x_k}\right|_{\bar{\mathbf{x}}} \operatorname{cov}\left[x_j, x_k\right] \tag{3}$$

$$Var\left[Y\left(x_0, x_1, ..., x_M\right)\right] \cong \sum_{j=0}^{M}\sum_{k=0}^{M}\left(\left.\frac{\partial Y}{\partial x_j}\right|_{\bar{\mathbf{x}}}\right)\left(\left.\frac{\partial Y}{\partial x_k}\right|_{\bar{\mathbf{x}}}\right)\operatorname{cov}\left[x_j, x_k\right] \tag{4}$$

In this paper, the method is referred to as the second moment method, for ease of reference. Kim and Reinschmidt [6] derived simple arithmetic equations for the calculation of Equations (3) and (4). The primary merit of the second moment method is its simplicity. Implementation of the second moment method requires only three input arguments: the means and the variances of individual cash flow variables and the correlation coefficients among them. Kim and Reinschmidt [6] also presented a user-friendly visual basic for application (VBA) function based on the second moment method.

Compared with Monte Carlo simulation, the second moment method has practical advantages. First of all, the second moment method does not require an extensive iteration of IRR calculation nor the posterior statistical calculations. There is also a computational difficulty in simulation. That is, if Monte Carlo simulation is used to generate random values of IRR using random values of x_j, the calculation must actually solve the polynomial for the lowest root in every iteration. During this process, it is possible that difficulties may arise in the solution due to certain combinations of the randomized x_j. The second moment method does not actually solve for the roots of the polynomial and therefore does not encounter this potential difficulty. Other characteristics of the three methods are briefly summarized in Table 1.

In this paper, the second moment method is chosen to carry out various what-if analyses for a stochastic cash flow.

2.2 The equivalent cash flow decomposition for probabilistic payback period analysis

Payback period is the time period required to recover the initial investments in capital investments. Payback may play an important role when a project has a high NPV and IRR in the long run or when the profits from an investment are from negative cash outflows (savings) rather than positive cash inflows. For example, the attractiveness of replacing existing equipment may be assessed through a payback analysis in a preliminary feasibility analysis for wind turbine projects.

Table 1. Comparison of probabilistic IRR methods.

Method	The second moment method	The Hillier method	Monte Carlo Simulation
Input requirements	• Means and covariances of all cash flow estimates.	• Means and covariances of all cash flow estimates.	• Probability distributions and covariances of all cash flow estimates.
Computing requirements	• Straightforward arithmetic calculation of Equations (3) and (4). • See [6] for detailed equations.	• Repetitive calculation.	• Random number generation from each random input variable. • Repetitive solution to the deterministic IRR equation.
Outputs	• Mean and the variance of the IRR.	• Cumulative distribution function of the IRR.	• Random numbers on the IRR.
Advantages	• The simplest method. • Ease of computation. • Posterior statistical analysis is not required.	• Complete distribution of IRR • Skewness in IRR distribution can be captured.	• Non-normal distributions can be used for input variables. • Skewness in IRR distribution can be captured.
Disadvantages	• Distribution of IRR needs to be approximated using the computed mean and variance.	• Input parameters need to be decided using the trial-and-error method. • Posterior statistical analysis is required.	• Knowledge in computer simulation is required. • Posterior statistical analysis is required.

For obvious reasons, monetary benefits from wind turbine projects often are spread over a long period of time during which major analysis input parameters are subject to various types of uncertainties (i.e., climate changes, new technologies, maintenance labor and equipment costs, or tax code changes). Therefore, it is reasonable for a potential sponsor to consider the payback period to confirm that the initial investment can be recovered within an acceptable time frame. If necessary, payback information can be used to decide a reasonable warranty period for equipment so that potential risks can be shared among the stakeholders.

Analytical tools for probabilistic payback period include the first passage time method [11] and the NPV-based annual cash flow method [8]. However, the first passage time method has several limitations which make the method less practical for most real world problems [7].

In contrast, the NPV-based annual cash flow method is computationally attractive due to its relative simplicity compared with simulation. The method is based on the observation that the payback period (T) of a stochastic cash flow is less than a particular time (t) can be determined from the probability that the net present value of the cash flow up to the time t is greater than zero. That is,

$$\Pr\{T < t\} = \Pr\{NPV(t,r) > 0 \mid t,r\} \tag{5}$$

where r is the interest rate. The NPV-based payback period method in Equation (5), however, has a critical limitation. That is, it can be applied only at the time points in which NPV can be computed. As a result, when a typical annual cash flow is used, payback period probabilities cannot be calculated during the period between two consecutive years.

Recently, a numerical solution to overcome the discontinuity problem of the NPV-based annual cash flow method has been proposed by Kim et al.[7]. The proposed method, the equivalent cash flow decomposition (ECFD) method, is a numerical technique to obtain a complete distribution of payback period using a discounted cash flow under uncertainty [7]. In short, the ECFD technique converts a typical annual cash flow layout into an economically and statistically equivalent sub-annual cash flow at a desired level of precision for the calculation of payback probability distribution [7].

The ECFD technique is a numerical solution that reconstructs the latent probability information of payback period between consecutive years. The ECFD technique fills in the missing portion in cumulative distribution function of payback period obtained by the NPV-based payback period method. In addition, a complete distribution of payback period generated by the ECFD technique can be easily represented with simple and quantitative statistical parameters such as mean and variance, which would contribute to easier communication.

It should be noted that a practical implementation of the ECFD is rather simple and straightforward. Given a stochastic cash flow layout, all it takes is to construct a sub-annual cash flow using a set of formulas, which can be easily programmed in virtually any programming language.

3. CASE STUDY

3.1 Wind Turbine Project

A wind turbine project is chosen to demonstrate a practical application of the probabilistic DCF techniques introduced previously. Feasibility analysis of wind turbine projects can be characterized as a high level of uncertainties stemming from, in large part, inherent variability of future year-to-year wind resource, possible fluctuations of electricity demand and price in the future, and the operation and maintenance costs throughout the service period. The risk that new technologies and innovations make the current power generation system less economical or even obsolete also exists. Most of all, typical wind turbine investments have a relatively long service life of over two to three decades, which makes any economic projection vulnerable to a significant degree of uncertainty.

In this paper, the data of an actual feasibility study report [10] for a wind power project at the City of Medford, MA is used. In the study report, two locations were chosen and two turbine height options, 32 meter and 40 meter, were compared for each location. As a result, four investment scenarios were compared using their 30-year annual cash flow projections. The cash flow of the investment scenario with the highest IRR is chosen in this paper. Table 2 shows the cash flow used in the following analysis. Note that the cash flow in Year 0 represents the initial investments for construction and installation of the turbine plus cost subsidiaries. In this particular project, $250,000 grant was added to other initial costs. Therefore, in the following analysis, variability of initial cost is imposed on the initial costs of $463,921 instead of the net cost of $213,921 in the table.

Three risk parameters are chosen to investigate the sensitivity of the IRR and PBP of the wind turbine project to the potential variations of the risk parameters. The three risk parameters used in the analysis are as follows:

- level of uncertainty in terms of the coefficient of variation (COV) of each cash flow variable: COV = 0.1, 0.2, 0.3;
- degree of correlation between cash flow variables: COR = 0.0, 0.5, 0.9. Exponentially-decaying correlation is assumed between annual revenues. That is, the correlation between the net revenues in two time periods is assumed to depend on the length of time separating these periods, as in :

  $$\rho_{jk} = \rho^{|k-j|}$$

  Here, ρ is a constant, first-order correlation coefficient and |k – j| is the absolute value of the time separating period k from period j;
- cost overrun in the initial investment in terms of the ratio of actual cost to the estimated cost: the initial cost factor (ICF) = 1.0, 1.1, 1.20.

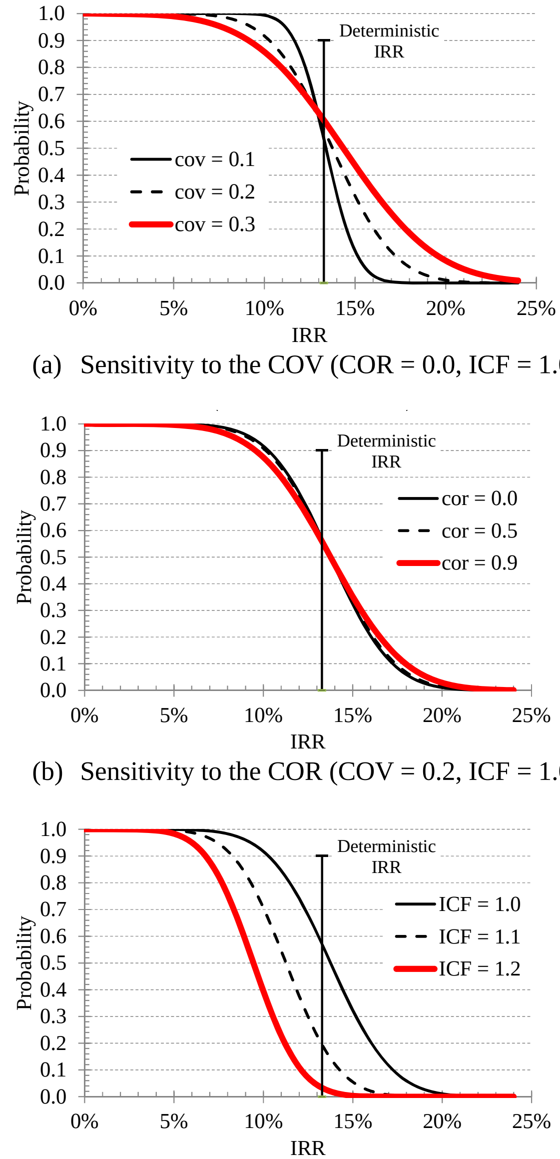
Table 2. A wind turbine cash flow estimates [10].

Year	Cash flow	Year	Cash flow
0	-$213,921		
1	$25,459	16	$31,685
2	$26,069	17	$32,636
3	$26,697	18	$33,615
4	$27,344	19	$34,623
5	$28,011	20	$35,662
6	$28,698	21	$36,732
7	$29,405	22	$37,834
8	$30,134	23	$38,969
9	$30,884	24	$40,138
10	$31,657	25	$41,342
11	$27,332	26	$42,582
12	$28,152	27	$43,860
13	$28,996	28	$45,175
14	$29,866	29	$46,531
15	$30,762	30	$47,927

3.2 Sensitivity of IRR

A series of sensitivity analyses was conducted using the probabilistic methods introduced previously. First, the sensitivity of IRR to the three risk factors is presented in Figure 1. From the results, following risk-based feasibility statements can be made.

- The expected value of IRR is not sensitive to the level of uncertainty and to the correlation in cash flow estimates, while very sensitive to the level of accuracy of the initial cost estimate. In Figure 1(a), the expected IRR at 50% probability of not exceeding is 13.5% at COV = 0.1, 14% at COV = 0.2, and 14.5% at COV = 0.3. Compared to the deterministic IRR of 13.3%, the increase at COV = 0.3 is not significant. In contrast, the expected IRR responses to the accuracy of estimates are rather abrupt. That is, 14% at ICF = 1.0, 11% at ICF = 1.2, and 10% at ICF = 1.2 as in Figure 1(b). Note that the IRR at ICF = 1.1 is about 75% of the deterministic IRR.
- The probability of getting the deterministic IRR is not very sensitive to the level of uncertainty and to the degree of correlation, while very sensitive to the level of accuracy of initial cost estimate. The probability of achieving the deterministic IRR decreases to 20% and 4% when ICF = 1.1 and ICF = 1.2, respectively. This result indicates that when the actual initial cost increases by 10%, which is considered within an acceptable range of typical EPC (Engineering, Procurement, and Construction) project estimates, the deterministically determined IRR can be hardly achieved.



(a) Sensitivity to the COV (COR = 0.0, ICF = 1.0)

(b) Sensitivity to the COR (COV = 0.2, ICF = 1.0)

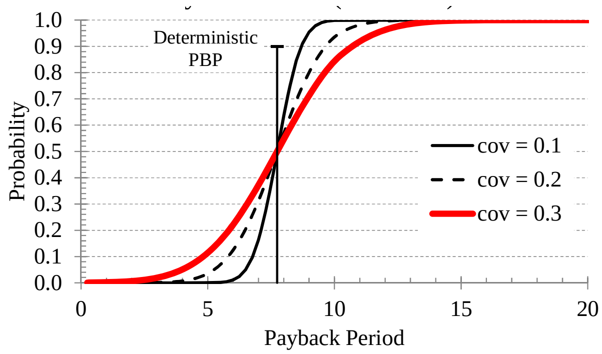
(c) Sensitivity to the ICF (COV = 0.2, COR = 0.0)

Figure 1. Sensitivity analysis for the IRR.

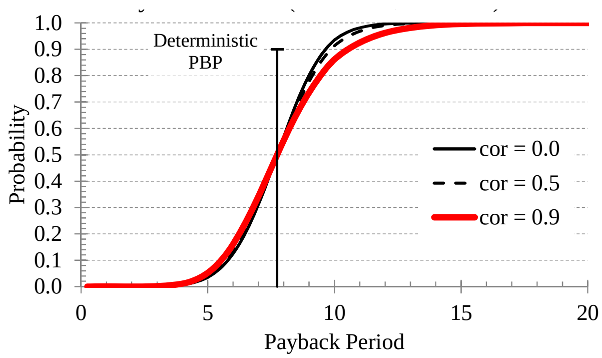
- At a specific level of acceptable risk, the IRR is sensitive to the COVs and the ICFs. For example, at 90% chance of not exceeding the IRR, the IRR decreases to 9% at COV = 0.3 and to 7% at ICF = 1.2, respectively.

3.3 Sensitivity of PBP

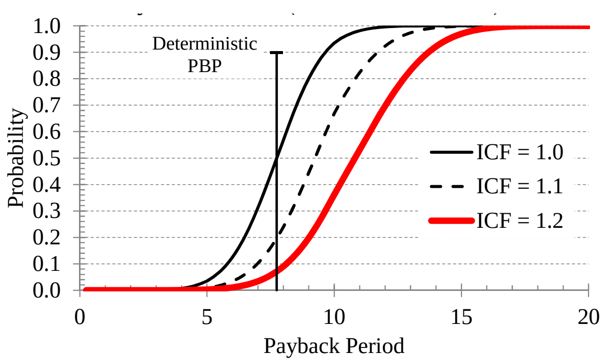
In a similar way, the sensitivity of PBP to the three risk parameters was investigated and the results are in Figure 2. The results indicate that, overall, similar patterns discussed in the sensitivity analysis of IRR are also observed. That is, the payback period is very sensitive to the accuracy of the initial cost regardless of the associated risk level, while it becomes more sensitive to the level of uncertainty in cash flow estimates as the target PBP deviates from the mean. Again, the degree of correlation coefficients does not influence the PBP significantly.



(a) Sensitivity to the COV (COR = 0.0, ICF = 1.0)



(b) Sensitivity to the COR (COV = 0.2, ICF = 1.0)



(c) Sensitivity to the initial cost (COV = 0.2, COR = 0.0)

Figure 2. Sensitivity analysis for the payback period.

3.4 Relationships between IRR and PBP

When multiple feasibility indicators are used concurrently, deterministic estimates of feasibility indicators may fail to properly represent a balanced evaluation of the multifaceted financial risks associated with the investment. For example, Table 3 shows a decision process when the decision criteria for the IRR and PBP of the wind turbine project are established as 15% and 10 years, respectively. The results indicate that the investment is acceptable according to the PBP criterion, but the IRR falls below the MARR. These kinds of mixed messages are not rare in multi-objective decision making problems. In practice, the final decision will be made according to the preference for a specific feasibility indicator or other strategic considerations.

Using the probabilistic analysis results in Figure 1 and 2, an integrative project evaluation based on the IRR and PBP becomes more viable, quantitative and informative.

Table 3. Multifaceted feasibility analysis – A deterministic case (COV = 0.2, COR = 0.0, ICF = 1.0).

Feasibility indicator	Decision criteria	Deterministic solution	Decision
IRR	15%	13.3%	No
PBP	10 years	7.74 years	Yes

Table 4. Multifaceted feasibility analysis – A probabilistic case (COV = 0.2, COR = 0.0, ICF = 1.0).

Feasibility indicator	Decision criteria	Probability of acceptance
IRR	15%	Pr{IRR > 15%} = 32.4%
PBP	10 years	Pr{PBP < 10 years} = 93.5%

For example, from Figure 1 (a) and Figure 2 (a), the probability of achieving IRR higher than the MARR and the probability of achieving PBP shorter than the MAPP can be determined as shown in Table 4. Such probabilistic information in quantitative terms is obviously desirable for decision makers because it will enable them to make better informed strategic decisions instead of being forced to accept a decisive outcome of 'yes' or 'no' as in Table 3.

The risk characteristics of the wind turbine project cash flow can be better understood by identifying a statistical relationship between the IRR and PBP. The procedure to be presented below is formulated based on the idea that IRR and PBP are inherently related because the IRR and PBP are computed based on the same cash flow layout.

In this paper, this hypothetical idea is empirically investigated using Monte Carlo simulation. First, the wind project cash flow layout in Table 1 is analyzed by simulation. A total of 20,000 iterations was carried out. For each iteration, the IRR and PBP are computed. The simulation results are graphically shown in Figure 3. For clarity of the graphics, only the first 1,000 randomly generated IRRs and PBPs are shown in the graphs. In the figure, deterministic solutions to the two feasibility indicator variables are shown in a solid circle.

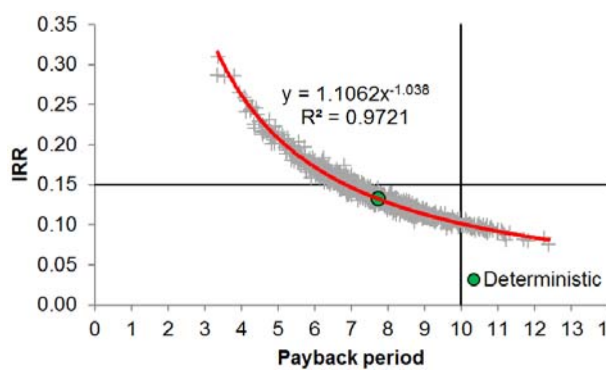


Figure 3. Relationship between the internal rate of return and the payback period.

Two interesting points can be made based on the results in Figure 3. First, the results clearly indicate that there is a strong correlation between the IRR and PBP of the wind turbine project. Five basic regression models (exponential, linear, logarithmic, polynomial, and power) are tested to the simulation results in Figure 3 and the model that provides the highest coefficient of determination (R^2) is shown in the figure. The results show that the power model best fits the simulated samples of the IRR and PBP.

The inversely proportional relationship between the PBP and IRR is intuitive because the PBP is likely to be shorter when the overall profitability of the project in terms of the IRR is higher, for example, when the initial investment is less than the expected value and/or when the revenues during the early years of operation tend to be higher than the expected values. However, the high R^2 value of 0.9721 of the regression model indicates that the actual PBP of the project, which can be observed as short as 3 years, is a good predictor of the IRR, which requires the entire 30 years cash flow layout. The results also illustrate that a well-known criticism against PBP becomes irrelevant to the wind turbine project. That is, the PBP has been criticized as less reliable than the IRR, NPV, or BCR because the PBP analysis is not based on the entire cash flow data. The regression model in Figure 3 clearly shows that PBP is a reliable early indicator of the IRR.

Second, the simulation results can be used to generate refined risk information encompassing multiple feasibility indicators. For example, the probability of satisfying both the MARR and the MAPB simultaneously can be computed from the results in Figure 3. That is, Pr{ (IRR > 15%) and (PBP < 10 years) } = Pr{IRR > 15%} because the PBP corresponding to all IRRs higher than 15% is less than 10 years. Unlike deterministic feasibility evaluations as in Table 3, probabilistic risk information in terms of individual feasibility indicators and the relationships among them can provide a useful tool for integrated evaluation of multifaceted financial

4. CONCLUSIONS

This paper tackles two practical challenges in the multifaceted feasibility analysis for the evaluation of stochastic cash flows. First, newly developed analysis methods are presented to conduct risk-based evaluations of two commonly used feasibility indicators: the internal rate of return and the payback period. The presented methods are robust, simple, and easy-to-implement because, in large part, they are formulated based on basic mathematics and fundamental statistics. Each of the methods can be used to obtain quick solutions to various what-if scenarios for the individual feasibility indicators. Comprehensive risk-based feasibility evaluation becomes more attainable because of the computational simplicity of the proposed methods.

Second, an integrated evaluation of multifaceted financial risk has been investigated by identifying the statistical relationship between the payback period and the internal rate of return. An empirical study has been conducted for a wind turbine project cash flow. The results indicate that statistical models can be established based on the stochastic properties of individual feasibility indicators. It has been demonstrated that the statistical model can be used to predict the actual IRR at the end of the 30-years analysis period as soon as the time that actual PBP is observed.

Using a cash flow estimate for a real wind turbine project, the sensitivities of IRR and PBP to three risk parameters in cash flow estimates were evaluated in terms of probability distribution. Additional information obtained from the probabilistic methods can be a valuable input to a strategic project evaluation.

The probabilistic cash flow analysis techniques presented in this paper can be efficiently used for other feasibility analyses, especially during the preliminary planning phase, in which the major design parameters are determined. In the wind turbine project, key design parameters such as turbine capacity, rotor diameter, and the height of the turbine can be decided through a sensitivity analysis under various combinations of decision factors. The simplicity of the proposed techniques enhances the ability of decision makers for making better informed decisions and makes risk-based cash flow analysis an viable option that can be implemented without simulation or other sophisticated statistical techniques.

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