

## 구형 조화 빔형성 응답을 이용한 음장 해석

### Investigation of Acoustic Fields Using Spherical Harmonic Beamforming Response

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#### 1. Introduction

An investigation on sound field using three dimensional microphone arrays and the spherical harmonics beamforming method has been introduced. The method for localizing a sound source with use of decomposition of sound field into spherical harmonics is described in this study. The sound field of interest has been analyzed within the framework of spherical harmonics and spherical Bessel functions. The space is presented in spherical coordinates.

The sound fields were simulated to check the degree of spherical harmonics and the order of Bessel functions which is necessarily adjusted for various situations.

The spherical waves and plane waves have been looked into describing in the direction of incoming and the wave fields is composed with the delta function which was later decomposed into spherical harmonics. The wave fields for both incoming spherical and plane waves are compared to the sound field on a sphere.

With use of spherical array the sound field can be spatially sampled and determined in practice, the numerical integration on the sphere should be done by extracting the angular part of sound field of sphere.

Two states are introduced to process spherical array signals measured in three-dimensional space. The first stage is to measure the acoustic field using the spherical microphone array. The second is to compute the measured signals with the beamforming algorithm steering the array to a

given direction.

Beampattern occurs either when the microphone array is steered in one direction, while the angle of the incoming wave field is varied or when the array steering direction can be changed while the incoming wave is fixed in direction. The beamforming functions a spatial filtering while suppressing the interference or other noise from other direction. The beamformer response is generally expressed as the output of the beamforming algorithm which is in beampattern and steered response.

The performance of the spherical (SPH) beamforming method on the optimized array and conventional delay-and-sum (DAS) beamforming has been compared. An array with 85 microphones is used to build a test model. The spatial resolution and the maximum sidelobe level of the SPH and DAS beamforming outputs show to be frequency dependent, having a wide mainlobe at low frequencies and a narrower mainlobe at high frequencies. Simulations results are compared to the ones from measurements with the test model showing agreement with all simulations except at low frequencies.

#### 2. Theoretical Background

##### 2.1 Acoustic Pressure on Sphere

###### (1) Wave Equation

Wave equation of sound in homogeneous media can be expressed as

$$\nabla^2 p(\vec{r}, t) = \frac{1}{c^2} \frac{\partial^2 p(\vec{r}, t)}{\partial t^2}, \quad (1)$$

where pressure  $p$ , position  $\vec{r}$ , time  $t$ , Laplace operator  $\nabla$ , and propagation speed  $c$ . The above equation can be Fourier transformed in frequency domain and the Helmholtz equation is

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driven as

$$(\nabla^2 + k^2)\psi(r, k) = 0, \quad (2)$$

where  $k$  is wave number and  $f$  is frequency.

The acoustic field can be described as

$$\psi = \psi_i + \psi_s, \quad (3)$$

Where the first term on right hand side  $\psi_i$  represents scattering term due to plane wave incoming and the second term  $\psi_s$  is the one due to the rigid sphere surface.

### (2) Spherical Fourier Transform

When spherical coordinate system has three coordinates  $(r, \theta, \phi)$  and a direction is given as  $s = (\theta, \phi)$ , the Fourier transform  $\tilde{f}_{nm}$  of a integrable function  $f(s)$  on a unit sphere is

$$\tilde{f}_{nm} = \int_{\Omega \in S^2} f(s) Y_n^{m*}(s) d\Omega = S\{f(s)\}, \quad (4)$$

and its inverse is given as

$$f(s) = \sum_{n=0}^{\infty} \sum_{m=-n}^n \tilde{f}_{nm} Y_n^m(s) = S^{-1}\{\tilde{f}_{nm}\}, \quad (5)$$

where  $Y_n^m$  is the spherical harmonics function of order and degree  $m$ .

### (3) Sound Field Decomposition

In case of incoming plane wave for a rigid surface sphere of radius  $a$ , the pressure is represented as

$$\psi_i(s_i, ka) = 4\pi \sum_{n=0}^{\infty} i^n j_n(ka) \sum_{m=-n}^n Y_n^m(s_j) Y_n^{m*}(s_i), \quad (6)$$

where  $s_j(\theta_j, \phi_j)$  is the direction of sound source and  $s_i(\theta_i, \phi_i)$  sensor element direction.  $j_n(ka)$  is the spherical Bessel function of the first kind. The scattering wave is

$$\begin{aligned} \psi_s(s_i, k, a) = \\ -4\pi \sum_{n=0}^{\infty} i^n \frac{j'_n(ka)}{h'_n(ka)} h_n(ka) \sum_{m=-n}^n Y_n^m(s_j) Y_n^{m*}(s_i), \end{aligned} \quad (7)$$

and sound pressure on a rigid surface of a sphere due to incoming plane wave is

$$\begin{aligned} \psi(s_j, s_i, ka) = \psi_i(s_i, ka) + \psi_s(s_i, ka) \\ = 4\pi \sum_{n=0}^{\infty} i^n b_n(ka) \sum_{m=-n}^n Y_n^m(s_j) Y_n^{m*}(s_i), \end{aligned}$$

(8)

then the mode amplitude coefficients for a sphere is given as

$$b_n(ka) = j_n(ka) - \frac{j'_n(ka)}{h'_n(ka)} h_n(ka), \quad (9)$$

where  $h_n(ka)$  is the spherical Hankel function of the first kind.

## 2.2 Beamforming

Spherical harmonic beamforming uses the orthonormality of spherical harmonic functions and decompose the acoustic field. The integrable function defined on a sphere can be described in spherical harmonic expansions and beampattern can be formed in need. Then the beamforming response  $y_j(k)$  is expressed as

$$y_j(k) = \sum_i w(k, s_j, s'_i) \psi(k, s'_i), \quad (10)$$

Where array response is  $w(k, s_j, s'_i)$  and  $\psi(k, s'_i)$  is pressure measured at  $i$ -th sensors..

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## Reference

- [1] J. Lee, S. Hong, J.-S. Choi and Y.-K. Kim, "Optimization of Sensor Position of a Rigid Surface Spherical Beamformer," in Proc. KSNVE 2010, pp. 128~129.
- [2] J. Lee, S. Hong, and J.-S. Choi, "Increase of Side-lobe Level Difference of Spherical Microphone Array by Implementing MEMS Sensor," in Proc. KSNVE 2011, pp. 816~820.