An Analytical Study on the Gas-Solid Two Phase Flows

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ABSTRACT

This paper addresses an analytical study on the gas-solid two phase flows in a nozzle. The primary purpose is to get recognition into the gas-solid suspension flows and to investigate the particle motion and its influence on the gas flow field. The present study is the primal step to comprehend the gas-solid suspension flow in the convergent-divergent nozzle. This paper try to made a development of an analytical model to study the back pressure ratio, particles loading and the particle diameter effect on gas-solid suspension flow. Mathematical model of gas-solid two phase flow was developed based on the single phase flow models to solve the quasi-one-dimensional mass, momentum equations to calculate the steady pressure field. The influence of particles loading and particle diameter is analyzed. The results obtained show that the suspension flow of smaller diameter particles has almost same trend as that of single phase flow using ideal gas as working fluid. And the presence of particles will weaken the strength of the shock wave; the bigger particle will have larger slip velocity with gas flow. The thrust coefficient is found to be higher for larger particles/gas loading or back pressure ratio, but it also depends on the ambient pressure.

Key Words : Gas-Solid Flow, Two Phase, Nozzle, Analytical Study

1. INTRODUCTION

The supersonic gas solid two phase flow in convergent-divergent nozzle has been studied recent years due to its application to many industrial equipment and national defense such as propulsive systems in rockets using solid fuels, pneumatic conveying using dilute suspension at a high velocity, or accelerate drugs to attain competent speeds to enter the human skin using a gas flows [1]. For high speed gas-solid suspension flow always reaches supersonic speeds [2].

High speed gas-solid two phase flow in convergent-divergent is always associated with lots of complex physics [3]. The flow field will add other forces like Basset /Magnus /Saffman force and other gradient-related forces due to the appearance of solid particles [4]. The experimental investigation also shows that some physical phenomena like particle-turbulence interaction, inter-particle collisions and particle- wall interaction can affect solid particles motion [5-7].The physical

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properties such as the size, density and shape of the particle, and the boundary conditions including wall roughness can also influence flow behavior [8].

In recent years, there has been much work on gas-solid two phase flow. Researchers such like Lourenco and Tsuji have studied on gas solid two phase flow with big particles in the pipe, the experimental and numerical results show that they have predicted the motion of the big particles in the pipe very well [9]. The result of experiment, using shock tube, which carried out by C.R.Jackson and W.E.Lear shows that the shocks generated by gas phase will influence the temperature of mixture very much [10]. The studies conducted by R.Ishii and Y.Umeda on the flows of gas-solid two phase flow in a convergent- divergent nozzle want to predict the particle trajectories and erosion [11]. However, it is not advisable to predict velocity of group of particles only using the single particle velocity. The primary objective of this paper is hence, to develop an analytical model to estimate the effect of different nozzle exit pressure ratios, particles/gas loading and the particle diameter on supersonic two phase flow features. The single phase flow model is extended to solve the one-dimensional mass, momentum, and entropy equations for the steady pressure field. The mass flow, Mach number, thrust coefficient and pressure were computed and analyzed.

2. FORMULATION OF FLOW FIELD

An analytical model is developed to study the gas-solid suspension flow. The following assumptions are made in order to model the flow.

2.1 ASSUMPTIONS

The particles used in this paper are very small, spherical in shape and of uniform size, they do not exchange mass with surroundings or react with each other.

(1) The specific heats of two phases are constant.

(2) The particles are uniformly dispersed in the gas, so that the value of temperature and velocity of both the phases are same.

2.2 GOVERNING EQUATIONS

The governing equations from [8] were used in present study are given as below.

The particles/gas loading (η) is constant defined by:

$$\eta = \rho_p / \rho_g \tag{1}$$

Where: ρ_p is density of particles, ρ_g is density of gas.

The density of mixture (ρ_m) is defined by:

$$\rho_m = \rho_p + \rho_g \tag{2}$$

The internal energy (U_{int}) of mixture is same as it for one-phase fluid:

$$U_{int} = \frac{C_g T}{\eta + 1} + \frac{\eta C_s T}{\eta + 1} \tag{3}$$

Where: C_g is specific heat of gas, C_s is specific heat of particles, T is temperature of mixture.

Similarly, the enthalpy (h_m) of mixture is:

$$h_m = u_{int} + \frac{p}{\rho_m} = \frac{C_g T}{\eta + 1} + \frac{\eta}{\eta + 1}\left(C_s T + \frac{p}{\rho_p}\right) \tag{4}$$

Where: p is pressure of mixture.

The specific heat ratio of mixture ($\bar{\gamma}$) can get from Eq.3 and Eq.4:

$$\bar{\gamma} = \frac{\partial h_m / \partial T}{\partial u_{int} / \partial T} = \frac{\gamma\left(1 + \frac{\eta C_s}{C_g}\right)}{1 + \frac{\gamma \eta C_s}{C_g}} \tag{5}$$

Where: γ is specific heat ratio of gas.

Assuming adiabatic flow, and by applying the first law thermodynamics, we get:

$$dQ = 0 = du_{int} - \frac{p d\rho_m}{\rho_m^2} \tag{6}$$

Rewriting the perfect gas law by mixture density, and using equation (3):

$$(C_g + \eta C_s)\frac{dT}{RT} = \left(\frac{1}{1 - \frac{\eta\rho_m}{(\eta+1)\rho_p}}\right)\frac{d\rho_m}{\rho_m} \tag{7}$$

Integrating and substituting in Eq.5 we get:

$$T\left(\frac{\rho_m}{1 - \frac{\eta\rho_m}{(\eta+1)\rho_p}}\right)^{-(\bar{\gamma}-1)} = C_1 \tag{8}$$

Using equations above and the perfect gas law by mixture density we get:

$$T\rho_g^{-(\bar{\gamma}-1)} = C_2 \tag{9}$$

$$p\rho_g^{-\bar{\gamma}} = C_3 \tag{10}$$

Where: C_1, C_2, C_3 is the constant.

Although both the gas and the solids behave thermodynamically as a single fluid, physically only the volume occupied by the gas is available for accommodating changes in pressure.

The speed of sound in gas with suspended particles will be different from that of gas phase alone. Hence it is necessary to compare the changes in sound speed at different particles/gas loading of two phase flow. Through Prandtl's qualitative analysis for the mixture of snow or sand in air, it can be observed that the speed of sound in mixture is less than that of the prefect gas [8]. It is because the density of the compressible mixture is larger than that of the prefect gas. The speed of sound is given by definition:

$$a = \sqrt{\frac{\partial p}{\partial \rho}} \tag{11}$$

The speed of sound of two phase flow for isentropic conditions can be calculated by:

$$a_s = \left(\frac{\partial p}{\partial \rho_m}\right)^{\frac{1}{2}} \tag{12}$$

Using the above equations, we can get a relation for the speed of sound in gas-solid two phase flow in terms of the speed of sound in gas alone as shown below:

$$\frac{a_s}{a} = \left(1 + \frac{\eta\rho_g}{\rho_p}\right)\left(\frac{\bar{\gamma}}{\gamma(1+\eta)}\right)^{\frac{1}{2}} \tag{13}$$

2.3 GEOMETRY AND DIMENSIONS

The geometry and dimensions of convergent-divergent nozzle are shown in Fig.1.

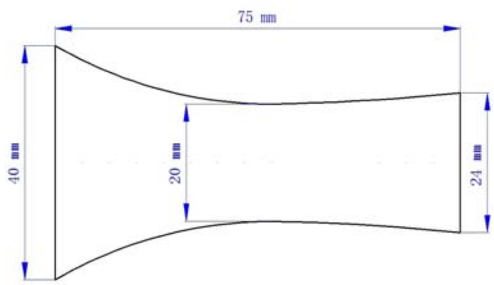

Fig .1 The Geometry and Dimensions of Nozzle

2.4 PROPERTIES OF GAS AND PARTICLE

Table .1 Properties of Gas and Particle

Gas	Particle
C_g = 1005 J/kg/K	C_p = 880 J/kg/K
ρ_g = 1.225 kg/m^3	ρ_p = 2719 kg/m^3
γ = 1.4	

The conditions and physical constants are listed in Table.1. The flow is in equilibrium. This paper employs on six kinds of particles /gas loading (η from Eq.3): 0, 0.1, 0.2, 0.3, 0.4 and 0.5, respectively.

3. RESULTS AND DISCUSSION

Based on the equations for single phase flow, the flow properties for single phase and gas-solid two phase flow in convergent-divergent nozzle were calculated.

3.1 EFFECT OF PARTICLE/ GAS LOADING IN TWO PHASE FLOW

The Fig.2 illustrates the Mach number variations for perfect gas single phase flow and gas-solid two phase flow. Mach numbers are compared for two cases, particles/gas loading equal to 0 and 0.2.

It can be observed that the Mach number for single phase flow is larger compared to the two phase flow at same location. It is seen that at same location, as particles/gas loading increases, the Mach number of two phase flow is decreased. Depending on the back pressure values, shock waves will occur in the supersonic region downstream. Fig.2 also shows the location of critical point of big particle/gas loading moves downstream. This is due to the fact that, as particles/gas loading η increase, the density of mixture also increases; the mixture mass in unit volume becomes larger. For same p_e/p_0, the driving force will be same for all the cases, and hence the flow with lower particles/gas loading will get more acceleration compared to that with higher particles/gas loading.

This also has an effect on shock wave behaviors. The reduction in Mach number across the shock for single phase flow (line AB in Fig.2) will be larger compared to that across the shock of two phase flow (line CD in Fig.4). The strength of the shockwaves in both cases (p_2/p_1, where p_2 is the downstream static pressure and p_1 is the upstream static pressure of the shock wave), is calculated and

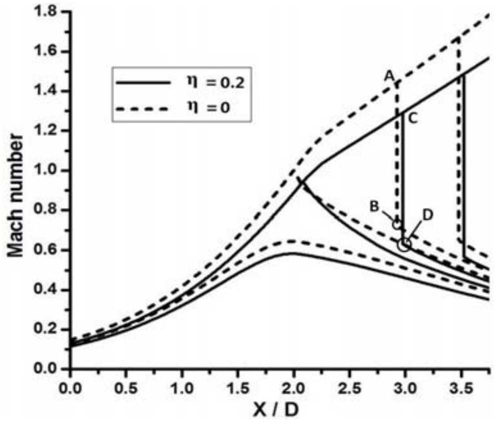

Fig.2 Mach Number Distributions Along Centerline

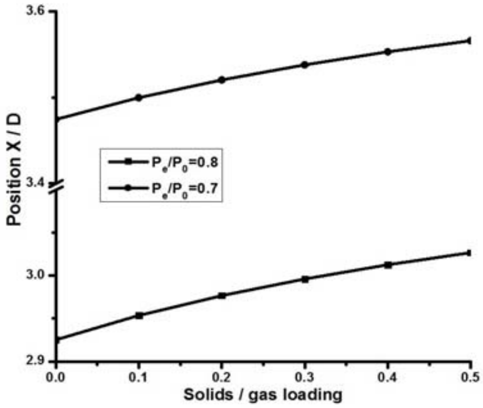

Fig. 3 Location of Shock Wave vs. η

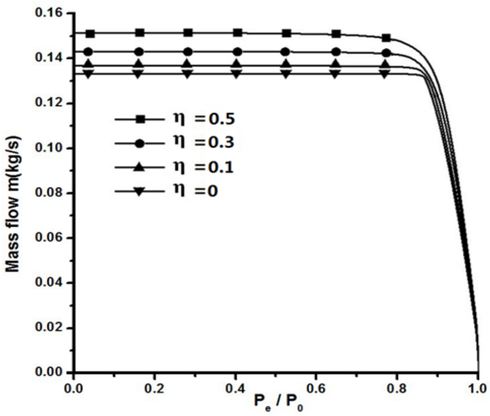

Fig. 4 Effect of η on Mass Flow Rate

found that the strength of shockwave is reduced when particles/gas loading η is increased. This means that the presence of solid particles will reduce the strength of the shock wave.

Fig.3 shows the effect of particles/gas loading on the location of the shock wave for two different pressure ratio, p_e / p_0 are 0.7, 0.8. It clearly shows that the shock wave moves downstream when particles/gas loading η increases. Compared to the single phase flow of perfect gas, the sound velocity of gas-solid two phase flow is small. Eq.12 also shows that at same p, ρ_m increase can lead to sound velocity decrease.

The variations of the mass flow rates of the two phase flow for various particles/gas loading is shown in Fig.4. However, the difference in the mass flow rates is not large. This is because the velocity of mixture decreases when the density of mixture is increases. The mass flow rate tendency of all the cases are same, the occurrence of critical flow conditions implies a maximum flow rate through the nozzle independent of a further reduction of the pressure at the nozzle exit. It is can obtain from the Fig.4 a continuous reduction of the exit pressure will lead to the

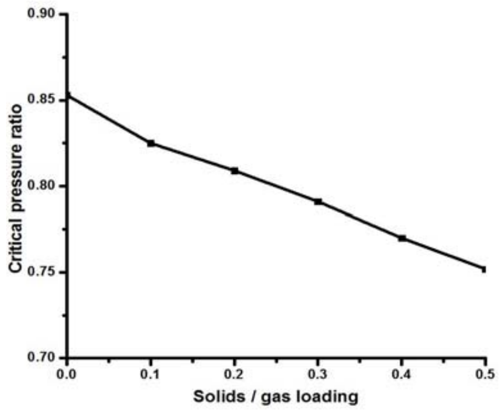

Fig. 5 Critical Pressure Ratio vs. η

critical flow conditions.

It is seen that chocking is attained gradually for higher particles/gas loading compared to the lower particles/gas loading and the critical pressure ratio is also decreased as shown in Fig.5. How the particle/ gas loading influence the critical pressure ratio is shown in Fig.5 in details.

3.2 EFFECT OF PARTICLE DIAMETER

The effect of particle diameter is analyzed here. The previous assumption of uniform velocities for the particle and gas phase becomes invalid here, as the gas phase velocity will be different from the particle velocity for larger particle diameters. While the big diameter particle will influence the turbulent flow [8], this paper only considers the particle which is small enough so that hardly impact on gas phase. The pressure gradient and related acceleration of the flow results in an increase of the slip velocity which is governed by the forces acting on the particles. Obviously, the slip velocity between gas and particles will be larger as the particle diameter increases.

From Newton's second law, the following equation can be derived [13]:

$$v_p \frac{dv_p}{dx} = \frac{3}{4} C_D \frac{\rho_g}{\rho_P d_P} \left(v_p - v_g\right)^2 \tag{14}$$

$$C_D = \begin{cases} \dfrac{3}{16} + \dfrac{24}{Re_P} & Re_P < 0.01 \\ \dfrac{24}{Re_P}\left[1 + 0,1315 Re_P^{(0.82 - 0.051 g Re_P)}\right] & 0.01 < Re_P \leq 20 \\ \dfrac{24}{Re_P}\left[1 + 0.1935 Re_P^{0.0305}\right] & 20 \leq Re_P \leq 260 \\ e^{1.6435 - 1.12421 g Re_P + 0.1558 (1 g Re_P)^2} & 260 \leq Re_P \leq 1500 \end{cases} \tag{15}$$

Where: v_p is particle velocity, v_g is gas velocity, x is distance from nozzle inlet along x-axis, C_D is drag coefficient of particle, d_p is diameter of particle.

Re_p is Reynolds number of particle given by:

$$Re_P = \frac{\rho_g |v_g - v_P| d_P}{\mu} \tag{16}$$

Where: μ is dynamic viscosity of gas given by Sutherland's law:

$$\mu = \mu_0 \left(\frac{T}{T_0} \right)^{\frac{3}{2}} \frac{T_0 + S}{T + S} \tag{17}$$

Where: μ_0 is reference value of viscosity, T_0 is reference temperature; S is Sutherland constant. For air at moderate temperatures and pressures, μ_0 =1.716*10-5 Pa • s, T_0=273.11K, S=110.56K.

The Particle Mach number variation for different particle diameters is shown in Fig.6.

It can obtain when the flow is accelerating, flow loaded with different particle diameters has different slip velocities. It shows in the Fig.6, the larger particle diameter has greater slip velocity. The two phase flow with lower particle diameter (d_p =5μm) shows almost same trend as that of single phase flow, as shown in Fig.6. When the gas was flows through the divergent section, its velocity increases rapidly. This causes the particles also to speed up and the slip velocity between gas and particles become larger.

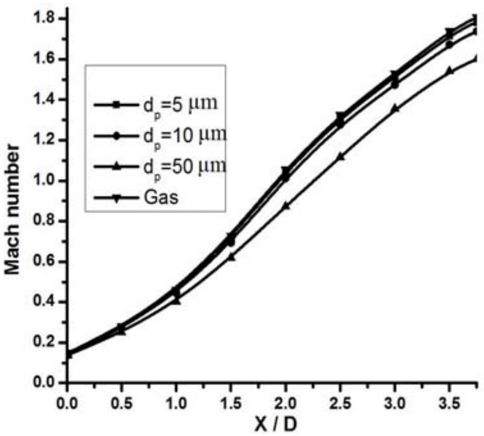

Fig. 6 Mach Number of Particle and Gas

3.3 THRUST COEFFICIENT

The thrust coefficient C_F can be calculated by [14]:

$$C_F = \sqrt{\gamma \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}} \times \sqrt{\frac{2\gamma}{\gamma-1} \left\{ 1 - \left(\frac{p_e}{p_0} \right)^{\gamma/(\gamma-1)} \right\}} + \sqrt{\gamma \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}} \times \left(\frac{p_e}{p_0} - \frac{p_a}{p_0} \right) / \left[\left(\frac{p_e}{p_0} \right)^{1/\gamma} \sqrt{\frac{2\gamma}{\gamma-1} \left\{ 1 - \left(\frac{p_e}{p_0} \right)^{\gamma/(\gamma-1)} \right\}} \right] \tag{18}$$

Where: C_F is thrust coefficient, p_a is ambient pressure, p_e is outlet pressure of nozzle, p_0 is inlet pressure of nozzle.

Taking the derivative of Eq.19 yields the following equation:

$$\frac{dC_F}{d(p_e/p_0)} = \sqrt{\gamma \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}} \times \sqrt{\frac{\gamma-1}{2\gamma}} \left(\frac{p_e}{p_0} \right)^{-\frac{1}{\gamma}} \left(\frac{p_a}{p_e} - 1 \right) \times \left\{ 1 - \frac{\gamma+1}{2} \left(\frac{p_a}{p_0} \right) \frac{\gamma-1}{\gamma} \right\} \left\{ 1 - \left(\frac{p_e}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \right\}^{-\frac{3}{2}} \Big/ \gamma \tag{19}$$

Equation 19 shows that thrust coefficient varies linearly with respect to inlet/outlet pressure ratio (p_e/p_0) and specific heat ratio (γ). When p_a/p_e<1 the flow is "under expanded," the flow becomes "properly expanded," for p_a/p_e=1 and when p_a/p_e >1 the flow becomes "over expanded".

The variation of thrust coefficient for different p_a/p_e, keeping p_a/p_0 as constant is shown in Fig.7. From the below figure it can be noticed that the variation in thrust coefficient can be divided into three sections. For low values of p_a/p_e ratio ($\leq$ 0.5) the thrust coefficient increases steeply as shown as section A in Fig.7. Then the thrust coefficient shows almost a constant value as p_a/p_e value increases up to a critical point where the p_a/p_e value equals to 1 (Section B). At that point C_F becomes maximum (Right hand side of Eq. 19 becomes zero). There after the thrust coefficient decreases with increase in the pressure ratio (p_a/p_e). From Fig.7 it can also be observed that for a constant value of p_a/p_e and p_0/p_e, the C_F increases as particles/gas loading η increases. The big particles/gas loading can get larger thrust coefficient.

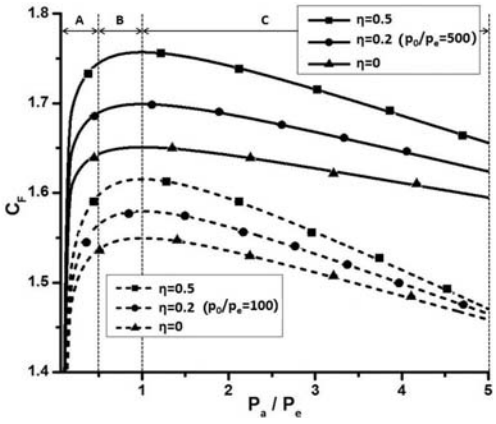

Fig. 7 C_F Variation with η

4. CONCLUSIONS

An analytical study is carried out to investigate the two-phase flow with solid/gas loading through typical convergent-divergent nozzle. The following conclusions are drawn from the analysis.

(1) As the particles/gas loading increase, the velocity and the sound speed of flow will decrease. The existence of particles will reduce the strength of the shock wave in the nozzle.

(2) It is observed that the particle diameter affects the flow velocity adversely, the larger the diameter, the slower the flow will be. This is attributed to the larger slip velocities between the gas phase and solid phase when the particle diameter is large. The two phase flow with lower particle diameter has almost same trend as that of the single phase flow with ideal gas as working fluid.

(3) Depending on the ambient pressure, the thrust coefficient is found to be higher for larger particles/gas loading or inlet/outlet pressure ratio.

NOMENCLATURE

a	speed of sound	[m/s]
a_s	speed of sound of mixture	[m/s]
C_D	drag coefficient of particle.	[----]
C_g	specific heat of gas	[J/kg·K]
C_s	specific heat of particle	[J/kg·K]
d_p	diameter of particle	[mm]
h_m	enthalpy	[J/kg]
Re_p	Reynolds number of particle	[----]
p_a	ambient pressure	[bar]
p_e	outlet pressure of nozzle	[bar]
p_0	inlet pressure of nozzle	[bar]
u_{int}	internal energy	[J/kg]
v_p	velocity of particle	[m/s]
v_g	velocity of gas	[m/s]
ρ_p	density of particle	[kg/m^3]
ρ_g	density of gas	[kg/m^3]
ρ_m	mixture density	[kg/m^3]
γ	specific heat ratio of gas	[----]
$\bar{\gamma}$	specific heat ratio of mixture	[----]
η	the particles/gas loading	[----]
μ	dynamic viscosity	[N·s/m^2]
T	temperature of mixture	[K]

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