신뢰도 계산의 여러 가지 알고리즘의 비교 연구

렌지얀, 장전해, 고창섭
전기전자 컴퓨터공학부, 충북대학교

A Comparative Study of Different Reliability Calculation Algorithms

Ziyan Ren, Dianhai Zhang, and Chang Seop Koh
College of Electrical and Computer Engineering, Chungbuk National University

Abstract - In this paper, three reliability calculation algorithms: Monte Carlo Simulation (MCS), Reliability Index Approach (RIA), and Sensitivity-based Monte Carlo Simulation (SMCS) are studied. Their efficiency and accuracy are validated by analytic test functions.

1. Introduction

In the engineering applications, the uncertainty in design variables is inevitable. Therefore, nowadays, the ability to accurately characterize and propagate these uncertainties is increasingly important. Usually, the reliability defined as the probability of a design being in the feasible region to evaluate its quality.

In the following paper, the merits and drawbacks of MCS, RIA, and SMCS are explored. The comparisons will give some guidances to reliability-based design optimization with the selection of proper reliability calculation method.

2. Reliability Calculation Methods

In order to simplify explanation, all the design variables are assumed as uncertain ones. For a design problem, the constraint function g(**X**)>0, defines feasible region; g(**X**)<0 defines infeasible region; and g(**X**)=0 defines limit state surface, where $\boldsymbol{X}(x_1,\ x_2,...,x_M)$ is design variable vector with independent components following Gaussian distribution, $X_i \sim N(\mu_i,\ \sigma_i)$.

2.1 Reliability Index Approach

For a given design, the Hasofer-Lind reliability index β [1] is defined as follows:

(1)Transform the X-design space into the normalized U-design space by using:

$$u_i = (X_i - \mu_i)/\sigma_i \tag{1}$$

(2)Redefine the limit state function in terms of the normalized design variables $\boldsymbol{U}(u_1,\ u_2,...,u_M)$, g(**U**)=0.

(3)β is the shortest distance from origin in the N-dimensional normalized **U**-design space to the redefined limit state function.

For the linear limit state function, the shortest distance can be calculated very easily; For nonlinear case, the computation of the minimum distance becomes an optimization problem:

$$\begin{aligned} &\text{minimize} \quad \sqrt{u_1^2 + u_2^2 + \ldots + u_M^2} \\ &s.t. \qquad\quad g(U) = 0 \end{aligned} \tag{2}$$

After searching the optimal solution of (2) called Most Probable Point of Failure (MPPF), $\boldsymbol{U}^{*}(u_1^*,\ u_2^*,...,u_M^*)$, then reliability index can be calculated as:

$$\beta = \sqrt{(u_1^*)^2 + (u_2^*)^2 + \ldots + (u_M^*)^2} \tag{3}$$

Finally, the reliability can be approximated as:

$$R = 1 - \Phi(-\beta) \tag{4}$$

where $\Phi(\bullet)$ is the standard normal Cumulative Distribution Function.

2.2 Monte Carlo Simulation

In the MCS, for the reliability of a design **X** with respect to the constraint g(**X**)>0, N different samples are generated in a certain confidence interval $[\mu_i - k\sigma_i, \mu_i + k\sigma_i]\ i = 1,2,...M.$ If n samples satisfy the constraint g(**X**)>0, then the reliability of the design **X** can be obtained as:

$$R(g(\boldsymbol{X}) > 0) = \frac{n}{N}. \tag{5}$$

2.3 Sensitivity-based Monte Carlo Simulation

Based on the requirement of reliability evaluation for real engineering application, the sensitivity-based MCS method is proposed. In SMCS, the sensitivity analysis is applied to calculate constraint function by constructing an approximated analytic function especially for the nonlinear performance constraints.

For a given determinant design $\mathbf{X}_0$, the constraint functions in uncertain regions are approximated to linear ones using their gradient vectors as follows:

$$g(\boldsymbol{X}) \cong g(\boldsymbol{X}_0) + \nabla g(\boldsymbol{X}_0) \bullet (\boldsymbol{X} - \boldsymbol{X}_0). \tag{6}$$

The gradient vector is calculated as follows [2]:

$$\begin{aligned} &\nabla g(\boldsymbol{X}_0) = \left.\frac{\partial g}{\partial [\boldsymbol{X}]^T}\right|_{A=C.} - [\lambda]^T \left(\left.\frac{\partial [R]}{\partial [\boldsymbol{X}]^T}\right|_{\nu = C.} - \frac{\partial [R]}{\partial \nu} \frac{\partial \nu}{\partial B^2} \frac{\partial B^2}{\partial [\boldsymbol{X}]^T} \right) \\ &[K + \overline{K}]^T [\lambda] = \frac{\partial g}{\partial [\boldsymbol{A}]} \end{aligned} \tag{7}$$

where R is the residual vector in Galerkin's approximation; other symbols have their usual meanings in Finite Element Method (FEM). Once the sensitivity is obtained from (7), the reliability analysis can be performed to an approximated analytic function.

3. Results and Conclusions

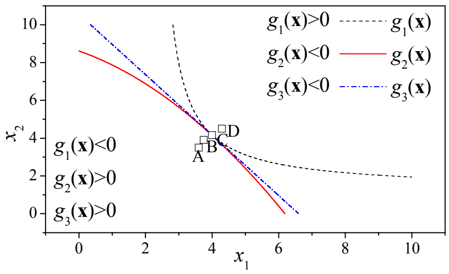
The reliability analysis are applied to the following analytic constraint functions as shown in Fig.1:

$$g_1(\boldsymbol{X}) = \frac{(x_1 - 2.25)(x_2 - 1.3)}{5} - 1 \geq 0 \tag{8-1}$$

$$g_2(\boldsymbol{X}) = \frac{80}{(x_1 + 2.47)^2 + 8x_2 + 5} - 1 \geq 0 \tag{8-2}$$

$$g_3(\boldsymbol{X}) = -1.6(x_1 - 4) - x_2 + 4.16 \geq 0 \tag{8-3}$$

where the design variables x1 and x2 $(0 \leq x_1, x_2 \leq 10)$ follows the same Gaussian distribution with standard deviation $\sigma = 0.3$. In the given design space, four testing points A(3.6, 3.5), B(3.75, 3.9), C(4.0, 4.16) and D(4.3, 4.5) are selected. In the MCS and SMCS, the confidence level of uncertain variable is set to 95%, and the number of MCS and SMCS trials is set to 2,000,000.



<Fig. 1> Limit state functions

For $g_1(X)$ and $g_2(X)$, testing points (B, C, D) and (A, B, C) are selected, respectively. The comparison results are shown in Table.1, Fig.2 and Table.2 Fig.3, respectively. From the comparisons, it is obvious that the SMCS can achieve almost same accuracy as MCS, while the accuracy of RIA is a little worse than MCS (Point C and D).

For the linear case, $g_3(X)$, as shown in Fig.4, the reliability obtained by MCS can coincide with the other two methods. As shown in Fig.1, due to the different range of the feasible region, the reliability of design C for three limit state functions should be different. However, the RIA find the same reliability index, as shown in Fig.5, which is not accurate for the nonlinear limit functions. Form the above analysis, we can get the following conclusions:

<Table 1> Reliability of constraint function 1

Method	B	C	D
MCS	9.1453E-2	0.484294	0.947406
SMCS	8.4862E-2	0.486787	0.940090
RIA	7.2881E-2	0.500	0.935162

<Table 2> Reliability of constraint function 2

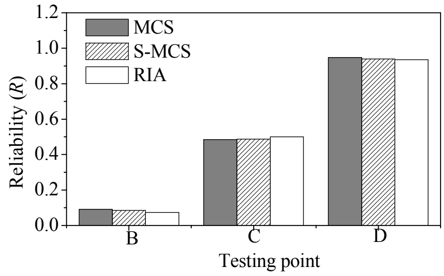
Method	A	B	C
MCS	0.998078	0.896754	0.484294
SMCS	0.993423	0.885806	0.486787
RIA	0.995587	0.881464	0.500

(1)The reliability from MCS is usually taken as a benchmark with its highest accuracy if the trials is as big enough. However, it is much more time-consuming for engineering applications;

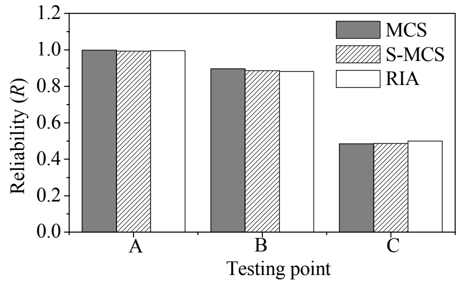
(2)Once the MPPF is found, the RIA is the most efficient method to calculate reliability. There are following drawbacks: searching of MPPF is related with optimization problem, if the reliability analysis is combined with design optimization, it will be time-consuming; the accuracy is worse especially for nonlinear limit state functions;

(3)Compared with MCS, the SMCS can save a lot of computational time with the help of sensitivity analysis; At the same time, the accuracy is no worse than the RIA.

We can demonstrate that, until now, the SMCS is the best choice for the engineering applications.



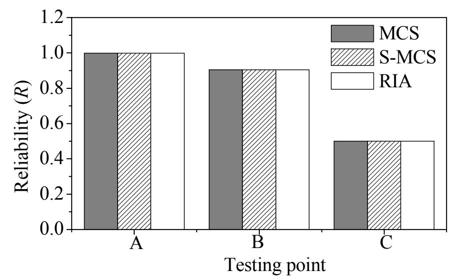
<Fig. 2> Reliability comparison of constraint 1



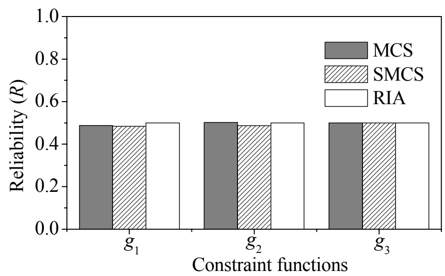
<Fig. 3> Reliability comparison of constraint 2

<Table 3> Reliability for constraint function 3

Method	A	B	C
MCS	0.998519	0.904420	0.499608
SMCS	0.998513	0.904583	0.499669
RIA	0.998578	0.904531	0.500



<Fig. 4> Reliability comparison of constraint 3



<Fig. 5> Reliability comparison of point C

[Reference]

[1]K. Deb, et, al.,"Reliability-based optimization using evolutionary algorithms," *IEEE Trans. Evol, Comput.,* vol.13, no.5, pp.1054-1073, Oct., 2009.

[2]J. S. Ryu, Y. Yao, C. S. Koh, S. Yoon, and D. S. Kim,"Optimal shape design of 3-D nonlinear electromagnetic devices using parameterized design sensitivity analysis," *IEEE Trans. Magn.,* vol.41, no.5, pp.1792-1795, May, 2005.