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A PATH ENUMERATION APPROACH FOR CRITICAL ANALYSIS IN PROJECT NETWORKS WITH FUZZY ACTIVITY DURATIONS

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ABSTRACT: A novel approach for analysis of criticality with respect to path and to activity in networks with fuzzy activity durations is proposed. After recalling the Yager ranking method, the relative degree of criticality of activities and paths are defined. An efficient algorithm based on path enumeration to compute the relative degree of criticality of activities and paths in networks with fuzzy durations is proposed. Examples of former researches are employed to validate the proposed approach. The proposed algorithm has been tested on real world project networks and experimental results have shown that the algorithm can calculate the relative degree of criticality of activities and paths in a reasonable time.

Keywords:Project scheduling and management; Fuzzy sets; Critical path analysis

1. INTRODUCTION

The critical path method (CPM) [21] is a network-based method designed to assist in the planning, scheduling and control of real world projects and CPM has become one of the tools that are most useful in practice. The activity durations in the CPM are deterministic and known; similarly the vast majority of the research efforts in project scheduling assume complete information about the scheduling problem to be solved, e.g. [8], although the activity durations are usually difficult to estimate precisely [19]. In fact, uncertainty is an attribute of information [38]. This is why the Program Evaluation and Review Technique (PERT) [26] and Monte Carlo simulation [30] based on the probability theory have been developed. So far in the literature, hundreds of papers have used these stochastic approaches and research in this area is still carried out [9]. In stochastic PERT the exact determination of the total project duration is generally intractable, except when the graph is a series-parallel one [13]. Thus, approximation techniques, like transforming the original network into a series-parallel one [10], have been proposed. Due to the uniqueness of projects [28], historical data about activity durations are not available. As activity durations have to be estimated by human experts, under unique circumstances, project management is faced with judgmental statements that are imprecise. In those situations, the fuzzy set scheduling literature recommends the use of fuzzy numbers for modeling durations, rather than stochastic variables [19]. Shipley et al. [31] and Lootsma [25] have compared the fuzzy approach with the stochastic approach.

Fuzzy critical path methods have been proposed since the late 1970s ([17], [27], [29]). In this approach, the problems of determining the possible values of the latest starting times and floats of activities in networks with fuzzy durations constitute important and challenging problems which have attracted intensively attentions. The possible values of the earliest starting times can be computed by means of a forward recursion procedure comparable to the one used in conventional CPM problems [3]. Unfortunately, the backward recursion issued from classical CPM is indeed not sound if durations are described by means of fuzzy numbers, in fact, the backward recursion takes the imprecision of some duration twice into account [13]. Several authors tried to cope with this problem. Kaufmann and Gupta [20] and Hapke et al. [18] proposed a backward recursion that relies on the 'optimistic' fuzzy subtraction and they provided good results for particular networks. Zielinski [39] has determined the possible values of the latest starting times of activities by proposing polynomial algorithms. Dubois et al. [12] have proposed an algorithm based on path enumeration to compute optimal intervals for latest starting times and floats. Fortin et al. [16] have provided a solution to the problem of finding the maximal floats of activities and Yakhchali and Ghodsypour [35] have proposed a hybrid genetic algorithm for the problem of finding the minimal floats of activities.

The criticality analysis in networks with fuzzy activity durations is a more realistic approach than the traditional ones. Chen [5] proposed an approach based on the extension principle and linear programming formulation to critical path analysis for a network with fuzzy activity durations. Chen and Hsueh [6] developed an approach to critical path analysis based on the linear programming and the Yager's ranking method. Chen and Huang [7] combined fuzzy set theory with the traditional methods to compute the critical degrees of activities and paths.

Chanas and Zielinski [4] applied Zadeh's extension principle to the classical criticality concept treated as a function of activity duration and proposed two methods for computing the path degree of criticality. Chanas et al. [2] introduced the notion of necessary criticality both with respect to paths and activities and proposed methods for calculating the degree of necessary criticality of a path. The idea of partitioning is used by Yakhchali and Ghodsypour [36] to develop an algorithm for determining various type of critical activity. The problems of the necessarily and possibly critical paths in the networks with imprecise activity and time lag durations have been discussed by Yakhchali et al. [33], [34]. Liberatore [24] presented an approach for fuzzy critical path analysis that is consistent with the extension principle.

This paper will provide a novel approach for critical analysis in project networks with fuzzy activity durations. The new definition of the relative degree of criticality of paths is proposed and based on it, the notion of the relative degree of criticality of activities is introduced. These degrees are computed by a path enumeration algorithm. The results of applying the proposed approach in two examples discussed in previous studies are compared to proof the validity of the approach.

2. Terminology and Representation

The project scheduling problems to be dealt with throughout this paper can be stated as follows. A set $V=\{1,2, \ldots, n\}$ of *activities* has to be executed where the dummy activities *1* and *n* represent the beginning and the termination of the project, respectively. Activities can be represented by an *activity-on-node* (*AON*) network $G=<V, E>$ with node set *V*, arc set *E*. Assume without loss of generality that the activities topologically numbered such that an arc always leads from a smaller to a higher node number.

Activity durations are determined by means of fuzzy numbers. Fuzzy numbers express uncertainty connected with the ill-known activity durations modeled by these numbers which generate possibility distributions (see **Error! Reference source not found.**) for the sets of values containing the unknown activity durations **Error! Reference source not found.**. A fuzzy number, $\tilde{a}$, is a normal convex fuzzy set in the space of real number with an upper semi-continuous membership function $\mu_{\tilde{a}}$. A fuzzy set is convex if and only if its membership function is quasiconcave, i.e., it fulfills the condition: $\mu_{\tilde{a}}(z) \geq min\{\mu_{\tilde{a}}(x), \mu_{\tilde{a}}(y)\}$ for each *x, y, z* such that $z \in [x, y]$ **Error! Reference source not found.**.

2. 1. Fuzzy Criticality

Fuzzy number $\tilde{d}_i$ imprecisely determines a duration time of an activity *i*, $i \in V$. The membership function $\mu_{\tilde{d}_i}$ generates a possibility distribution for the duration time of the activity *i*, $i \in V$. It would be accurate to say that the ascertain of the from "v_i is $\tilde{d}_i$", where v_i is a variable and $\tilde{d}_i$ is a fuzzy number, generates the possibility distribution of v_i with respect to the following formula (see **Error! Reference source not found.**):

$$Poss(v_i = x) = \mu_{\tilde{d}_i}(x), \quad z \in \Re_+ \qquad (1)$$

The notation of *configuration* denoted by Ω has been defined by Buckley **Error! Reference source not found.** to relate the fuzzy case to the deterministic case of classical PERT/CPM problems. A configuration is tuple Ω $(d_1, d_2, \ldots, d_n)$ of activity durations such that $d_i \in \Re_+$, $i \in V$. The (joint) possibility distribution over configurations, denoted by $\pi(\Omega)$, is determined by the following formula:

$$\pi(\Omega) = min_{i \in V}\, \mu_{\tilde{d}_i}(d_i), \quad \Omega \in \Re_+^n \qquad (2)$$

The following formula determines the possibility that an activity, $k \in V$, is critical **Error! Reference source not found.**.

$$Poss(k \text{ is critical}) = \sup_{\Omega: k \text{ is critical in } \Omega} \pi(\Omega) \qquad (3)$$

Let us denote the set of all paths in *G* from *1* to *n* by *P*. $P(k)$ will denote the set of all paths from *P* which contain the given activity *k*, $P(k) = \{p \mid p \in P \text{ and } k \in p\}$. Before we pass on to the basic consideration, let us recall Yager ranking index, which will be helpful in formulating and proving an algorithm.

2. 2. Defuzzification

The activity durations are defuzzyfied into crisp ones by Yager ranking index. Yager **Error! Reference source not found.** provided a procedure for ordering fuzzy sets based on the area compensation. "Area compensation" is robust and possesses the properties of linearity and additivity **Error! Reference source not found.**. Yager ranking index is defined as following:

$$I(\tilde{d}) = \int_0^1 0.5(d_\alpha^L + d_\alpha^U)\, d\alpha \qquad (4)$$

where (d_α^L, d_α^U) is the α*-cut* of a given convex fuzzy number, $\tilde{d}$.

This index is calculated for the given convex fuzzy activity duration $\tilde{d}$ from the extreme values of its α*-cut*, d_α^L and d_α^U, rather than its membership function. Thus, it is not necessary to know the exact form of the membership functions of the fuzzy durations, although the most of the ranking methods require the explicit form of the membership.

Moreover, the Yager's ranking method has been used by several authors in the literature for fuzzy critical analysis, e.g. **Error! Reference source not found.**. Based on this index, the fuzzy problem is transformed to a problem with crisp activity durations in the following.

2. 3. The Relatively Critical Degree

The length of a path in network with crisp durations is defined as the sum of all the durations associated with the activities (nodes) belonging to that path. The following definition of the length of a path is based on the Yager ranking indices.

Definition 1(**Error! Reference source not found.**): The length of a path *p*, denoted by L_p and $p \in P$, in *G* with activity durations being fuzzy numbers is defined as the sum of the Yager ranking indices of all fuzzy activity durations on this path.

Based on *Definition 1*, the length of a path *p*, $p \in P$, is determined by formula (5):

$$L_p = \sum_{\forall i \in p} I(\tilde{d}_i) \qquad (5)$$

The idea of Definition 2 came from the project networks with crisp durations, the longest path is the critical path.

Definition 2: The length of longest path, denoted by L_{max}, in networks with fuzzy durations is defined as the length of path with maximum path length, $L_{max} = max_{p \in P} L_p$.

The length of longest path is unique, but in some networks there exist various longest paths. The relatively critical degree of the longest path is set as "*1*". For other paths, the relatively critical degree of a given path *p*, $p \in P$, is defined as following.

Definition 3: The relatively critical degree of a given path *p*, denoted by Rcd_p, is the ratio of the Yager ranking index of the path to the length of longest path. That degree is calculated by formula (6):

$$Rcd_p = \frac{L_p}{L_{max}} \tag{6}$$

These two definitions slightly differ from the ones used by Chen **Error! Reference source not found.**. Definition 4 introduces the relatively critical degree of a given activity *k*, $k \in V$. The relatively critical degree of *k* will be denoted by Rcd_k.

Definition 4: The relatively critical degree of a given activity *k*, $k \in V$, is maximum of the relatively critical degree of all paths from *P* which contain the given activity *k*. This index is computed by formula (7):

$$Rcd_k = \max_{p \in P(k)} Rcd_p \tag{7}$$

Two examples are provided to compare the relatively critical degree of an activity with the degree of possible criticality of activities in **3. 3. Comparisons**.

3. The Path Enumeration Approach for Critical Analysis

Initially in the path enumeration approach, all fuzzy activity durations are defuzzyfied into crisp ones by Yager ranking index. Then the length of all paths, $l_p \; \forall p \in P$, are calculated by the proposed algorithm (Algorithm 1) and the longest paths are determined. Based on Definition 3 and Definition 4, the relatively critical degree of all paths and activities are computed.

3. 1. The Path Enumeration Algorithm

Algorithm 1 computes all paths in the network *G*, calculates the length of paths and determines the relatively critical degree of paths and activities. In Algorithm 1, the set of immediate successors of an activity *j*, *j* ∈ *V*, is denoted by $succ(j)$, $succ(j) = \{k \mid (j,k) \in E\}$.

Fuzzy activity durations are defuzzyfied by Yager ranking index in the lines 1 to 3 of the algorithm. Paths are made by the recursive procedure '*Enumeration Procedure*'. The call to the procedure in the line 6 of Algorithm 1 will constructs the set *P*. The procedure updates the length of the longest path and saves the current path and its length. The relatively critical degree of paths is calculated in the line 8. Analogously, the algorithm in line 11 computes the relatively critical degree of activities.

The number of tested paths depends on the topology of the network that is potentially exponential but in practice the algorithm runs can find the answers in a reasonable time on realistic problems. This is discussed in section **3. 4. Computational Experience**.

Algorithm 1: Determining the relatively critical degree of activities and paths

Input: A network $G = \langle V, E \rangle$
Output: The relatively critical degree of activities and paths

1: **for** $i \in V$ **do**
2: ... $d_i = I(\tilde{d}_i)$
3: **end for**
4: $p \leftarrow \{1\}$
5: $L_{max} = -\infty$
6: call *Enumeration Procedure* (*1*)
7: **for** $p \in Paths$ **do**
8: ... $Rcd_p = L_p / L_{max}$
9: **end for**
10: **for** $i \in V$ **do**
11: ... $Rcd_k = max\{Rcd_p \mid p \in P(k)\}$
12: **end for**

***Enumeration Procedure** (j)*

a: **if** $j = n$ **then**
b: ... $Paths \leftarrow Paths \cup \{p\}$
c: ...**if** $L_{max} < L_p$ **then**
d: ...... $L_{max} = L_p$
e: ...**end if**
f: **else**
g: ...**for** $k \in succ(j)$ **do**
h: ...... $p \leftarrow p \cup \{k\}$
i: ......call *Enumeration Procedure* (k)
j: ...**end for**
k: **end if**

3. 2 Numerical example

To compare the proposed approach with former researches, a project network in Figure 1 which was proposed by Chanas and Zielinski **Error! Reference source not found.** is investigated. Chanas and Zielinski **Error! Reference source not found.** used the activity-on-arc (AOA) convention, but in this paper the proposed network was converted to the activity-on-node (AON) convention, to adopt the paper's network representation. Two following examples have the same network as shown in Figure 1.

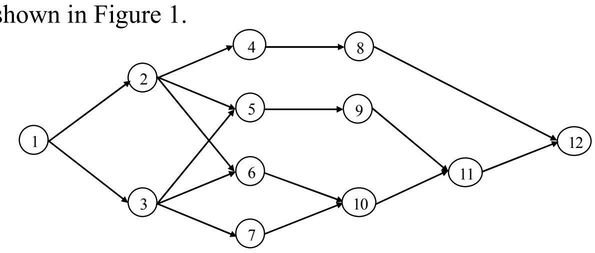

Figure 1: A project network

The notion of fuzzy numbers of the *L-R type* is recalled before the basic consideration. A fuzzy number $\tilde{A}$ is called a fuzzy number of the *L-R type* if its membership function $\mu_{\tilde{A}}$ has the following form **Error! Reference source not found.**:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 & \text{for } x = [\underline{a}, \overline{a}], \\ L(\frac{\underline{a} - x}{\alpha_A}), & \text{for } x < \underline{a}, \\ R(\frac{x - \overline{a}}{\beta_A}), & \text{for } x > \overline{a}, \end{cases} \tag{8}$$

where *L* and *R* are continuous nonincreasing functions, defined on $[0, +\infty)$, strictly decreasing to zero in those subintervals of interval $[0, +\infty)$ in which they are positive, and fulfilling the conditions *L(0)=R(0)=1*. The parameters α_A and β_A are nonnegative real numbers. The fuzzy number of *L-R type* is denoted by $A = (\underline{a}, \overline{a}, \alpha_A, \beta_A)_{L-R}$ **Error! Reference source not found.**.

Example 1: Assume that the network in Figure 1 is given. The activities durations are fuzzy numbers of *L-R type* as shown in Table 1.

Table 1: The fuzzy activity durations of Figure 1

$\tilde{d}_i$	$L_i(x)$	$R_i(x)$
$\tilde{d}_1 = (0,0,0,0)_{L_1-R_1}$	$L_1(x) = max(0,1-x)$	$R_1(x) = max(0,1-x)$
$\tilde{d}_2 = (1,1.5,1,1)_{L_2-R_2}$	$L_2(x) = max(0,1-x^2)$	$R_2(x) = max(0,1-x)$
$\tilde{d}_3 = (2,3,0,2)_{L_3-R_3}$	$L_3(x) = e^{-x}$	$R_3(x) = max(0,1-x)$
$\tilde{d}_4 = (2,3,1,2)_{L_4-R_4}$	$L_4(x) = max(0,1-x^4)$	$R_4(x) = e^{-x}$
$\tilde{d}_5 = (9,9,1,1)_{L_5-R_5}$	$L_5(x) = max(0,1-x^4)$	$R_5(x) = e^{-x}$
$\tilde{d}_6 = (5,5,1,1)_{L_6-R_6}$	$L_6(x) = max(0,1-x)$	$R_6(x) = max(0,1-x^4)$
$\tilde{d}_7 = (6,7,0,2)_{L_7-R_7}$	$L_7(x) = e^{-x^2}$	$R_7(x) = max(0,1-x^2)$
$\tilde{d}_8 = (8,9,2,4)_{L_8-R_8}$	$L_8(x) = max(0,1-x^4)$	$R_8(x) = max(0,1-x^2)$
$\tilde{d}_9 = (3,4,2,0)_{L_9-R_9}$	$L_9(x) = max(0,1-x)$	$R_9(x) = max(0,1-x^4)$
$\tilde{d}_{10} = (4,4,2,2)_{L_{10}-R_{10}}$	$L_{10}(x) = max(0,1-x^2)$	$R_{10}(x) = max(0,1-x^4)$
$\tilde{d}_{11} = (6,9,2,3)_{L_{11}-R_{11}}$	$L_{11}(x) = max(0,1-x^2)$	$R_1(x) = max(0,1-x)$
$\tilde{d}_{12} = (0,0,0,0)_{L_1-R_1}$	$L_1(x) = max(0,1-x)$	$R_{11}(x) = e^{-x^2}$

The Yager's ranking indices for activities in Table 1, $I(\tilde{d}_i)\quad i \in V$, are calculated as shown in Table 3, in fact the results of lines 1 to 3 in Algorithm 1. The algorithm constructs all paths for Figure 1 and computes the length of paths as listed in Table 2. According to Definition 2, the path {1-3-5-9-11-12} is longest path and $L_{max} = 23.2627$. Then the relatively critical degree of paths, $Rcd_p\quad p \in P$, is calculated through the line 8 in the algorithm as shown in Table 2.

Table 2: The length of paths and the relatively critical degrees of paths in Example 1

Paths	Path length	Relatively critical degree
1-2-4-8-12	13.3000	0.5717
1-2-5-9-11-12	21.4294	0.9212
1-2-6-10-11-12	18.6127	0.8001
1-3-5-9-11-12	23.2627	1.0000
1-3-6-10-11-12	20.4460	0.8789
1-3-7-10-11-12	22.4627	0.9656

Eventually, the algorithm based on Definition 4 computes the relatively critical degree of activities, $Rcd_i\quad i \in V$, as listed in Table 3.

Table 3: The Yager's ranking indices of activities and the relatively critical degrees of activities in Example 1

Activities	Yager's ranking index	Relatively critical degree
1	0.00000	1.0000
2	1.16667	0.9212
3	3.00000	1.0000
4	3.10000	0.5717
5	9.10000	1.0000
6	5.15000	0.8789
7	7.16667	0.9656
8	9.03333	0.5717
9	3.00000	1.0000
10	4.13333	0.9656
11	8.16267	1.0000
12	0.00000	1.0000

Example 2: The project network is similar to that in Example 1 (Figure 1). Assume that the activity durations are fuzzy numbers of the same *L-L type*, where $L(x) = max\,(0,1-x^2)$ as following:

$\tilde{d}_1 = (0,0,0,0)_{L-L}$ $\quad \tilde{d}_2 = (1,1.5,1,1)_{L-L}$ $\quad \tilde{d}_3 = (2,3,0,2)_{L-L}$

$\tilde{d}_4 = (2,3,1,2)_{L-L}$ $\quad \tilde{d}_5 = (9,9,1,1)_{L-L}$ $\quad \tilde{d}_6 = (5,5,1,1)_{L-L}$

$\tilde{d}_7 = (6,7,0,2)_{L-L}$ $\quad \tilde{d}_8 = (8,9,2,4)_{L-L}$ $\quad \tilde{d}_9 = (3,4,2,0)_{L-L}$

$\tilde{d}_{10} = (4,4,2,2)_{L-L}$ $\quad \tilde{d}_{11} = (6,9,2,3)_{L-L}$ $\quad \tilde{d}_{12} = (0,0,0,0)_{L-L}$

Once again, Algorithm 1 calculates the Yager's ranking indices for activities, as shown in Table 5, to compute the length of paths and the relatively critical degree of paths. The results are listed in Table 4 in the algorithm outputs order.

Table 4: The length of paths and the relatively critical degrees of paths in Example 2

Paths	Path length	Relatively critical degree
1-2-4-8-12	13.2500	0.5803
1-2-5-9-11-12	20.9166	0.9161
1-2-6-10-11-12	18.0833	0.7920
1-3-5-9-11-12	22.8333	1.0000
1-3-6-10-11-12	20.0000	0.8759
1-3-7-10-11-12	22.1667	0.9708

The relatively critical degrees of activities are similarly computed by Algorithm 1 as shown in Table 5.

Table 5: The Yager's ranking indices of activities and the relatively critical degrees of activities in Example 2

Activities	Yager's ranking index	Relatively critical degree
1	0.0000	1.0000
2	1.2503	0.9161
3	3.1670	1.0000
4	2.8333	0.5803
5	9.0000	1.0000
6	5.0000	0.8759
7	7.1667	0.9708
8	9.1664	0.5803
9	2.8333	1.0000
10	4.0000	0.9708
11	7.8330	1.0000
12	0.0000	1.0000

3. 3. Comparisons

To validate the proposed approach, the results of the examples are compared with former researches. Although the definition of the relatively critical degree of a path differs from the definition of the relative path degree of criticality proposed by Chen **Error! Reference source not found.**, they compute exactly same values. Chen's approach is based on linear programming formulation. In this approach, a pair of linear programs parameterized by possibility level α is formulated to calculated the lower and upper bounds of the fuzzy total duration at α, then the fuzzy critical paths are identified by enumerating different values of α and the relative degree of criticality of paths are computed by applying the Yager ranking method.

The proposed approach in this paper can determine the same degree of criticality of paths without using linear programming formulation. The proposed approach determines the relative degree of criticality of all paths in only one execution, but Chen's approach determines the relative degree of criticality of fuzzy critical paths which are identified by enumeration different values of α. In addition, the proposed approach calculates the relative degree of criticality of all activities at the same time.

Chen **Error! Reference source not found.** compared the relative degree of criticality of paths with Chanas and Zielinski's results **Error! Reference source not found.**, this comparison is omitted for the sake of brevity.

Table 6 compares the relatively critical degree of activities, proposed in this paper, with the degree of possible criticality of activities, proposed in former researches in Example 1 and 2.

Table 6: The comparison of the relatively critical degrees of activities with degree of possible criticality in Example 1 and 2

	Example 1.		Example 2.	
Activities	Relatively critical degree	Degree of possible criticality	Relatively critical degree	Degree of possible cri
1	1.0000	1.0000	1.0000	1.0000
2	0.9212	0.6269	0.9161	0.7500
3	1.0000	1.0000	1.0000	1.0000
4	0.5717	0.6269	0.5803	0.7024
5	1.0000	1.0000	1.0000	1.0000
6	0.8789	0.3854	0.8759	0.4375
7	0.9656	0.9941	0.9708	0.9796
8	0.5717	0.6269	0.5803	0.7024
9	1.0000	1.0000	1.0000	1.0000
10	0.9656	0.9941	0.9708	0.9796
11	1.0000	1.0000	1.0000	1.0000
12	1.0000	1.0000	1.0000	1.0000

The results of Table 6 show that the definition of the relatively critical degree of activities is theoretically sound.

3. 4. Computational Experience

Algorithm 1 has been tested on realistic project scheduling problems for the experimental validation of its efficiency. For this reason, the algorithm computes the relatively critical degrees of paths and activities in project networks that have been generated by ProGen **Error! Reference source not found.**. Kolisch et al. **Error! Reference source not found.** are supposed to be representative of real project scheduling problems. On those problems, activity durations are precisely defined, thus the activity durations are converted to triangular fuzzy numbers. The choice of fuzzy numbers is not important for the test due to the fact that the algorithm complexity only depends on the network topology. The tested networks can be downloaded from the web site http://129.187.106.231/psplib/.

Different test sets of projects are chosen from this web site as shown in Table 7. Table 7 presents the performance of the proposed algorithm on libraries of project networks, with respectively, 32, 62, 92 and 122 activities (on 480, 480, 480 and 600 instances of project networks, respectively).

Table 7: The execution times and the number of paths are evaluated by Algorithm 1

Nb of activiteis	32	62	92	122
Nb of networks tested	480	480	480	600
Minimal execution time	0.168	0.3271	0.4761	0.6553
Average execution time	0.6014	1.3701	2.4133	3.7385
Maximal execution time	2.5272	6.3469	11.5661	15.9495
Minimal nb of paths	18	33	48	65
Average nb of paths	56.9792	127.35	221.425	325.6933
Maximal nb of paths	204	563	961	1277

The proposed algorithm has been programmed in MATLAB (R2006b) and run on a personal computer with 1.60 GHz processor (Intel Centrino 1.7) and 512 MB of RAM. The overall execution times, expressed in seconds, are measured. The results of Table 7 are presented in Figure 2 and Figure 3.

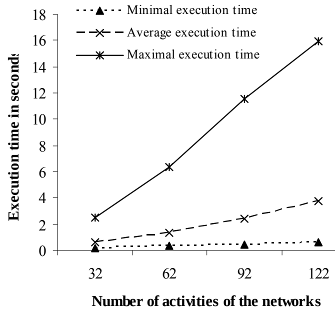

Figure 2: Execution times of Algorithm 1

These tests show that Algorithm 1 can calculate the relatively critical degrees of paths and activities in big project networks with acceptable execution times.

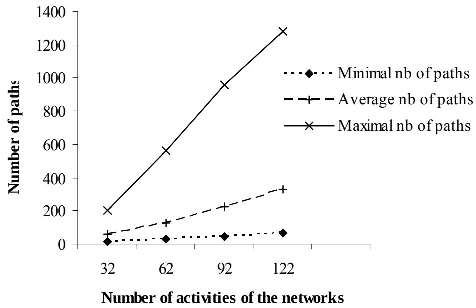

Figure 3: Number of paths are evaluated by Algorithm1

4. Conclusions

Activities durations, especially at the beginning of projects, are ill-known, so they can be modeled by means of fuzzy numbers, representing the possible values of these durations. With such modeling, the standard criticality analysis collapses. Basically this research effort aims at analyzing the criticality of activities and paths together. In this paper, a novel approach for critical analysis in networks with fuzzy activity durations was proposed. This approach was summarized in an effective algorithm, relies on a path enumeration technique. The algorithm computes the length of all paths and calculates the relatively critical degree of activities and paths, without using linear programming formulation. The definition of the relatively critical degree of an activity introduced in this paper is practical and theoretically sound.

Extensive computational results are reported using a problem set consisting of 2040 instances with up to 120 activities. The results show that the algorithm can calculate the relatively critical degree of activities and paths in a reasonable time on realistic problems. Hence, the proposed algorithm can be used in future project planner software that will cope with uncertainty.

As topics for further research, it seems possible to further develop some heuristics, like Branch and Bound, based on the proposed algorithm.

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