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# BOT REAL OPTION VALUATION UNDER PERFORMANCE BONDING

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**ABSTRACT:** Build-operate-transfer (BOT) projects are privatized infrastructure undertakings that face long-term investment risks and uncertainties. To ensure these projects can be completed on time and operated according to performance specifications, governments usually require BOT concessionaires to furnish performance bonds as a security. However, in order to attract investment, governments often provide abandonment rights for concessionaires to deal with investment risks and uncertainties. In the context of real options, these abandonment rights will increase project value, but the furnish of performance bonds will reduce this value. Currently in the BOT context, there is no real option model that can handle explicitly the impact of performance bonds on project value. In this paper, a real option valuation model is derived to deal with this important issue. The Taiwan high-speed rail project is used as a case study to show the applicability of the proposed model.

*Keywords: Build-Operate-Transfer, Real Options, Valuation, Performance bonds*

## 1. INTRODUCTION

Build-Operate-Transfer (BOT) infrastructure projects are long-term undertakings that involve irreversible sunk investments and are prone to substantial project risk and uncertainty. To help managing BOT risk and uncertainty, host governments around the world have offered a variety of policy tools for BOT concessionaires, including loan repayment guarantee, minimum-revenue guarantee, the rights to invest incrementally, the rights to abandon project prematurely, and the like. The existence of these policy tools and the idiosyncratic nature of infrastructure investments mean that traditional valuation methods, such as the discounted cash flow model, are no longer satisfactory for BOT project valuation. Recently, many studies have used real option valuation methods to handle this issue. The real option approach is a concept originated from financial options. Financial options are a type of contracts that give the holders of options rights to purchase or sell specific quantity financial products in certain time for certain prices, which are defined as the exercise or strike prices of the options. Basically, there are two types of options, namely call option and put option. A call option gives the holder of the option the right to buy an asset by a certain date for a certain price, while a put option gives the holder of the option the right to sell an asset by a certain date for a certain price. In addition, options can be either American or European. American-styled options can be exercised at any time up to the option expiry date, whereas European-styled options can be exercised only on the expiry date. The underlying assets of the financial options are financial securities, while the underlying assets of real options are real assets. Real options gave the holders the rights to decide whether or not to invest on specific underlying assets in certain time, and thus provide a type of investment flexibility for managing BOT investment risks and uncertainties.

Many real options valuation methods have been proposed for the valuation of complex BOT projects. For example, Smit [1] provides a real-option-based game theory model to explore airport expansion investment issues. Garvin and Cheah [2] propose a real-option pricing model for analyzing tollroad investments. Rose [3] shows how to evaluate complex interacting real options in tollroad investments. Huang and Chou [4] develop a real-option approach for evaluating minimum-revenue guarantees. Huang and Pi [5, 6] develop a European-styled sequential compound call option approach for evaluating multi-stage infrastructure projects that involve market competition, technological obsolescence, and dedicated asset investment.

These real-option approaches are powerful in dealing with different types of valuation issues. However, they do not consider the existence of performance bonds in BOT undertakings. In order to ensure that BOT projects are performed according to preset construction schedules and other performance specifications, host governments often require BOT concessionaires to furnish performance bonds as security. These performance bonds should affect the value of projects that involve the rights to abandon prematurely, because the concessionaires will face penalties when they exercise the rights. Due to the prevalence of the abandonment rights in BOT projects, how performance bonding would affect BOT project value becomes an important issue.

In this paper, a simple real-option valuation model is proposed to handle this valuation issue. The risk-neutral pricing theory is applied to derive a closed-form solution for a European-style call option with performance bonding. The Taiwan high-speed rail project is chosen for a numerical implementation of the proposed valuation model. Results show that performance bonding would destroy the value of flexibility provided by the option to abandon prematurely. This has an important policy implication in BOT risk management.

The rest of this paper is organized as follows. Section 2 provides the valuation model. Section 3 presents the influence of performance bonding on project value. Section 4 presents the case study. Section 5 presents the policy implication of the research findings. Section 5 concludes this paper.

## 2. THE VALUATION MODEL

As in Fig. 1, a BOT project usually goes through three major stages, namely preconstruction (Stage I), construction (Stage II), and operation (Stage III). The concessionaire signs the concession contract with the host government at time $t_0$, and is obligated to start construction at $t_1$. The parameter $K$ denotes the total construction cost of the project. If the concessionaire decides to invest $K$ at $t_1$ and complete the construction of the project on schedule at $t_2$, then the project will be operated until the expiry of the concession period at $t_n$. If the concessionaire decides not to invest at $t_1$, then a performance bond, whose value is denoted by $B$, will be executed as a compensation for the host government.

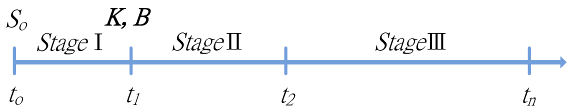

**Figure 1.** A typical BOT project lifecycle.

In this contractual setting, the concessionaire has a European call option, that is, the rights to decide whether to invest at $t_1$ or not. When the concessionaire signs the concession contract at $t_0$, the project has an asset value of $S_0$, which is uncertain and will be re-evaluated from $t_0$ to $t_1$ when new project information is available. A rational concessionaire will decide to invest at $t_1$ if and only if the re-evaluated asset value $S_{t_1}$ is higher than $K$, which is, by definition, the exercise price of the call option.

This European call option differs from a traditional one because the concessionaire will face a penalty of $B$ when deciding not to invest at $t_1$. In a no-arbitrage, risk neutral environment (see Harrison and Kreps [7] and Harrison and Pliska [8]), the payoff of this specific call option is provided by:

$$C_t = E^Q\{\max[S_t - K, -B]\} = E^Q\{\max[S_t - (K-B), 0]\} - B \tag{1}$$

where $E^Q$ denotes a conditional expectation operator under the risk-neutral environment $Q$, and $S_t$ denotes the time-$t$ re-evaluated project asset value. To find the value of this option, further assume that the project asset value exhibits the following stochastic behavior (see Black and Scholes [9] and Merton [10]):

$$\frac{dS_t}{S_t} = (r-q)dt + \sigma dz_t^Q \tag{2}$$

where $r$ denotes risk-free rate of return, $q$ denotes dividend payout rate, and $z_t^Q$ denotes a standard Brownian motion under the risk-neutral environment. The parameter $\sigma$ denotes asset value volatility, which is assumed to have a deterministic value.

**Theorem.** *(A Pricing Formula for a European Call Option with Performance Bonding)*

*Based on the foregoing project settings, the value of the European call option at $t_0$ is provided by*

$$\begin{aligned} C_0 &= S_0 e^{-\int_{t_0}^{t_1} q(u)du} N(d_1) - (K-B)e^{-\int_{t_0}^{t_1} r(u)du} N(d_2) - Be^{-\int_{t_0}^{t_1} r(u)du} \\ &= S_0 e^{-\int_{t_0}^{t_1} q(u)du} N(d_1) - Ke^{-\int_{t_0}^{t_1} r(u)du} N(d_2) - Be^{-\int_{t_0}^{t_1} r(u)du} N(-d_2) \end{aligned} \tag{3}$$

*where*

$$d_1 = \frac{\ln(\frac{S_0}{K-B}) + \int_{t_0}^{t_1}[r(u) - q(u) + \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} = d_2 + \sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}$$

$$d_2 = \frac{\ln(\frac{S_0}{K-B}) + \int_{t_0}^{t_1}[r(u) - q(u) - \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}$$

***Proof.***

According to Cox and Ross [11], Harrison and Kreps [7], and Harrison and Pliska [8], the fair price of the European call option is provided by:

$$\begin{aligned} C_0 &= e^{-\int_{t_0}^{t_1} r(u)du} E^Q\{\max[S(T) - K, -B]\} \\ &= e^{-\int_{t_0}^{t_1} r(u)du}\{E^Q[\max[S(T)-(K-B),0]] - B\} \\ &= e^{-\int_{t_0}^{t_1} r(u)du} E^Q\{S(T)\cdot 1_{\{S(T)>(K-B)\}}\} \\ &\quad - (K-B)e^{-\int_{t_0}^{t_1} r(u)du} E^Q\{1_{\{S(T)>(K-B)\}}\} - Be^{-\int_{t_0}^{t_1} r(u)du} \end{aligned} \tag{4}$$

The solution of the stochastic differential equation (S.D.E.) in (2) is

$$S_t = S_0 e^{\int_{t_0}^{t_1}[r(u)-q(u)-\frac{1}{2}\sigma^2(u)]du + z^Q\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}$$

(see, for example, Shreve [12]). Substituting this solution into (4) yields:

$$\begin{aligned} C_0 &= S_0 e^{-\int_{t_0}^{t_1} q(u)du} E^Q\left\{e^{-\int_{t_0}^{t_1}\frac{1}{2}\sigma^2(u)du + z^Q\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} \cdot 1_{\{S(T)>(K-B)\}}\right\} \\ &\quad - (K-B)e^{-\int_{t_0}^{t_1} r(u)du} E^Q\{1_{\{S(T)>(K-B)\}}\} - Be^{-\int_{t_0}^{t_1} r(u)du} \end{aligned} \tag{5}$$

To eliminate the uncertain term in the expectation operator $E^Q$, one can use Girsanov's Theory to change the probability measure. First define the Radon-Nikodym derivative as:

$$\frac{dR}{dQ} = -\int_{t_0}^{t_1}\frac{1}{2}\sigma^2(u)du + z^Q\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du} \tag{6}$$

Then the Brownian motion term in (2) can be re-written as $dz_t^Q = dz_t^R + \sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}$, and the stochastic asset process becomes $\frac{dS_t}{S_t} = \left(r - q + \sigma^2\right)dt + \sigma dz_t^R$, where $dz_t^R$ represents a standard Brownian motion under the measure $R$, using the underlying asset value as numéraire (see, for example, Shreve [12]). By Ito's lemma, the solution of the stochastic asset value process under measure $R$ is $S_t = S_0 e^{\int_{t_0}^{t_1}[r(u)-q(u)+\frac{1}{2}\sigma^2(u)]du + z^R\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}$. Substituting this solution into (5) yields

$$
\begin{aligned}
C_0 &= S_0 e^{-\int_{t_0}^{t_1} q(u)du} E^Q\left\{\frac{dR}{dQ}\cdot 1_{\{S(T)>(K-B)\}}\right\} \\
&\quad - \left(K-B\right)e^{-\int_{t_0}^{t_1} r(u)du} E^Q\left\{\ 1_{\{S(T)>(K-B)\}}\right\} - Be^{-\int_{t_0}^{t_1} r(u)du} \qquad (7)\\
&= S_0 e^{-\int_{t_0}^{t_1} q(u)du} E^R\left\{\ 1_{\{S(T)>(K-B)\}}\right\} \\
&\quad - \left(K-B\right)e^{-\int_{t_0}^{t_1} r(u)du} E^Q\left\{\ 1_{\{S(T)>(K-B)\}}\right\} - Be^{-\int_{t_0}^{t_1} r(u)du} \\
&= S_0 e^{-\int_{t_0}^{t_1} q(u)du} P^R\left(\ln S(T) > \ln(K-B)\right) \\
&\quad - \left(K-B\right)e^{-\int_{t_0}^{t_1} r(u)du} P^Q\left(\ln S(T) > \ln(K-B)\right) - Be^{-\int_{t_0}^{t_1} r(u)du}
\end{aligned}
$$

where

$$
\begin{aligned}
&P^R\left(\ln S(T) > \ln(K-B)\right) \\
&= P^R\left(\ln S_0 + \int_{t_0}^{t_1}[r(u)-q(u)+\frac{1}{2}\sigma^2(u)]du + z^R\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du} > \ln(K-B)\right) \\
&= P^R\left(-z^R < \frac{\ln\left(\frac{S_0}{K-B}\right) + \int_{t_0}^{t_1}[r(u)-q(u)+\frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}\right) = P^R\left(-z^R < d_1\right)
\end{aligned}
$$

A similar process yields

$$
P^Q\left(\ln S_t > \ln(K-B)\right) = P^R\left(-z^Q < d_2\right) \qquad (8)
$$

where

$$
d_2 = \frac{\ln\left(\frac{S_0}{K-B}\right) + \int_{t_0}^{t_1}[r(u)-q(u)-\frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}
$$

Accordingly, the time-$t_0$ value of the call option is provided by

$$
\begin{aligned}
C_0 &= S_0 e^{-\int_{t_0}^{t_1} q(u)du} P^R\left(-z^R < d_1\right) \qquad (9)\\
&\quad -\left(K-B\right)e^{-\int_{t_0}^{t_1} r(u)du} P^Q\left(-z^Q < d_2\right) - Be^{-\int_{t_0}^{t_1} r(u)du}
\end{aligned}
$$

where both $z^R$ and $z^Q$ are standard Brownian motions. Finally, using the fact that $z \sim N(0,1)$ and $1-N(d_2)=N(-d_2)$, one has the pricing formula in (3). □

## 3. INFLUENCE OF PERFORMANCE BONDING ON PROJECT VALUE

**Proposition.** *From the pricing formula (3), the time-$t_0$ value of the European call option $C_0$ is monotonic and strictly decreasing in B.*

***Proof.***

According to (3),

$$
\begin{aligned}
\frac{\partial C}{\partial B} &= \frac{\partial}{\partial B}\left\{S_0 e^{-\int_{t_0}^{t_1} q(u)du} N(d_1) - Ke^{-\int_{t_0}^{t_1} r(u)du} N(d_2) - Be^{-\int_{t_0}^{t_1} r(u)du} N(-d_2)\right\} \\
&= S_0 e^{-\int_{t_0}^{t_1} q(u)du} \frac{\partial}{\partial B} N(d_1) - Ke^{-\int_{t_0}^{t_1} r(u)du} \frac{\partial}{\partial B} N(d_2) \\
&\quad - e^{-\int_{t_0}^{t_1} r(u)du} N(-d_2) - Be^{-\int_{t_0}^{t_1} r(u)du} \frac{\partial}{\partial B} N(-d_2) \qquad (10)
\end{aligned}
$$

By chain rule,

$$
\frac{\partial}{\partial B} N(d_i) = \frac{\partial N(d_i)}{\partial d_i}\cdot\frac{\partial d_i}{\partial B} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_i^2} \frac{1}{\left(K-B\right)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}; \ i = 1,2
$$

and

$$
\frac{\partial}{\partial B} N(-d_2) = \frac{\partial N(-d_2)}{\partial(-d_2)}\cdot\frac{\partial(-d_2)}{\partial B} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(-d_2)^2} \frac{-1}{\left(K-B\right)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}
$$

Therefore,

$$
\begin{aligned}
\frac{\partial C}{\partial B} &= S_0 e^{-\int_{t_0}^{t_1} q(u)du} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \frac{1}{\left(K-B\right)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} - e^{-\int_{t_0}^{t_1} r(u)du} N(-d_2) \\
&\quad - Ke^{-\int_{t_0}^{t_1} r(u)du} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \frac{1}{\left(K-B\right)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} \\
&\quad - Be^{-\int_{t_0}^{t_1} r(u)du} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(-d_2)^2} \frac{-1}{\left(K-B\right)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} \\
&= S_0 e^{-\int_{t_0}^{t_1} q(u)du} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \frac{1}{\left(K-B\right)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} - e^{-\int_{t_0}^{t_1} r(u)du} N(-d_2) \qquad (11)\\
&\quad - \left(K-B\right)e^{-\int_{t_0}^{t_1} r(u)du} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \frac{1}{\left(K-B\right)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}
\end{aligned}
$$

Replacing $d_1$ by $d_2 + \sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}$ and by some simple calculation, $\frac{\partial C}{\partial B} = -e^{-\int_{t_0}^{t_1} r(u)du} N\left(-d_2\right) < 0$ (12)

This confirms the proposition. □

## 4. A CASE STUDY

In this section, the Taiwan high-speed rail project was chosen for the numerical implementation of the foregoing valuation model using a MATLAB-based program.

### 4.1 Project Profile

The Taiwan high-speed rail project is a large-scale BOT project, with a total construction cost of NT$446.58 billion. The concessionaire of the project, the Taiwan High-speed Railroad Company (THSRC), signed a concession contract with the Taiwan government in July 1998. The contract provides the concessionaire exclusive rights to build and operate the project for 35 years, from July 1998 to July 2033. At the expiry of the concession period, the project will be transferred from the THSRC to the host government. According to the contract, the concessionaire should start the construction of the project on March 2000, which indicates that the preconstruction stage of the project is less than 2 years. The concessionaire can terminate the project prematurely, but a performance bond is required to guarantee that the project is performed accordingly. The value of the performance bond is NT$15 billion.

### 4.2 Valuation Parameters

According to the financial model of the project disclosed by the government, the project has an initial asset value $S_0$ of NT$385.27 billion at $t_0$. The value was calculated by the discounted value of the project's earnings before interest, tax and depreciation (EBITDA), without considering the effects of financing and taxation. The government set the service rate of the project by a 10% rate of return on investment. This rate of return is generally referred as the reasonable rate of return of the project in price regulation. In this base-case valuation, we use the government's original setup of the rate of return. It is possible that the rate of return would be affected changing interest rates and other factors during the life cycle of the project (for example Polk, Thompson, and Vuolteenaho [13]). Based on the 10% discount rate, the time-$t_1$ discounted value of the project's construction cost is NT$400.47 billion.

The risk-free interest rate of the project is 6.35%, which was estimated from the monthly spot rates of the 10-year Treasury bonds reported by the Central Bank between January 1993 and July 1998. The asset volatility of the project is 0.3687, estimated from the one-year stock price series of eight transportation-related companies reported in the Taiwan Economic Journal Database (TEJD). This case study does not consider the effect of dividend payout on project value, that is $q$=0. Table 1 summarizes the foregoing base-case valuation parameters.

**Table 1.** Base-case project valuation parameters.

| Variable | Value |
|---|---|
| Time-t0 discounted value of the underlying project asset ($S_0$) | NT$385.273. billion |
| Time-t1 discounted value of the construction cost ($K$) | NT$400.469 billion |
| Performance bond value ($B$) | NT$15 billion |
| Risk-free interest rate ($r$) | 6.35% |
| Asset return volatility ($\sigma$) | 0.3687 |
| Dividend payout rate ($q$) | N.A. |

### 4.3 Valuation Outcome

Notwithstanding the effect of the performance bond (that is $B$=0), the base-case valuation parameters produce a project value of NT$ 83.32 billion. This value is higher than the net present value (NPV) of the project, which is NT$ 65.711 billion based on the 10% discounted rate. This result indicates that the option to abandon the project prematurely at $t_1$ is valuable.

When the effect of the performance bond is considered (that is $B$=15), however, the project value is reduced from NT$ 83.32 billion to NT$ 76.26 billion, or about 8.5% less. Although the reduced project value is still higher than the project NPV, the value reduction is substantial. As a result, the effect of performance bonding on project value should be considered; otherwise, the valuation outcome may lead to wrong investment decisions.

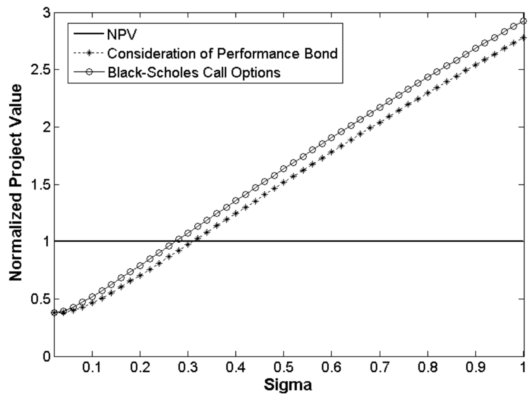


**Figure2.** Sensitivity analysis with respect to asset return volatility.

### 4.4 Sensitivity Analysis

Figure 2 shows that the flexibility for the concessionaire to abandon the project prematurely becomes more valuable when the asset return volatility increases. This result is reasonable because asset return volatility is an effective measure of project uncertainty and cash flow risk caused by, for example, unexpected project events or a higher level of market competition. Note that with the performance bond, the project value will fall below the project NPV when the asset return volatility falls below 0.3, but the base-case project value will still stay above the project NPV. This is a case in point showing how ignoring the effect of performance bond would lead to wrong decisions.

Figure 3 and Figure 4 further show the sensitivity of the project value with respect to the performance bond value. The value of flexibility provided by the option to abandon will be totally destroyed when the project bond value is set above 10% of the total construction cost. This result has an important policy implication.

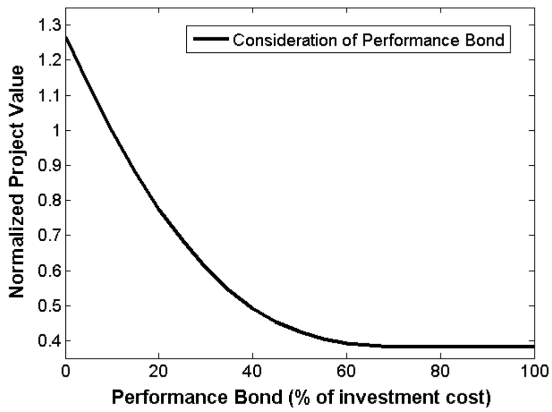


**Figure3.** Sensitivity analysis with respect to performance bond value.

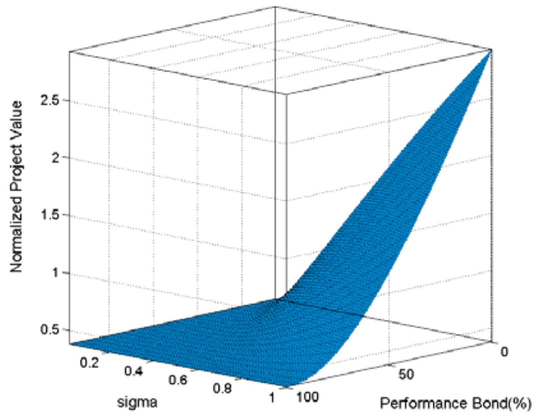


**Figure4.** Sensitivity analysis with respect to asset return volatility and performance bond value.

## 5. DISCUSSION

According to Huang [14, ch5], the reason for performance bonding in BOT projects is different from that of conventional construction projects. When abandoning a BOT project prematurely, the concessionaire will face large financial losses and risk reputational damage to project consortium members. Therefore, according to Huang [14], the concessionaire's promise to furnish a performance bond for a BOT project can be seen as a kind of reciprocal arrangement to demonstrate the long-term commitment of project consortium members. From this perspective, the cost of performance bonding is justified by reciprocity and thus mutual trust between the concessionaire and the host government.

However, the foregoing case study shows that the option to abandon prematurely can increase project value, but performance bonding can destroy the value. According to Dixit and Pindyck [15], when a capital investment involves irreversible sunk cost, it should remain flexible for managing project risk and uncertainty. The option to abandon prematurely is a type of flexibility addressed by Dixit and Pindyck [15], and it is prevalent in BOT undertakings. Therefore, when flexibility is important for BOT risk management, performance bonding should be assessed carefully:

- if the bonding requirement warrants a substantial increase in the financial cost of the project, and
- if the bonding value will destroy the value of flexibility and thus affect the concessionaire's willingness to invest.

## 6. CONCLUSION

Real-option theory is popular in dealing with complex valuation problems in BOT undertakings. However, previous real-option valuation methods did not deal with the impact of performance bonding on BOT project values in the presence of voluntary, premature abandonment rights granted to BOT concessionaires. This paper derived a valuation model to deal with this issue. Sensitivity analysis from the proposed valuation model show that the value of flexibility created by the option to abandon decreases when the value of performance bond increases. Further numerical implementation using the Taiwan high-speed rail project data show that the value of flexibility could be totally destroyed even when the bonding value is moderate. This result makes performance bonding more difficult to justify when flexibility is important for BOT risk management.

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