고속 병렬형 로봇의 역동역학 해석
Inverse dynamics analysis of a Delta-type Parallel-kinematic Robot

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Key words: Delta-type Parallel-kinematic Robot, Lagrangian equation, high speed robot.

1. Introduction

Parallel robots have received increasing attention due to their inherent advantages over conventional serial counterparts, such as high rigidity, high load capacity, high velocity, good dynamic characteristics, and high precision. In recent years, many 3-DOF parallel manipulators have been designed; extensive research work was focused on the well-known DELTA robot with three translational DOF (Tsai's manipulator which was similar to DELTA robot but not a version of DELTA). Although the parallel manipulator with limited DOF have been investigated extensively on a kinematic scope, investigations on their dynamics are relatively few, and the limited DOF parallel platforms have been investigated to some extent, very little has been reported in the literature on the closed-form dynamics. We will use direct Lagrangian method, to derive the motion of equation. Based on the kinematic analysis, the dynamic model is derived by the Lagrangian approach. The derived inverse dynamics result has been simulated with different trajectories. The inverse dynamic equations will be used for the real-time computed torque control.

2. Kinematic and Jacobian analyses

A loop-closure equation can be written for each limb:

$$\overline{A_iB_i} + \overline{B_iC_i} = \overline{OP} + \overline{PC_i} - \overline{OA_i} \qquad (1)$$

Expressing Eq.1 in the (x_i, y_i, z_i) coordinate frame, and solving for θ_{1i}, θ_{2i}, θ_{3i} yield

$$\theta_{3i} = \arccos\left(c_{yi}/l_2\right),\quad \theta_{2i} = \arccos(\kappa),\quad \theta_{1i} = 2\arctan t_{1i}$$

where $\kappa = \left(c_{xi}^2 + c_{yi}^2 + c_{zi}^2 - l_1^2 - l_2^2\right) \Big/ \left(2l_1l_2\sin\theta_{3i}\right)$

$a = l_2\sin\theta_{3i}\sin\theta_{2i}$, $b = l_1 + l_2\sin\theta_{3i}\cos\theta_{2i}$,

$t_{1i} = b \pm \sqrt{b^2 - \left(c_{zi}^2 - a^2\right)} \Big/ \left(c_{zi} + a\right)$.

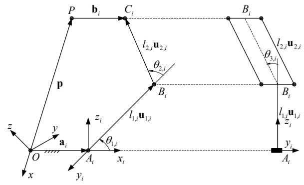


Fig. 1: Description of the joint angles

Differentiating Eq.1 with respects to time and expressing the resulting equation in the (x_i, y_i, z_i) coordinate frame provides

$$\overline{V_P} = \overline{\omega_{1i}} \times \overline{A_iB_i} + \overline{\omega_{2i}} \times \overline{B_iC_i} \qquad (2)$$

where ω_{ni} is the angular velocity of the nth link of the ith leg in the (x_i, y_i, z_i) coordinate frame.

Expressing the vector of Eq.1 in the (x_i, y_i, z_i) coordinate frame, and substituting the results into Eq. 2, we obtain the relation between the velocity of the moving platform and actuator joints

$$\mathbf{J}_x\mathbf{v}_p = \mathbf{J}_q\dot{\mathbf{q}} \qquad (3)$$

where $J_q = l_1\begin{bmatrix} \sin\theta_{11}\sin\theta_{31} & 0 & 0 \\ 0 & \sin\theta_{22}\sin\theta_{32} & 0 \\ 0 & 0 & \sin\theta_{23}\sin\theta_{33} \end{bmatrix}$.

$$J_x = \begin{bmatrix} j_{1x} & j_{1y} & j_{1z} \\ j_{2x} & j_{2y} & j_{2z} \\ j_{3x} & j_{3y} & j_{3z} \end{bmatrix}$$

$$j_{ix} = \cos(\theta_{1i} + \theta_{2i})\sin\theta_{3i}\cos\phi_i - \cos\theta_{3i}\sin\phi_i$$

$$j_{iy} = \cos(\theta_{1i} + \theta_{2i})\sin\theta_{3i}\sin\phi_i + \cos\theta_{3i}\cos\phi_i$$

$$j_{iz} = \sin(\theta_{1i} + \theta_{2i})\sin\theta_{3i}$$

3. Inverse dynamics analysis

In this section, the inverse dynamics of Delta robot is presented. For the computed torque control in high speed applications, the closed form solution of inverse dynamics is essential. Using the Lagrange equations of the first type, the closed-form dynamics equations can be derived.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}=Q_j+\sum_{i=1}^{k}\lambda_i\frac{\partial \Gamma_i}{\partial q_j}\ \text{for } j=1\,to\,n$$

where n is the number of generalized coordinates, k is the number of constraint functions, $n-k$ is the number of actuated joint variables, Γ_i denotes the i^th constraint function, and λ_i is the Lagragian multiplier.

The Lagrangian multipliers are derived as

$$2\sum_{i=1}^{3}\lambda_i\left(p_x+h\cos\phi_i-r\cos\phi_i-a\cos\phi_i\cos\theta_{1i}\right)=\left(m_p+3m_b\right)\ddot{p}_x$$

$$2\sum_{i=1}^{3}\lambda_i\left(p_y+h\sin\phi_i-r\sin\phi_i-a\sin\phi_i\cos\theta_{1i}\right)=\left(m_p+3m_b\right)\ddot{p}_y$$

$$2\sum_{i=1}^{3}\lambda_i\left(p_z-a\sin\theta_{1i}\right)=\left(m_p+3m_b\right)\ddot{p}_z+\left(m_p+3m_b\right)g_c.$$

Once the Lagrangian multipliers are found the actuator torque can be determined as

$$\tau_1=\left(\frac{1}{3}m_a a^2+m_b a^2\right)\ddot{\theta}_{11}+\left(\frac{1}{2}m_a+m_b\right)g_c a\cos\theta_{11}$$
$$-2a\lambda_1\left[\left(p_x\cos\phi_1+p_y\sin\phi_1+h-r\right)\sin\theta_{11}-p_z\cos\theta_{11}\right]$$

$$\tau_2=\left(\frac{1}{3}m_a a^2+m_b a^2\right)\ddot{\theta}_{12}+\left(\frac{1}{2}m_a+m_b\right)g_c a\cos\theta_{12}$$
$$-2a\lambda_2\left[\left(p_x\cos\phi_2+p_y\sin\phi_2+h-r\right)\sin\theta_{12}-p_z\cos\theta_{12}\right]$$

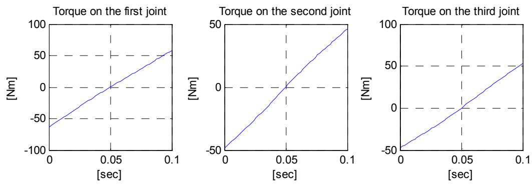
$$\tau_2=\left(\frac{1}{3}m_a a^2+m_b a^2\right)\ddot{\theta}_{13}+\left(\frac{1}{2}m_a+m_b\right)g_c a\cos\theta_{13}$$
$$-2a\lambda_3\left[\left(p_x\cos\phi_3+p_y\sin\phi_3+h-r\right)\sin\theta_{13}-p_z\cos\theta_{13}\right]$$

The mass properties and kinematic parameters of Delta-type parallel-kinematic are listed in table 1.

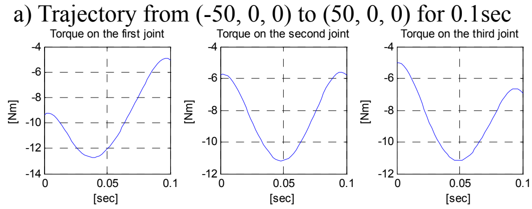
Table 1. Design parameters

No	Values
Upper robot arm m_a [g]	82.9
Parallelogram m_b [g]	48.2
Moving platform m_p [g]	83.6
Radius of the fixed base a [mm]	60
Radius of the moving platform b [mm]	22
Upper arm length l_1 [mm]	100
Parallelogram length l_2 [mm]	224

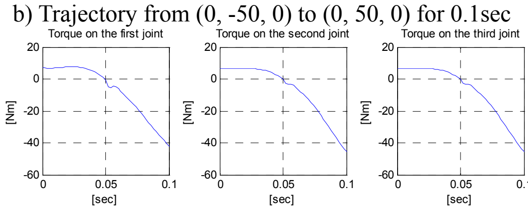
Fig. 2 presents the actuator forces obtained by the analytical inverse dynamics for three different cubic trajectories with average velocity 1000mm/sec.



a) Trajectory from (-50, 0, 0) to (50, 0, 0) for 0.1sec



b) Trajectory from (0, -50, 0) to (0, 50, 0) for 0.1sec



c) Trajectory from (0, 0, 188) to (0, 0, 288) for 0.1sec

Fig.2: Simulations results of the inverse dynamics

4. Conclusion

In this the inverse dynamics of the Delta-type Parallel-kinematic Robot has been derived. The derived inverse dynamic equations will be used for real-time computed torque control.

후기

본 연구는 2010 년도 지식경제부 생산기술 사업화 과제의 지원을 받은 것으로 이에 감사 드립니다.

References

1. Clavel R, “Delta, a Fast Robot with Parallel Geometry,” 18th Int. Sym. on Industrial Robots, Sydney, Australia, 91-100, 1988.
2. Tsai L. W, “Kinematics of a Three-DOF Platform Manipulator with Three Extensible Limbs,” Advances in Robot Kinematics, 401-410, 1996.
3. Liu X. J, “Optimal Kinematic Design of a Three Translational DoFs Parallel Manipulator,” Robotica, Vol. 24, No. 2, 239-250, 2006.
4. Lee K. M., Shah D. K., Kinematic Analysis of **A** Three degree-of-Freedom In-Parallel Actuated Manipulator, *IEEE J. of Robotics and Automation,* 1988, Vo1.4, No.3, 354-360.