Analytical Modeling of the Pseudo-Colloid Migration with the Band release Boundary Condition in the Fractured Porous Media

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1. Introduction

The concept of deep geological disposal of high-level radioactive waste has been widely accepted at many countries. The repositories aim mainly to prevent the radionuclides form migrating to the biosphere through any one of many pathways. Fractures can act as main pathways for radionuclide transport because of their relatively high permeabilities. Many papers have already dealt with the problem of the raionuclide transport in various fractured porous systems, but without discussing daughter products. However, natural radionuclides may decay to radioactive daughter muclides, which may travel farther than the parent nuclides. It is considered the multi-member decay chain of the actinide nuclide with the band release inlet boundary condition in a fractured porous rock.

2. Mathematical Modelling

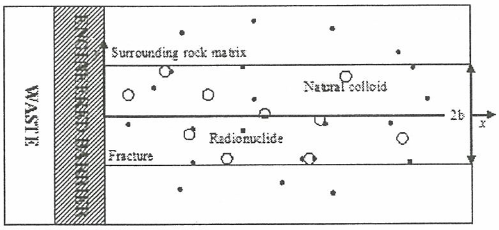

Fig. 1. The Pseudo-colloidand the solute migration in a fractured porous media

We consider the concentration of a radioactive solute, $C_{2i}(x,t)$ in the liquid phase in a thin fracture of a fractured porous rock. Also present in a fracture are the natural colloids, on which the solute can be sorbed to form the pseudo-colloids as shown in Fig.1 In the one-dimensional convective-diffusive transport within a fracture, and it is assumed that the colloids are too large to diffuse into the rock matrix. The governing equation of the pseudo-colloid migration is:

$$\epsilon_1\xi_1\frac{\partial C_{1i}(x,t)}{\partial t}+\epsilon_1\xi_1 v_1\frac{\partial C_{1i}(x,t)}{\partial x}-\epsilon_1\xi_1 D_1\frac{\partial^2 C_{1i}(x,t)}{\partial x^2}$$
$$+\epsilon_1\xi_1\lambda_i C_{1i}(x,t)-\epsilon_1\xi_1\lambda_{i-1}C_{1(i-1)}(x,t)+\epsilon_1 S_{1i}(x,t)-\epsilon_1 S_{2i}(x,t)=0$$
$$x>0,\ t>0 \quad \cdots\cdots (1)$$

where ϵ_1 is the ratio of the liquid volume to total volume in a fracture, $C_{1i}(x,t)$ is the amount of solutes sorbed on the colloid per an unit volume of the solid colloid, [kg/m^3], v_1 is the colloid velocity in a fracture, [m/yr], D_1 is the colloid dispersion coefficient, [m^2/yr], λ_i is the decay constant, [1/yr], ξ_1 is the constant volume fraction of natural colloids in a fracture, $\epsilon_1 S_{1i}(x,t)$ is the rate of sorption of pseudo-colloid to stationary fracture, and $\epsilon_1 S_{2i}(x,t)$ is that of desorption of solute from the pseudo-colloid.

For the same solute in liquid in a fracture

$$\epsilon_1\frac{\partial C_{2i}(x,t)}{\partial t}+\epsilon_1 v_2\frac{\partial C_{2i}(x,t)}{\partial x}-\epsilon_1 D_{2i}\frac{\partial^2 C_{2i}(x,t)}{\partial x^2}+\frac{q_i(x,t)}{b}$$
$$+\epsilon_1\lambda_i C_{2i}(x,t)-\epsilon_1\lambda_{i-1}C_{2(i-1)}(x,t)-\epsilon_1 S_{3i}(x,t)+\epsilon_1 S_{2i}(x,t)=0$$
$$x>0,\ t>0 \quad \cdots\cdots (2)$$

where v_2 is the solute velocity, [m/yr], D_{2i} is the solute dispersion coefficient, [m^2/yr], $\epsilon_1 S_{3i}(x,t)$ is the rate of solute sorption on the stationary fracture, b is the fracture half-width, [m], and $q_i(x,t)$ is the diffusive solute flux into the rock matrix, [kg/m^2·yr],

$$q_i(x,t)=-\epsilon_p D_{pi}\left.\frac{\partial N_i(x,y,t)}{\partial y}\right|_{y=b} \quad \cdots\cdots (3)$$

where y is the distance from the center of a fracture, [m], ϵ_p is the rock porosity, D_{pi} is the radionuclide diffusion coefficient in the rock matrix pores, [m^2/yr], and $N_i(x,y,t)$ is the radionuclide concentration in the pore water in the rock, [kg/m^3]:

$$R_{pi}\frac{\partial N_i(x,y,t)}{\partial t}-D_{pi}\frac{\partial^2 N_i(x,y,t)}{\partial y^2}$$
$$+R_{pi}\lambda_i N_i(x,y,t)-R_{p(i-1)}\lambda_{i-1}N_{i-1}(x,y,t)=0$$
$$x>0,\ t>0,\ y>0 \quad \cdots\cdots (4)$$

where R_{pi} is the solute retardation coefficient in the rock matrix.

For solute species sorbed on the stationary fracture

$$(1-\epsilon_1)\frac{\partial C_{3i}(x,t)}{\partial t}+(1-\epsilon_1)\lambda_i C_{3i}(x,t) \quad , x>0,\ t>0 \cdots(5)$$
$$-(1-\epsilon_1)\lambda_{i-1}C_{3(i-1)}(x,t)+\epsilon_1 S_{3i}(x,t)=0$$

where $C_{3i}(x,t)$ is the concentration of a sorbed solute on the stationary fracture, [kg/m^3].

For pseudo-colloids on the solid in the fracture;

$$(1-\epsilon_1)\xi_2\frac{\partial C_{1i}(x,t)}{\partial t}+(1-\epsilon_1)\xi_2\lambda_i C_{1i}(x,t) \quad , x>0,\ t>0 \cdots(6)$$
$$-(1-\epsilon_1)\xi_2\lambda_{i-1}C_{1(i-1)}(x,t)-\epsilon_1 S_{1i}(x,t)=0$$

where ξ_2 is the constant volume fraction of the sorbed colloid on the solid phase in the fracture.

Linear sorption equilibrium between the solute species in the fracture and the same species sorbed on the colloid is assumed. Both the solute species and the colloids in the fracture are assumed to undergo the linear sorption equilibrium with the fracture solids:

$$K_{d1}=\frac{\xi_2}{\xi_1},\quad K_{d2i}=\frac{C_{3i}(x,t)}{C_{2i}(x,t)},\quad K_{d3i}=\frac{C_{1i}(x,t)}{C_{2i}(x,t)} \cdots (7)$$

The initial and boundary conditions are

$$N_i(x,\infty,t)=0, \qquad x>0,\ t>0 \cdots\cdots\cdots\cdots (8)$$

$$N_i(x,b,t)=\frac{C_{1i}(x,t)}{K_{d3i}}, \qquad x>0,\ t>0 \cdots\cdots\cdots\cdots (9)$$

$$N_i(x,y,t)=0, \qquad x>0,\ t>0 \cdots\cdots\cdots\cdots (10)$$

$$v_2 C_{2i}(0,t)-D_{2i}\left.\frac{\partial C_{2i}(x,t)}{\partial x}\right|_{x=0}=\frac{f_i(t)}{A}, \quad t>0 \cdots\cdots (11)$$

$$C_{1i}(\infty,t)=0, \qquad t>0 \cdots\cdots (12)$$

$$C_{1i}(x,0)=0, \qquad x>0 \cdots\cdots (13)$$

where $f_i(t)$ is the band release function, and A is the fracture cross-sectional area, [m^2].

The semi-analytical solution can be found using the Laplace Transform.

$$\tilde{N}_i=\sum_{m=1}^{i}\frac{\tilde{C}_{1m}}{K_{d3m}}\left(\prod_{r=m}^{i-1}\frac{R_{pr}}{D_{p(r+1)}}\lambda_r\right)\sum_{n=m}^{i}\frac{\exp[-\Phi_n(y-b)]}{\prod_{\substack{l=m\\ l\neq m}}^{i}(\Phi_n^2-\Phi_l^2)} \cdots (14)$$

$$\tilde{C}_{1i}=\sum_{j=1}^{i}u_{ji}(s)a_j(s)\exp[\alpha_j(1-\gamma_j)x] \cdots\cdots\cdots\cdots (15)$$

3. Numerical Illustration

It is investigated the pseudo-colloid and radionuclide transport in a fractured porous medium with the band release B.C.

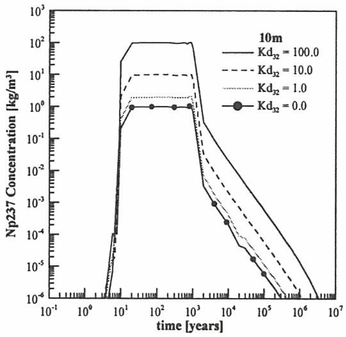

Fig. 2. the concentration profile of the Np237 with or without colloids

Fig.2 illustrates the effect of the amount of colloid in a fracture. The inlet boundary condition is applied the band release with decay, $e^{-\lambda_i t}\{h(t)-h(t-T)\}$. The total radionuclide concentration of NP237 which is the sum of solute and pseudo-colloid concentration are plotted. For this analysis, retardation and sorption coefficient of each radionuclide is assumed same, that is, R_{pi} for all nuclides is 100.0 and $Kd2_i$ is 1.0. $Kd3_i$ is changed 0~100. As amount of colloid in the liquid phase in the fracture become increased, the transport is accelerated and the total concentration become high. And as expected, the radionuclide travel more distance as same time. Therefore it is indicated that the influence of the colloid in the fracture is significantly important in the geologic system.

4. Conclusions

In this paper, it is developed the pseudo-colloid migration with the band release inlet boundary conditions with multi-member decay chains in a fractured porous matrix. It is obtained a semi-analytical solution for the multi-member decay chains as a canonical form. As one can expected, the colloid has significantly important influence to the radionuclide transport in the geologic system and the decay chain also isn't neglecting.

5. Acknowledge

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