고체지구조석계산에 있어 차수2항과 3항의 바른 계산

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Comparison of degree 2 and degree 3 terms in the body tide calculation

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Tide is a phenomenon which has been recognized for more than a thousand years, and its careful analysis started in the nineteenth century and flourished afterwards. Nowadays, with reliable model of the earth's physical properties and the digital computer, has it been possible to calculate earth tide. The 'solid earth tide' – which is also known as 'body tide', exists on the earth's surface with amplitude up to 54 cm for lunar tide. In this study, set of Love numbers have been re-calculated and used to predict the body tide with terms up to degree 3.

The tidal potential on the earth due to a body of mass M at distance r can be expressed as

$$W = \sum_{n=2}^{\infty} W_n = GM \sum_{n=2}^{\infty} \frac{a^n}{r^{n+1}} P_n(\cos\psi) \quad , \tag{1}$$

where ψ is the zenith angle of the tide-raising body, and a is the radius of the earth. Since the sun and the moon are quite far away from the earth, the first leading term in the above series dominates. Although direct calculation using the zenith angle of tide raising body is possible, it is more accurate and convenient to decompose this tidal potential using the 'addition theorem':

$$\begin{aligned} P_n(\cos\psi) = {} & P_n(\cos\theta) P_n(\cos\theta') \\ & + 2\sum_{m=1}^{n} \frac{(n-m)!}{(n+m)!} \left[P_{nm}(\cos\theta)\cos m\lambda \, P_{nm}(\cos\theta')\cos m\lambda' \right. \\ & \left. + P_{nm}(\cos\theta)\sin m\lambda \, P_{nm}(\cos\theta')\sin m\lambda' \right] . \end{aligned} \tag{2}$$

For the term of degree n = 2, each decomposed terms for the perturbed earth' s gravity potential associated with the tidal deformation can be expressed as

$$\Delta C_{20} = \frac{M}{M_e}(1+k_2)(\frac{a}{r'})^3 P_{20}(\cos\theta') , \quad (3a)$$

$$\Delta C_{21} = \frac{M}{M_e}(1+k_2)(\frac{a}{r'})^3 P_{21}(\cos\theta')\cos\lambda' , \quad \Delta S_{21} = \frac{M}{M_e}(1+k_2)(\frac{a}{r'})^3 P_{21}(\cos\theta')\sin\lambda' , \quad (3b,c)$$

$$\Delta C_{22} = \frac{M}{M_e}(1+k_2)(\frac{a}{r'})^3 P_{22}(\cos\theta')\cos 2\lambda' , \quad \Delta S_{22} = \frac{M}{M_e}(1+k_2)(\frac{a}{r'})^3 P_{22}(\cos\theta')\sin 2\lambda' \quad (3d,e)$$

where M and M_e are the masses of the tide−raising body and the earth. k_2 is the potential Love number of degree n = 2. For this study, set of Love numbers have been re−calculated through our own computer program using the IASPEI earth model(see separate report for the details of Love number calculation). In Table 1, the calculated Love numbers for the four different kinds of tides, which have spatial dependence as spherical harmonics of same degree n = 2 but different order (1 for diurnal and 2 for semidiurnal), are listed.

Table 1. Love numbers h_2, k_2, and l_2 for diurnal and semidiurnal tides

	h_2	k_2	l_2
Lunar diurnal	0.60618	0.29927	0.08397
Solar diurnal	0.60623	0.29930	0.08397
Lunar semi diurnal	0.60345	0.30036	0.08408
Solar semi diurnal	0.60867	0.30046	0.08409

The coefficients ΔC_{20} , ΔC_{21} , ΔS_{21} , ΔC_{22} , and ΔS_{22} are the changes of the coefficients C_{20}, C_{21}, S_{21}, C_{22}, and S_{22} in the spherical harmonic representation of the earth gravity field.

$$V = \frac{GM_e}{r}[1+\sum_{n=1}^{\infty}\sum_{m=0}^{n}(\frac{a}{r})^n (C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)P_{nm}(\cos\theta)] . \quad (4)$$

Then the gravity potential perturbation of degree n at any arbitrary position due to the tidal force can be evaluated as $\Delta V_n = (1+k_n)W_n$. Also, the corresponding earth tidal deformations $(u_r, u_\theta, u_\lambda)$ are acquired as $(h_n \frac{W_n}{g}, \frac{l_n}{g}\frac{\partial W_n}{\partial\theta}, \frac{l_n}{g\sin\theta}\frac{\partial W_n}{\partial\lambda})$. Other associated perturbations can be acquired accordingly. First, tidal variation of the height of the ocean surface is evaluated as $u_n = (1+k_n-h_n)\frac{W_n}{g}$, because the ocean bottom would rise by an amount of $u = h\frac{W}{g}$. Considering the r dependence of the three terms of u_n, the gravity tide is evaluated as $\Delta g_n = -(1+\frac{2}{n}h_n - \frac{n+1}{n}k_n)\frac{\partial W_n}{\partial r}$. Tidal variation of the deflection of vertical is evaluated as $(\Delta\xi, \Delta\eta) = \frac{1+k_n-h_n}{g}(\frac{1}{a}\frac{\partial W_n}{\partial\theta}, \frac{1}{a\sin\theta}\frac{\partial W_n}{\partial\lambda})$. It should be noted that the decomposition as in Eq. (3a−e), makes it possible for each different frequency component of tidal perturbations to be treated separately with the corresponding Love number of degree n. A sample output for the gravity perturbation (at Chuncheon, Korea, during the week of KSEEG 2010 fall meeting) is shown in Fig. 1.

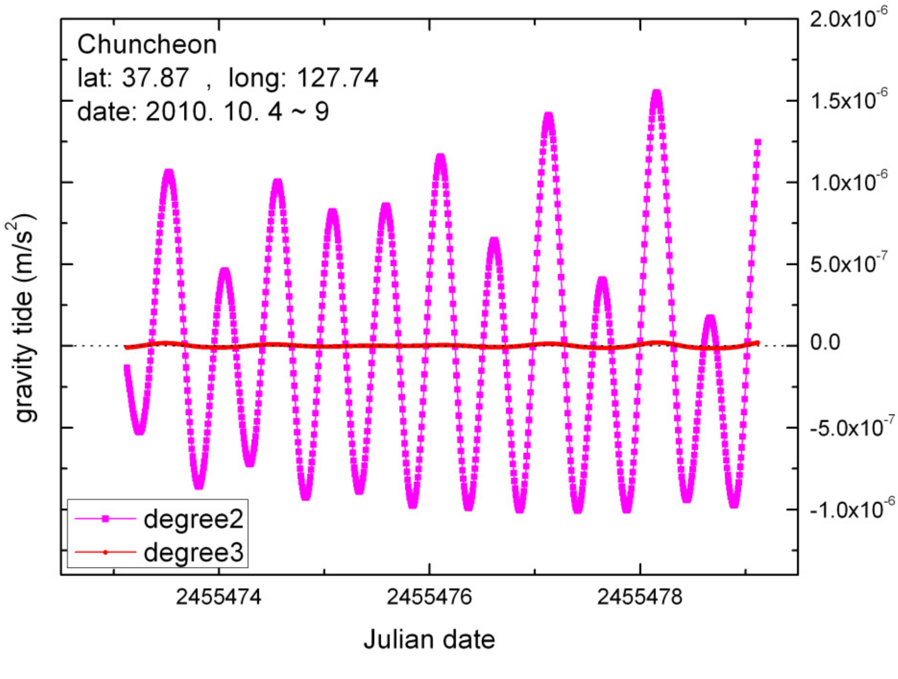

Fig. 1 Gravity tide of degree 2 and degree 3 at the specified location.

In Fig. 1, the small gravity tide perturbation of degree n = 3 term, which is calculated by using the corresponding Love numbers, are drawn together with the main term of degree n = 2. In Table 2, are listed the calculated Love numbers of degree n = 3. It is reported here that conceptual errors and wrong Love numbers were embedded in a FORTRAN program to calculate body tide, which has been widely used in Korea for more than a decade.

Table 2. Love numbers h_3, k_3, and l_3 for diurnal and semidiurnal tides

	h_3	k_3	l_3
Lunar diurnal	0.28933	0.09240	0.01463
Solar diurnal	0.28934	0.09241	0.01456
Lunar semi diurnal	0.28975	0.09253	0.01453
Solar semi diurnal	0.28979	0.09254	0.01453

Love numbers have been re−calculated using the IASPEI earth model through our own computer program. The body tide of the earth can be more accurately predicted by using an algorithm to decompose the tidal potential into terms of different degree and order with corresponding different Love numbers. Terms of gravity tide of degree 2 and degree 3 can be correctly evaluated by using our new computer program.