A Mathematical Definition of Cognitive Science

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Abstract

Formally, we may define cognitive science as the convergent study between symbolic and connectionist approaches at macro and micro levels. Since what we refer to as the human mind is regarded as a mathematical product of the human brain and the computing machine, we can obtain two mathematical dynamical projections: one from the set of human brains to the set of mind, the other from the set of computing machines to the set of mind. Then, we are having a new projection from the classical models to the quantum mind.

Problem

One of the most fundamental difficulties for cognitive scientists is to define cognition in terms of science. In a broader sense, the term *cognition* may be defined differently according to the field concerned. In psychology, cognition may be defined as a mental function in a mental world. In neuroscience, it may be defined as a physical function in a physical world. In artificial intelligence, it may be defined as a computational function in an artificial world. The critical point here is that cognitive scientists would unify cognition, with respect to the mind, the brain, and the computer, into a single frame. This does not mean that the three systems are not different from each other. Rather, cognition in the mind may be considered as a certain level of processing of the brain and the computer. For example, Newell(1990) offered the description of the mind as the control system that guides the behaving organism in its interactions with the dynamic real world. Yet, such a definition does not distinguish the mind from the brain.

Defining Cognition

Cognitive scientists generally would assume cognition to be a collection of goal-oriented behaviors such as reasoning, learning, language processing, problem solving among others. The concept of cognition, however, has no single definition or well-defined description, and therefore, offers no uniform scope.

Cognition is by origin said to be a concept of computation. Computation has been a key word and methodology constructing major theories of cognitive science.

Marr's Proposition

Trying to understand perception by studying only neurons is like trying to understand bird flight by studying only feathers.

Following D. Marr(1982), there are two important formalization for understanding cognition: representation and process.

Representation

A *representation* is defined as a formal system for making explicit certain entities or types of information, together with a specification of how the system does this. For example, the Arabic, Roman and binary numeral systems are all formal systems for representing numbers.

Process

A *process* is defined as a mapping from one formal system as representation to another.

Marr provided a fundamental frame for analysis in cognitive science. He suggested that there are three levels at which the cognitive process must be studied and that we should focus on computational theory.[1] We agree with Marr in terms of functional analysis, yet we disagree with him in terms of structural analysis. For Marr's three levels of analysis cannot explain the relationship between behaviors and neurons. In other words, they do not offer a bridge between mental activities and brain regions. On this point, Arbib's thesis of schemas is more convincing (Arbib 1992). According to him, schemas are the categories of computational analysis that intermediate between cognitive behaviors and physical neurons. Hence, the concept of schemas mediated between two analyses: functional and structural, or higher level and lower level analysis, respectively (Fig. 1).

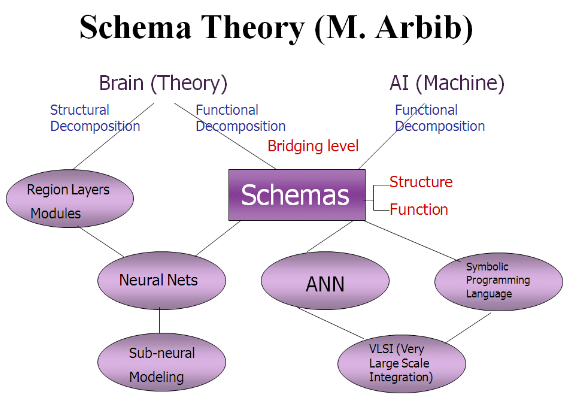

Fig.1 Schema Theory

Behavior is defined as a mapping from a set of times to a set of spaces. Thus cognition as an action can be defined as a mapping from a set of times to a set of mental spaces as a mind, where the mind can be restricted to natural or artificial (Fig.3). The domain of cognition may be upon the discrete or indiscrete numbers. Thus, the properties of cognition include the progression of a modified model of the world with its attendant adaptability, flexibility and generality, and dynamic planning (Arbib 1989).

Minds, brains and computing machines are dynamical cognitive systems. They are thus best understood from the perspective of dynamics. A *dynamical* system is system that changes in time; what changes is the *state* of the system. A *mathematical dynamical system* consists of the space of all states of the system and a rule called the dynamic for determining the state.

A dynamical system consists of a state space, X, and a mapping D: $X \times T \rightarrow X$, called the dynamics. Each vector, $x \in X$, is a state of the system. The dynamic, D, describe the way the system changes over time, T. Dynamic, D, can be given as the solution of a different equation that describes the way the system changes.

Defining Cognitive Science

Cognitive science is an emerging and converging field of study whose boundaries are not yet well-defined. D. Norman(1981) defines cognitive science as the search for understanding of cognition both in general and in the abstract, be it real or hypothetical, human or non-human, natural or artificial. According to the definition by H. Simon, cognitive science is the study of intelligence and intelligent systems, such as humans, computers, and the abstract, with particular reference to intelligent behavior as computation (Simon & Kaplan 1989).

Yet, the mathematical modeling of real world is not straightforward. We begin with a real world problem. Then we need to model the original problem by a mathematical problem. The final step is to interpret the mathematical solution in terms of the original problem (Fig.2).

From this assumption, we arrive at two basic models for the mind in real world and in formal world. The current mathematical models in cognitive science imply that the human mind is regarded as a product of the human brain

[1] Marr's three levels are as follows: (1) computational theory; (2)representation and algorithm; (3)hardware implementation.

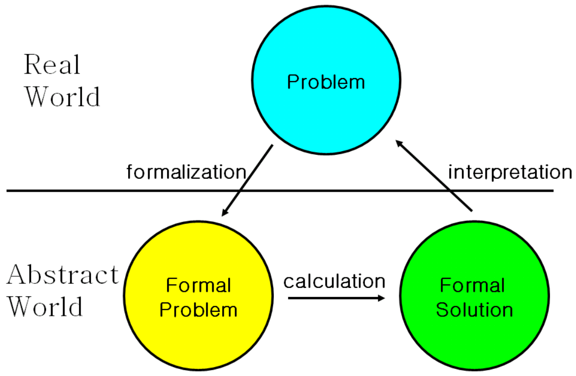

Fig.2 Mathematical Modeling of Real World

and the computing machine (Fig.3). Here, mind can be defined in terms of product as follows:

$$\text{Mind} = \text{Brain} \times \text{Machine}.$$

In other words, the product of Brain and Machine is the relation such that couple (x, y) belongs to Mind if and only if both $(x, y) \in$ Brain and $(x, y) \in$ Machine, where x means a state of Brain, y a state of Machine. Thus we can obtain two mathematical projections:

1. Projection from Mind to Machine: Symbolic modeling

Symbolic modeling can be defined in terms of projection as follows:

$$\text{Projection S: Mind} \rightarrow \text{Machine}.$$

This symbolic modeling is concerned with mind as computing machine. The *symbolism* is used for studies that model human thought and behavior in terms of symbolic manipulation of sequential automaton like units. The symbolic school views the mind as a serially symbolic manipulating system.

Simon and Newell formulated the physical symbol system hypothesis: A physical symbol system has the necessary and sufficient means for general intelligent action (Simon & Newell 1976). Here, the term *physical* refers to two features; (1) such system clearly obey the laws of physics, (2) they are realizable by engineered systems made of engineered components. The term *symbol* is not restricted to human symbol systems. According to this school, the study of formal logic and computers leads to these notions and suggests that intelligence may reside in physical symbol systems (Herken 1995).

2. Projection from Mind to Brain: Connectionist modeling

Connectionist modeling can be defined in terms of projection as follows:

$$\text{Projection C: Mind} \rightarrow \text{Brain}.$$

This connectionist modeling is concerned with mind as computing brain. Many properties of the human mind involve parallel processing. The *connectionism* is used for studies that model human thought and behavior in terms of networks of neuron like units working parallel and in distributed manner. This connectionist school views cognitive systems as a parallel distributed processing system. This brain style theory of processing takes its inspiration from the neural networks, which executes all operations simultaneously in a parallel manner.

McCulloch and Pitts formulated the neural network hypothesis: to psychology, however defined, specification of the network would contribute all that could be achieved in that field- even if the analysis were pushed to the ultimate psychic units or "psychons," for a psychon can be no less than the activity of a single neuron (McCulloch & Pitts 1943).

In this connectionist modeling, (1) the use of networks of active computing elements, with programs residing in the structure of interconnections, (2) massive parallelism, with no centralized control, (3) the encoding of semantic units either by single network units or by patterns of activity in a population of such units (Arbib 1987; Smolensky 1996).

From this assumption, we arrive two mathematical models for an artificial mind: the Turing machine for the symbolic approach and the neural network for the connectionist one (Fig.3).

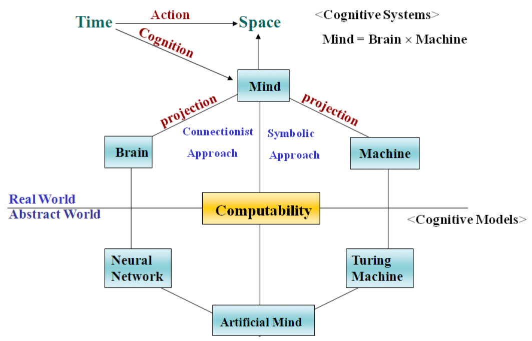

Fig.3 Structure of Cognitive Science

Turing Machine. Let be a finite set of symbols including Blank 0 and Stroke 1, and let $q_1, q_2, \ldots$ be symbols of states not in S. Then a Turing machine on S is a finite set of quintuples (q_i, s, t, Φ, q_j), where s and t are in S and Φ is one of the symbols R(move one right) or L(move one left), such that no two distinct quintuples have the same first two members. The symbol q_i stands for the state i. Formally, a Turing machine is a mapping TM such that for some natural number n.

$TM:\{0,1,2,\ldots,n\}\times\{0,1\}\rightarrow\{0,1\}\times\{L,R\}\times\{0,1,2,\ldots,n\}$.

Neural Network. Neural network is a finite set of quintuples (V, X, Y, E, g), where V is a finite set of neurons, $X\cap V=\emptyset$ is a set of inputs, $Y\subseteq V$ is a set of outputs, $E=V\times V$, $(V\cup X, E)$ is a weighted graph, and $g:V\rightarrow F$ is a neuron assignment function (F is the set of neuron functions). The connection is defined by (E, W), where W is a set of weights.

Since the 1980s, the debate concerning these two schools has been intensive and fierce. Yet, after the late of 1990s, implications of the debate indicate that no cognitive scientist could totally deny the complementary success of the two modeling.

Both models lead to the question of hybrid models in cognitive science. Rather, both are different but compatible approaches for understanding the mind in various forms (Holyoak & Spellman 1993; Dyer 1991).

For more advanced understanding as a whole, we need to consider cognition in terms of both connectionism and symbolism (Fig.4). Thus, the current cognitive science can be defined in terms of projections S and C as follows:

Projection S ∩ Projection C.

Quantum Cognitive Science

All the classical computing system can not be free from Gödel's incompleteness theorems. His incompleteness theorems imply the necessity of the non-deterministic quantum computation beyond the deterministic computation (Blaha 2005).

If we offer a new projection from all the classical models to the mind at the quantum level, then we construct a quantum cognitive science. How to capture the quantum computing model inherent in cognitive science is very interesting and challenging (Goertzel 1993). Given the current models in cognitive science, we are interested in how to gain some insights about the definition of quantum cognitive science from quantum computing.

Quantum computing is theoretically based on quantum system with finite dimensional Hilbert spaces, especially the state space of a qubit, $\mathbb{C}^2$. The quantum logic by G. Birkhoff and J. von Neumann is based on the lattice of closed subspaces of a Hilbert space. Since the distribution law does not hold in the quantum logic of $\mathbb{C}^2$, therefore, the quantum logic of $\mathbb{C}^2$ is different from that of $\mathbb{C}$, hence different from Turing machines equivalent to the first order logic (Dunn 2005).

Classical cognitive science can be defined in terms of projections as follows:

Projection S ∩ Projection C.

Then quantum cognitive science can be defined as follows:

Quantum Projection S ∩ Quantum Projection C.

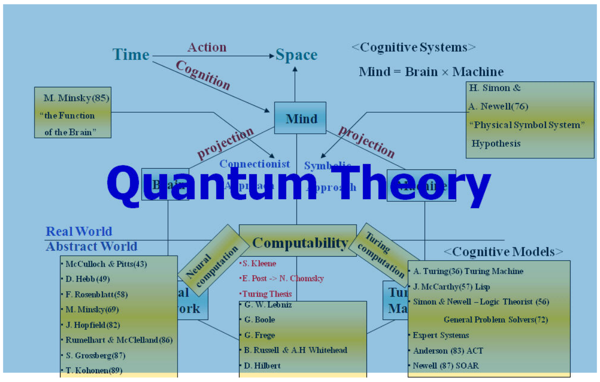

Fig.4 Quantum Cognitive Science

Can cognition be the results of quantum computing on the part of neurons? A new cover of logic has been developing ever since the discovery of quantum theory. A hypothesized quantum approach is also intended as architectures of mind (Stapp 2004; Hiley 1997; Penrose 1994; Goertzel 1994; Hodgson 1991; Lockwood 1989).

Proposition

1) mind = f(quantum physical reality)

2) quantum physical reality = g(mind),

for some mappings f and g.

This means:

1) mind is defined in some way by quantum physical reality

2) quantum physical reality is defined in some way by mind.

Kak (1992) suggests building a quantum neural computer for solving artificial intelligence problems. Quantum theoretical concepts are relevant to the understanding of the human mind. The non-computational aspects of cognition may be related to the non-computational processes which may be involved in the reduction of the wavefunction to macroscopic observables (Penrose 1994). Penrose also asserts that the microtubules serve as quantum information-processing devices, even though the microtubules are orders of magnitude too large. Who knows what the future of cognitive science will bring?

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