# A Temporal Diversity using Vector Perturbation

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## Abstract

In this paper, we propose a temporal diversity scheme based on the vector perturbation for a multiuser multipleinput multiple-output (MIMO) broadcasting system. Our method is inspired by the fact that precoding process of the vector perturbation designed to suppress the multiuser interference causes a reduction of the transmitted signal power. In order to boost up the transmit power, we employ a non-integer based vector perturbation together with temporal transmit diversity. We show from the simulation on  $10 \times 10$  multiuser MIMO system that the proposed method outperforms the epetition coded vector perturbation scheme as well as the standard vector perturbation.

## I. INTRODUCTION

In the multiuser multiple–input multiple–output (MIMO) system, a basestation with multiple antennas transmits independent messages to multiple mobile users, each with one or more receive antennas. As a practical method for achieving near sum capacity in multiuser MIMO channel [1]–[2], Hochwald et. al. proposed a method referred to as vector perturbation (VP) technique [3]. VP exploits a judiciously designed integer offset referred to as perturbation vector into the information vector to improve the effective signal–to–noise (SNR) of the system with no virtual overhead in the receiver.

While the VP provides considerable gain for each mobile user in the downlink, in many practical scenarios, further protection from the fading is needed to meet complicated requirements (packet error rate, throughput, coverage, etc.) of the future communication standard. Classical ways to deal with this problem include adjustment of modulation format or code rate reduction in error correction coding. Although these methods provide improvement in reliability at the expense of data rate loss, they may not be always applicable due to the lack of efficiency (e.g., adjusting modulation format from QPSK to BPSK) or cost caused by the decoder structure modification (e.g., rate 1/2 to rate 1/4). An alternative wayo deal with this issue is to employ the temporal diversity [4]. Indeed, by employing a repetition coding scheme which coherently combines multiple received signals, improvement in SNR proportional to the diversity order (number of repetition in transmission) can be achieved with minimal cost in the system.

The technique proposed in this paper is a temporal diversity strategy of the VP based on a deliberately designed precoding for two or more transmission periods. The key motivation of the proposed method lies on the fact that the VP, although it removes the multiuser interference, induces the reduction in the effective transmit power. By employing an aggressive perturbation strategy together with the temporal transmit diversity proposed method alleviates the transmit power reduction, which directly brings considerable gain in performance. From the simulation on realistic multiuser MIMO scenario, it is shown that the proposed method outperforms the repetition coded VP by a large margin.

The rest of this paper is organized as follows. In Section II, we present the proposed temporal diversity method. The performance results are provided in Section III and we conclude in Section IV.

# II. VP BASED TRANSMIT DIVERSITY

#### A. Relaxation of Integer based Perturbation

In this section, we propose a simple transmit diversity technique based on the VP that provides additional gain over the repetition coded VP. The repetition coding refers to an approach transmitting the same data vector over two consecutive symbol periods. By combining two (or more) received signals coherently, the repetition coding can achieve a time diversity gain proportional to the number of repetition [4]. In essence, our proposed method is distinct from the repetition coding in the sense that different symbols, i.e., the information vector and the perturbation vector, are being transmitted for two consecutive intervals.

To be specific, during an even transmission period (t = 2n, n = 0, 1,  $\cdot \cdot \cdot$ ), precoded vector of the original information u is transmitted, and during an odd period (t = 2n + 1, n = 0, 1,  $\cdot \cdot \cdot$ ), precoded version of perturbation vector  $\ell$  is transmitted (see Fig. 1). By eliminating the constraint that the perturbation vector should be a complex number based on the integer, the search space becomes much wider, and hence  $\gamma$  of the proposed scheme can be made much smaller than that of the standard VP. It is worth pointing out that although the perturbation vector  $\ell$  is made up of integers ( $\ell$  re,  $\ell$  im  $\in \mathbb{Z}^{K}$ ), most of them in real scenario are either 0 or  $\pm 1$  (the probability of  $\pm 2$  is extremely small). Hence, choosing  $\ell$  from the perturbation set U<sup>K</sup> = {-1, 0, 1}<sup>K</sup> provides virtually same performance as choosing  $\ell$  from  $\mathbb{Z}^{K}$ . Now, if elements of U are made smaller than unity, a reduction of  $\gamma$  can



Fig 1 The transmitter structure of the proposed transmit diversity method (dotted box includes additional block due to the proposed method).

be achieved but the key assumption of the VP is violated so that the modulo operation at the receiver will not work.

The proposed method addresses this issue by transmitting the perturbation vector  $\ell$  during the odd transmission period. In order to reduce the error probability in  $\ell$  vector transmission, it is desirable to use as simple modulation as possible. Considering that elements of perturbation vector are a complex number ( $\ell = \ell \operatorname{re} + j \ell \operatorname{im}$ ), QPSK becomes the simplest modulation format can be employed. In this respect, the desirable choice of the perturbation set becomes  $U^{K} = \{-\varepsilon, \varepsilon\}^{K}$ . Recalling that  $\ell$  information is being transmitted in the odd transmission period,  $\varepsilon$  should not necessarily be an integer. In fact, as supported by the simulation in Fig. 3,  $\varepsilon \sim 0.25$  achieves the minimum of average  $\chi$ , which is clearly better than the caseusing  $\varepsilon = 0$  and 1.

#### B. Temporal Diversity with Non-integer based VP

The transmitter architecture of the proposed method is shown in Fig. 1, respectively. In the even period (t = 2n), the precoded data vector is being transmitted

$$\mathbf{x}_{2n} = \frac{1}{\sqrt{\gamma}} \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \alpha \mathbf{I})^{-1} (\mathbf{u}_{2n} + \tau \ell_{2n})$$
(1)

where  $\gamma = \| H^H (HH^H + \alpha I)^{-1} (u_{2n} + \tau \ell_{2n}) \|^2$ .  $\ell 2n$  is selected to minimize  $\chi$  so that

$$\ell_{2n} = \arg\min_{\ell' \in \mathbf{U}^K} \gamma(\ell')$$
  
=  $\arg\min_{\ell' \in \mathbf{U}^K} \|\mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \alpha \mathbf{I})^{-1} (\mathbf{u}_{2n} + \tau \ell')\|^2.$ 
(2)

In searching  $\ell_{2n}$  from  $U^{K} = \{-0.25, 0.25\}^{K}$ , SE is being employed. The received signal vector y2n is approximately expressed as

$$\mathbf{x}_{2n+1} = \frac{1}{\sqrt{\gamma'}} \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \alpha \mathbf{I})^{-1} (\mathbf{u}_{2n+1} + \tau' \ell_{2n+1})$$
(3)

Since non-integer based perturbation vector  $\ell_{2n}$  is used, it is certain that  $\tau \ell_{2n}$  cannot be removed by the modulo operation. In the odd period (t = 2n+1), therefore,  $\ell_{2n}$  is mapped to the QPSK



Fig 2 Average as a function of  $\varepsilon$  for K = 10 and M = 10 system with 16-QAM modulation when U = {- $\varepsilon$ ,  $\varepsilon$ } is used as a perturbation set.

symbol vector  $(u_{2n}+1 = fqpsk(\ell_{2n}))$ , precoded by the VP, and then transmitted as depicted in Fig. 1. The transmitted signal is

$$\mathbf{x}_{2n+1} = \frac{1}{\sqrt{\gamma'}} \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \alpha \mathbf{I})^{-1} (\mathbf{u}_{2n+1} + \tau' \ell_{2n+1})$$
(4)

where  $\gamma' = \| H^H (HH^H + \alpha I)^{-1} (u_{2n+1} + \tau' \ell_{2n+1}) \|^2$ . Same as  $\ell_{2n}$  vector generation in even period, the sphere encoding is used to get  $\ell_{2n+1}$  and  $\gamma'$  as

$$\ell_{2n+1} = \arg \min_{\ell' \in \mathbf{U}^K} \gamma'(\ell')$$
  
= 
$$\arg \min_{\ell' \in \mathbf{U}^K} \|\mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \alpha \mathbf{I})^{-1} (\mathbf{u}_{2n+1} + \tau'\ell')\|^2.$$
(5)

Note that the perturbation set  $U^k$  in the odd period should be  $Z^K$  (in fact,  $U^K = \{-1, 0, 1\}^K$ ) since the detection of  $x_{2n+1}$  should rely on the modulo operation. Then the received signal vector is approximately

$$\mathbf{y}_{2n+1} = \frac{1}{\sqrt{\gamma'}} (\mathbf{u}_{2n+1} + \tau' \ell_{2n+1}) + \mathbf{n'}.$$
(6)

# IV. SIMULATION RESULTS AND DISCUSSION

In this section, we observe the performance of the proposed method along with the repetition coding and non-diversity family including the original VP [3], channel inversion and regularized channel inversion [5]. We consider 16 and 64-QAM transmission over 10 × 10 multiuser MIMO channel (i.e., 10 transmit antennas and 10 users) with Ravleigh fading (hij ~ CN(0, 1)). Fig. 3(a) provides symbol error rate (SER) performance for 16-QAM modulation. Due to the gain obtained from the diversity, we clearly observe that temporal diversity schemes outperform non-diversity schemes over the entire simulation range. While the repetition coding scheme provides approximately 3 dB gain over the standard VP, the proposed method brings additional 1.5 dB gain in all SNR regime under test. Fig. 3(b) show results when 64-QAM modulation is employed. Similar to the 16-QAM scenario, we can observe that the proposed method brings similar performance gain  $(\sim 1.5 \text{ dB gain from the repitition coded VP}).$ 

In this paper, we investigated the temporal diversity scheme based on VP for the multiuser MIMO system. Motivated by the fact that the reduction of  $\gamma$  offers an increase in SNR, we employed an aggressive strategy (non-integer perturbation vector together with temporal diversity) to achieve reduced  $\gamma$ . We have shown that the proposed scheme brings performance gain compared to the repetition coded VP with minimal computational overhead. Although our focus is only on the scenario where receiver has single antenna, extension to the scenario having multiple receiver antennas is possible by joint application of block diagonalization and VP or treating multiple receiver antennas as independent ones for distinct users.

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Fig 3 SER performance of  $10 \times 10$  multiuser MIMO systems with (a) 16–QAM and (b) 64–QAM modulation.