중주파수 응답해석을 위한 축소 기법

Model Order Reduction for Mid-Frequency Response Analysis

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Abstract

Most of the studies use model order reduction for low frequency (LF) response analysis due to their high computational efficiency. In LF response analysis, one of model order reduction, algebraic substructuring (AS) retains all LF modes when using the modal superposition. However, in mid-frequency (MF) response analysis, the LF modes make very little contribution and also increase the number of retained modes, which leads to loss of computational efficiency. Therefore, MF response analysis should consider low truncated modes to improve the computational efficiency. The current work is focused on improving the computational efficiency using a AS and a frequency sweep algorithm. Finite element simulation for a MEMS resonator array showed that the performance of the presented method is superior to a conventional method.

keywords : mid-frequency analysis, model order reduction, algebraic substructuring, MEMS resonator

1. Introduction

So far, a substructuring-based model order reduction including algebraic substructuring [Gao et al. 2008] has been developed to improve the efficiency in a linear dynamic analysis of large systems. Numerical techniques that consider low- and high-frequency mode truncations are also required for the mid-frequency response analysis. There have been two recently-introduced methods to compensate for the low- and high-truncations; one is the frequency sweep algorithm [Bennighof et al. 1998], and the other is the mode acceleration method [Qu 2001]. The convergence of frequency sweep algorithm was verified when it was applied to compensate for both the truncation errors [Ko et al. 2008].

In this work, the finite element models of practical resonator arrays were used for the investigation of performance of presented method. A general purpose finite element package ANSYS [ANSYS 2007] was adopted as a conventional method for comparison with the present method.

2. Mid-Frequency response analysis

The discretized model of a structure for a continuous single input and single output second order system can be written as

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$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = bu(t),\ \ y(t) = l^T x(t) \tag{1}$$

with the initial conditions $x(0) = x_0$ and $\dot{x}(0) = v_0$. Here, t is the time variable, is a state vector, N is the degree of freedom (DOF). $M, K, D \in \Re^{N \times N}$ are the system mass, stiffness and damping matrices respectively, where D is approximated by $\alpha M + \beta K$. For frequency response analysis of $[\omega_{\min}, \omega_{\max}]$ near a specific mid frequency mode, the frequency response function of the dynamic system can be given as

$$H(\omega) = l^T [\gamma_1 K^{\sigma} + \gamma_2 M]^{-1} b \tag{2}$$

where $\gamma_1 = \gamma_1(\omega) = 1 + i\omega\beta$, $\gamma_2 = \gamma_2(\omega, \sigma) = -\omega^2 + \sigma + i\omega(\alpha + \sigma\beta)$, and $K^{\sigma} = K - \sigma M$.

In algebraic substructuring (AS) among substructuring based model order reductions, first the transformation matrix L is obtained from the shifted eigensystem in the Craig Bampton form, and next the S matrix, which is composed of m substructure modes, is obtained. The subspace spanned by the columns of the matrix $A_m = L^{-1}S$ is called *AS subspace*. Projecting the frequency response function $H(\omega)$ of (2) onto the AS subspace yields

$$H_m(\omega) = l_m^T [\gamma_1 K_m^{\sigma} + \gamma_2 M_m]^{-1} b_m = l_m^T G_m^{-1}(\omega) b_m \tag{3}$$

where G_m is the dynamic matrix, $K_m^{\sigma} = A_m^T K^{\sigma} A_m$, $M_m = A_m^T M A_m$, $l_m = A_m^T l$, and $b_m = A_m^T b$. m is much smaller than N. The frequency response function of Eq. (3) is represented by the summation of $H_n(\omega)$ and $H_t(\omega)$, which are computed by p_n and p_t , respectively:

$$H_m(\omega) = H_n(\omega) + H_t(\omega) \tag{4}$$

From (3), mode superposition yields

$$H_n(\omega) = l_m^T p_n(\omega) = l_m^T \Phi_n (\gamma_1 \Theta_n^{\sigma} + \gamma_2 I)^{-1} \Phi_n^T b_m \tag{5}$$

where (Θ_n^{σ} , Φ_n) are the eigenpairs of the eigensystem of (K_m^{σ}, M_m). n is very small as compared to m. An iterative scheme, the frequency sweep algorithm (FS) is used for error compensation by

$$H_t(\omega) = l_m^T p_t^{\ell}(\omega),\ \ p_t^{\ell}(\omega) = p_t^{\ell-1}(\omega) + \frac{1}{\gamma_1}\left[(K_m^{\sigma})^{-1} - \Phi_n (\Theta_n^{\sigma})^{-1} \Phi_n^T\right] r_m^{\ell-1}(\omega) \tag{6}$$

The iteration (6) guarantees its convergence by satisfying the condition that the *contraction ratio* ξ is smaller than one when the cutoff values for the eigenvalues of the normal modes are determined by

$$\lambda_{\min}^{\sigma} = -d_{\max} / \xi \ \text{ and } \ \lambda_{\max}^{\sigma} = d_{\max} / \xi \ , \tag{7}$$

where $d(\omega, \sigma) = |-\gamma_2 / \gamma_1|$ and $d_{\max} = \max\{d(\omega_k, \sigma),\ 1 \le k \le n_f\}$, in which n_f is the number of sampling frequencies.

3. Results

The finite element model of a single resonator is constructed by brick elements with its order of 24,960 as shown in Fig. 1(a). From eigenvalue computation, the extensional wine glass mode, whose natural frequency is 638.6 MHz, is shown in Fig. 1(b), and the corresponding eigenvalue of the mode is the 281 th small eigenvalue. Frequency responses are computed at 201 sampling points.

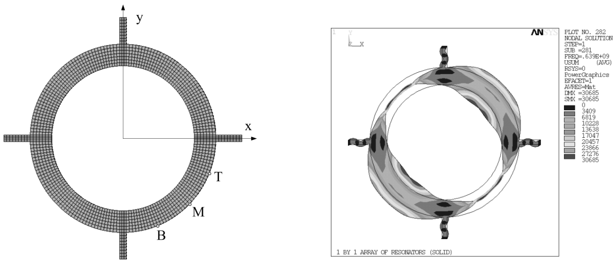

Figure 1 (a) Finite element model (b) its extensional wine glass mode, and (c) frequency response functions of the single resonator

Table 1 Performance comparison

	AS+FS	*Modal ANSYS*
m	469	
n	5	287
FS time	0.19 sec.	
Total time	10.34 sec.	162 sec.

According to Fig. 1(c), the frequency response functions of AS+FS and Modal ANSYS are clearly agree with those of Direct ANSYS. It is indicated in Table 1 that the time of FS iteration is negligible, and AS+FS uses 5 modes in 469 AS subspace instead of 287 modes in 24,960 FE subspace. Approximately 16 times more time was required for Modal ANSYS than for AS+FS.

A 4 by 4 resonator array is modeled, as shown in Fig. 2(a), the order of its finite element model is 371,280 and, resonant mode is represented in Fig. 2 (b). Frequency responses are computed at 201 sampling points.

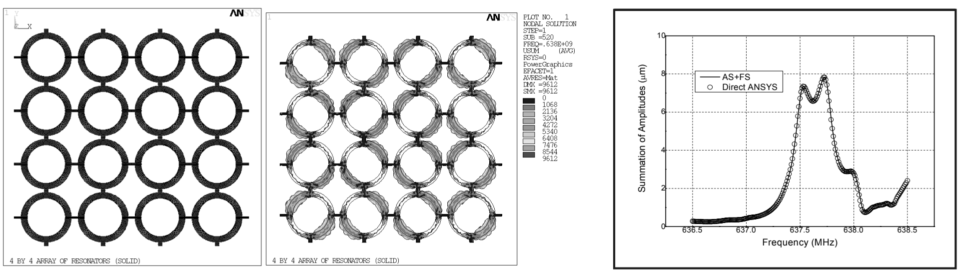

Figure 2 (a) Finite element model, (b) a resonant mode close to the extensional wine glass mode, and (c) Frequency response functions of the 4 by 4 resonator array

Table 2 Performance comparison

	AS+FS	*Modal ANSYS*
M	7,101	
N	90	4,335
FS time	7.29 sec.	
Total time	226.49 sec.	> 10 hours

According to Fig. 2(c), the frequency response functions of AS+FS are in good agreement with those of Direct ANSYS. It is indicated in Table 2 that the time of FS iteration is short, and AS+FS uses 90 modes in 7,101 AS subspace instead of 4,335 modes in 371,280 FE subspace. For this case, AS+FS spent less than a hundredth of the elapsed time of Modal ANSYS. We also found that the frequency response curve of 4 by 4 array has wider bandwidth than that of single resonator.

4. Conclusion

The performance of the presented method is superior to a conventional method through finite element simulation for a MEMS resonator array.

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