잡음 및 진동제어시스템을 위한 Filtered -x LMS 알고리즘

김수용* • 지석근*

군산대학교*

Filtered-x LMS Algorithm for noise and vibration control system

soo-yong kim* • suk-kun Jee*

Kunsan National University*

E-mail: ksy8910@kunsan.ac.kr

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Abstract

Filtered-x LMS algorithm maybe the most popular control algorithm used in DSP implementations of active noise and vibration control system. The algorithm converges on a timescale comparable to the response time of the system to be controlled, and is found to be very robust. If the pure tone reference signal is synchronously sampled, it is found that the behavior of the adaptive system can be completely described by a matrix of linear, time invariant, transfer functions. This is used to explain the behavior observed in simulations of a simplified single input, single output adaptive system, which retains many of the properties of the multichannel algorithm.

Key Words : Filtered-x LMS algorithm, noise and vibration control system, linear,transfer functions,properties

I. Introduction

1.1 Concept of active control of noise and vibration

Active control of noise and vibration has received much attention in recent research and industrial applications]. It's an approach based on a number of controlled "secondary" sources so that the field generated by these sources interferes destructively with the field caused by the original "primary" source. The extent of the destructive interference is possibly depends on the geometric arrangement of the primary and secondary sources(produced by controller) and their environment, and on the spectrum of the field produced by the primary source. It is often desirable to make this controller adaptive since the characteristics of the noise and vibration source and the environment are time varying, the frequency content, amplitude, phase, and sound velocity of the undesired noise are nonstationary . In order to construct a practical adaptive controller, some measurable error criterion must be defined which the controller is required to minimize. Although the minimization of total radiated power, under acoustic free field conditions, or total acoustic potential energy, for enclosed sound fields, have been proposed in theoretical formulations, to determine the best possible attenuation which can be achieved with an active control system, these quantities are generally not practically measurable. One error criterion which can be directly measured is the sum of the squares of the outputs of a number of sensors.

The signal processing problem is then to design an adaptive algorithm to minimize the sum of the squares of a number of sensor outputs by adjusting the magnitude and phases of the sinusoidal inputs to a number of secondary sources. Perhaps the most popular adaptive control algorithm used in DSP implementations of active noise and vibration control systems is the filteredX least-mean-square (LMS) algorithm. There are several reasons for this algorithm's popularity. First, it is well-suited to both broadband and narrowband control tasks, with a structure that can be adjusted according to the problem at hand. Second, it is easily described and understood, especially given the vast background literature on adaptive filters upon which the algorithm is based [14][15], Third, its structure and operation are ideally suited to the architectures of standard DSP chips, due to the algorithm's extensive use of the multiply/accumulate (MAC) operation. Fourth, it behaves robustly in the presence of physical modeling errors and numerical effects caused by finite-precision calculations. Finally, it is relatively simple to set up and tune in a real-world environment.

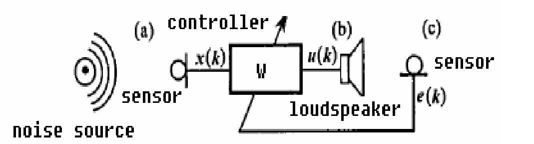

Fig 1 principle of the active noise control

II. Types of ANC Systems

Active noise control systems are based on one of two methods. Feedforward control is where a coherent reference noise input is sensed before it propagates past the canceling speaker. Feedback control is where the active noise controller attempts to cancel the noise without the benefit of an upstream reference input.

Feedforward ANC systems are the main techniques used today.

2.1 The Feedforward System

In applications where the primary noise is periodic (or nearly periodic) and is produced by rotating or reciprocating machines, the input microphone can be replaced by a nonacoustic sensor such as a tachometer, an accelerometer, or an optical sensor. This replacement eliminates the problem of acoustic feedback.

The block diagram of a narrowband feedforward active noise control system is shown in Figure 1. The nonacoustic sensor signal is synchronous with the noise source and is used to simulate an input signal that contains the fundamental frequency and all the harmonics of the primary noise. This type of system controls harmonic noises by adaptively filtering the synthesized reference signal to produce a canceling signal. In many cars, trucks, earth moving vehicles, etc., the revolutions per minute (RPM) signal is available and can be used as the reference signal. An error microphone is still required to measure the residual acoustic noise. This error signal is then used to adjust the coefficients of the adaptive filter.

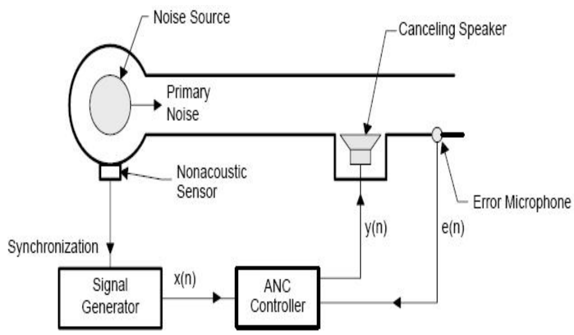

Fig 2. Narrowband Feedforward ANC System

Generally, the advantage of narrowband ANC systems is that the nonacoustic sensors are insensitive to the canceling sound, leading to very robust control systems. Specifically, this technique has the following advantages:

Environmental and aging problems of the input microphone are automatically eliminated. This is especially important from the engineering viewpoint, because it is difficult to sense the reference noise in high temperatures and in turbulent gas ducts like an engine exhaust system.

The periodicity of the noise enables the causality constraint to be removed. The noise waveform frequency content is constant. Only adjustments for phase and magnitude are required. This results in more flexible positioning of the canceling speaker and allows longer delays to be introduced by the controller.

The use of a controller-generated reference signal has the advantage of selective cancellation; that is, it has the ability to control each harmonic independently.

It is necessary to model only the part of the acoustic plant transfer function relating to the harmonic tones. A lower-order FIR filter can be used, making the active periodic noise control system more computationally efficient.

The undesired acoustic feedback from the canceling speaker to the input microphone is avoided.

2.2 The Feedback ANC System

Feedback active noise control was proposed by Olson and May in 1953 [6]. In this scheme, a microphone is used as an error sensor to detect the undesired noise. The error sensor signal is returned through an amplifier (electronic filter) with magnitude and phase response designed to produce cancellation at the sensor via a loudspeaker located near the microphone. This configuration provides only limited attenuation over a restricted frequency range for periodic or band-limited noise. It also suffers from instability, because of the possibility of positive feedback at high frequencies. However, due to the predictable nature of the narrowband signals, a more robust system that uses the error sensor's output to predict the reference input has been developed (see Figure 2). The regenerated reference input is combined with the narrowband feedforward active noise control system.

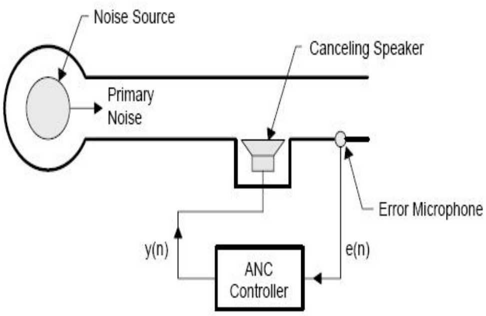

Fig 3. Feedback ANC System

One of the applications of feedback ANC recognized by Olson is controlling the sound field in headphones and hearing protectors. In this application, a system reduces the pressure fluctuations in the cavity close to a listener's ear. This application has been developed and made commercially available.

2.3 The Multiple-channel ANC System

Many applications can display complex modal behavior. These applications include:

Active noise control in large ducts or enclosures

Active vibration control on rigid bodies or structures with multiple degrees of freedom

Active noise control in passenger compartments of aircraft or automobiles.

When the geometry of the sound field is complicated, it is no longer sufficient to adjust a single secondary source to cancel the primary noise using a single error microphone. The control of complicated acoustic fields requires both the exploration and development of optimum strategies and the construction of an adequate multiple-channel controller. These tasks require the use of a multiple-input multiple-output adaptive algorithm. The general multiple-channel ANC system involves an array of sensors and actuators. A block diagram of a multiple-channel ANC system for a three-dimensional application is shown in Figure 3.

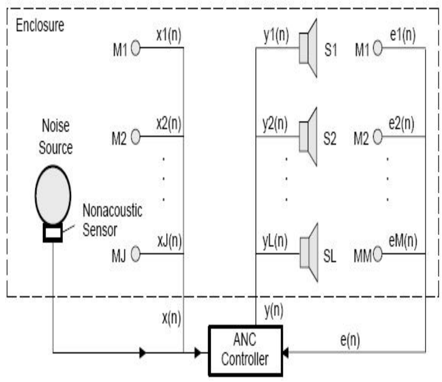

Fig 4. Multiple-channel ANC System for a 3-D Enclosure

III. Filtered-x LMS Algorithm

3.1 Filtered-x LMS algorithm

For multiple-input-multiple-output systems the multiple error lms algorithm is a generation of filtered-x LMS algorithm.

Assume:

There are L sensors and M actuators, and $L \geq M$, then

$$e_l(n) = d_l(n) + \sum_{m=1}^{M}\sum_{j=0}^{J-1} c_{lmj} \cdot \sum_{i=0}^{I-1} w_{mi}(n-j)x(n-i-j) \quad (1)$$

Where

$e_l(n)$: the sampled output of the lth error sensor;

$d_l(n)$: the desired signal from this sensor

$w_{mi}(n)$: the ith coefficient of an adaptive FIR controller at the nth sample;

c_{lmj}: modeling the transfer function between this input and the output of the lth sensor as a jth-order FIR filter, c_{lmj} is its jth coefficient.

So, the total error can be defined as

$$J = E\left\{\sum_{l=1}^{L} e_l^2(n)\right\} \quad (2)$$

Where $E\{\cdot\}$ denotes an expectation value.

If the reference signal x (n) is at least partly correlated with each $d_l(n)$,it is possible to reduce the value of J due to the primary source alone, by driving the secondary sources with a filtered version of the reference signal, as indicated above.

It is physically clear that the total error will be a quadratic function of each of these filterscoefficients (although this is also demonstrated analytically below). The optimum set of filter coefficients required to minimize J may thus be evaluated adaptively using gradient descent methods. The differential of the total error with respectto one coefficient is

$$\frac{\partial J}{\partial w_{mi}} = 2E\left\{\sum_{l=1}^{L} e_l(n)\frac{\partial e_l(n)}{\partial w_{mi}}\right\} \quad (3)$$

Assuming for the moment that each $w_{mi}(n)$ is time invariant, differentiating (1) with respect to one of these coefficients gives

$$\frac{\partial e_l(n)}{\partial w_{mi}} = \sum_{j=0}^{J-1} c_{lmj}x(n-i-j) \quad (4)$$

This sequence is the same as the one which would be obtained at the lth sensor if the reference signal, delayed by i samples, were applied to the mth actuator. Let this be equal to $r_{lm}(n-i)$, a filtered reference.

If each coefficient is now adjusted at every sample time by an amount proportional to the negative instantaneous value of the gradient, a modified form of the well-known LMS algorithm is produced [12]

$$w_{mi}(n+1) = w_{mi}(n) - \alpha\sum_{l=1}^{L} e_l(n)r_{lm}(n-i) \quad (5)$$

Where

α: the convergence coefficient.

The assumption of time invariance in the filter coefficients is equivalent, in practice, to assuming that the filter coefficients w_{mi} change only slowly compared to the timescale of the response of the system to be controlled. This timescale is defined by the values of the coefficients c_{lmj}.

3.2 Analysis of the algorithm on time domain

In order to analytically demonstrate the shape of the error surface, and so determine the optimum, Wiener, set of filter coefficients, it is convenient to consider the case in which the filter coefficients are exactly time invariant. In this case, (1) may be written

$$e_l(n) = d_l(n) + \sum_{m=1}^{M}\sum_{i}^{I-1} w_{mi} \cdot \sum_{j}^{J-1} c_{lmi}x(n-i-j)$$

$$= d_l(n) + \sum_{m=1}^{M}\sum_{i=0}^{I-1} w_{mi}r_{lm}(n-i) \quad (6)$$

where the filtered reference signal $r_{lm}(n)$ is defined as above. This equation may be written as

$$e_l(n) = d_l(n) + r_l^T w \qquad (7)$$

Where

$$r_l^T = [r_{l1}(n), r_{l1}(n-1), \cdots r_{l1}(n-I+1), \vdots r_{l2}(n), r_{l2}(n-1), \cdots r_{l2}(n-I+1), \vdots r_{l3}(n), \cdots \vdots r_{lM}(n), \cdots r_{lM}(n-I+1)]$$

$$w^T = [w_{10}, w_{11}, \cdots w_{1I-1} \vdots w_{20}, w_{21}, \cdots w_{2I-1} \vdots w_{30}, \cdots\cdots \vdots w_{M0}, w_{M1}, \cdots w_{MI-1}]$$

So if

$$e^T = [e_1(n), e_2(n), \cdots e_L(n)] \quad \text{and}$$

$$d^T = [d_1(n), d_2(n), \cdots d_L(n)]$$

Then $e = d + Rw$ (8)

Where

$$R^T = [r_1, r_2, r_3, \cdots r_L].$$

The error criterion can now be written as

$$J = E\left\{\sum_{l=1}^{L} e_l^2(n)\right\} = E\{e^T e\} = E\{d^T d\} + 2w^T E\{R^T d\} + w^T E\{R^T R\} w \qquad (9)$$

The quadratic natureof the error surface can now be clearly identified, and it can be confirmed that the surface has a unique global minimum by examining the positive definiteness of $E\{R^T R\}$. By setting the differential of this expression with respect to w to zero, the optimum Wiener set of coefficients may be obtained as

$$w_{opt} = -[E\{R^T R\}]^{-1} E\{R^T d\} \qquad (10)$$

This set of filter coefficients gives a minimum error criterion equal to

$$J_{\min} = J_0 - E\{d^T R\}[E\{R^T R\}]^{-1} E\{R^T d\} \qquad (11)$$

Where $J_0 = E\{d^T d\}$ is the value of the error criterion with no control applied.

3.3. analysis of the algorithm on the frequency domain

When the reference signal is a pure tone, it is more convenient to use a frequency domain analysis to determine the optimum set of filters which minimizes the sum of the squared errors. In this case, the amplitude and phase of each signal in the steady state can be described by a complex number, thus,

$$E_l(\omega_0) = D_l(\omega_0) + \sum_{m=1}^{M} C_{lm}(\omega_0) W_m(\omega_0) \qquad (12)$$

where

ω_0: the frequency of the reference signal,

E_l: represents the complex response of the l th secondary source,

D_l: the response due to the primary source alone,

W_m: the complex output of the m th secondary source, which is the same as the complex response of the m th filter in the controller;

C_{lm}: the complex response of the l th sensor to the m th source at the frequency ω_0.

This may be written as

$$E = D + CW \qquad (13)$$

Where

$$E^T = [E_1, E_2, \cdots E_L]$$

$$D^T = [D_1, D_2, \cdots D_L]$$

$$W^T = [W_1, W_2, \cdots W_L]$$

$$C = \begin{pmatrix} C_{11} & \cdots & C_{1M} \\ \vdots & \ddots & \vdots \\ C_{L1} & \cdots & C_{LM} \end{pmatrix}$$

The explicit dependence on ω_0 has been dropped for convenience.The error criterion this case is the sum of the moduli of the individual errors which may be written as

$$J = E^H E = D^H D + W^H C^H D + D^H CW + W^H C^H CW \qquad (14)$$

In which the superscript H denotes the Hermitian transpose.

By setting the differential of J with respect to the real and imaginary parts of W to zero, we obtain the optimum responses of the filters:

$$W_{opt} = -[C^H C]^{-1} C^H D \qquad (15)$$

which gives a minimum error criterion of

$$J_{\min} = D^H [I - C[C^H C]^{-1} C^H] D \qquad (16)$$

This frequency domain analysis has been used to calculate the optimum filter responses and the minimum total error for the pure tone simulations described in [16]. These are found to be close to the steady-state results of this simulation of the adaptive time domain algorithm.

Although these formulations allow the optimum steady state filter coefficients to be evaluated, they cannot properly describe the convergence properties of the algorithm. This is because steady state assumptions are made in both the time domain and frequency domain formulations, and if the controller adapts on a timescale significantly smaller than the delays in the system to be controlled, then these assumptions will be violated.

Ⅳ. CONCLUSION

In this paper, filtered-x LMS algorithm used for single channel as well as multichannel active control of noise and vibration system has been presented. The analysis on the multiple error LMS algorithm which is the generation of the single channel filtered-x LMS algorithm involved the derivation of the algorithm with the assumption that the adaptive filters were only varying slowly compared to the timescale of the response of the system to be controlled. However, simulations of the algorithm using a sinusoidal reference, and a practical implementation in an active sound control application has shown that the algorithm is able to converge in a time comparable to the response time of the system to be controlled. The simulations of the algorithm also indicate that the total error converged to a value close to the optimum least mean sum of squares solution, and that it was robust to errors made in the assumed response of the system to be controlled and to uncorrelated measurement noise.

Similar behavior is also shown by a simplified, single input, single output version of the algorithm, which corresponds tothe filtered x LMS algorithm [9]. The pole positions of the equivalent transfer function, derived using the approach of Glover [14], can, however, be easily evaluated in this case. These can be used to analytically derive an expression for the optimum convergence coefficient, which, in this case, agrees well with computer simulations, and is approximately equal to the reciprocal of the delay in the system to be controlled, measured in samples. The equivalent transfer function can also be used to analytically demonstrate that there is a ±90″ phase condition on the estimate of the system response in the limit of slow adaption.

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