ITERATIVE FACTORIZATION APPROACH TO PROJECTIVE RECONSTRUCTION FROM UNCALIBRATED IMAGES WITH OCCLUSIONS

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ABSTRACT

This paper addresses the factorization method to estimate the projective structure of a scene from feature (points) correspondences over images with occlusions. We propose both a column and a row space approaches to estimate the depth parameter using the subspace constraints. The projective depth parameters are estimated by maximizing projection onto the subspace based either on the Joint Projection matrix (JPM) or on the the Joint Structure matrix (JSM). We perform the maximization over significant observation and employ Tardif's Camera Basis Constraints (CBC) method for the matrix factorization, thus the missing data problem can be overcome. The depth estimation and the matrix factorization alternate until convergence is reached. Result of Experiments on both real and synthetic image sequences has confirmed the effectiveness of our proposed method.

Keywords: factorization, subspace method, projective reconstruction, missing data

1. INTRODUCTION

The factorization approach is an attractive method for recovering the structure from motion which has many interesting applications. The projective factorization initially proposed by Sturm and Triggs [1–3] enables us to estimate the joint projection matrix (JPM) and the joint structure matrix (JSM) simultaneously, in which framework the measurement matrix containing consistent set of the projective depth estimated by means of the fundamental matrix and the epipoles is factored to obtain projective structure. Among various method [4] subsequently elaborated by many others the subspace based method [5–7] is an extension of [1–3] to utilize whole set of the image observation for estimating the depth parameters. The most of existing projective factorization methods employ the singular value decomposition (SVD) for the matrix factorization, and hence they inherit simplicity and numerical stability from it.

Despite its attractive features, however, there exists very important issue that should be solved for practical use, i.e. missing data problem. If some measurements are missing the SVD method can never be applied directly, therefore the missing data problem has been remaining one of the most persistent difficulty since the factorization approach was initially proposed. Furthermore, this problem of course creates difficulties for the depth estimation because, especially in the subspace approach, the depth parameter is estimated by maximizing projection of the observation onto subspace.

In this paper we propose a subspace based approach to reconstruct the JPM and the JPM in the face of uncalibrated images with occlusions. Our proposed method alternates the depth estimation and the subspace computation until convergence is reached. For the former we propose two methods: a column space approach based on the JPM and a row space approach based on the JSM. We show how the subspace constrains are imposed on incomplete data to estimate the projective depth. In our formulation the maximization to enforce the constraints is performed over the significant observation, hence the missing data problem can be overcome. For the latter, the subspace computation, we reformulate Camera Basis Constrains proposed by Tardif et al. [8], where they are imposed for *gluing* partial reconstructions, and the JPM is estimated from the design matrix constructed by integrating the constraints. We present a performance evaluation among both of the proposed methods and the bundle adjustment [4], and the results show that our method yields practical quality. We also compare both of the column space approach and row space approach, and the result shows that the latter is more robust to the choice of image coordinate system.

The rest of this paper proceeds as follows. After a brief review of the Sturm/Triggs factorization in Section 2, Section 3 presents the formulation of depth estimation as a maximization of projection onto subspace. Section 4 shows how this subspace can be estimated by means of CBC translated from partial reconstructions. Experimental results and brief discussion are presented in Section 5, followed by conclusion in Section 6.

2. BACKGROUND

Consider a scene consisted of n 3D points $\boldsymbol{X}_j = [X_j\ Y_j\ Z_j\ 1]^T$, $j = 1, \ldots, n$ observed m uncalibrated cameras with 3×4 projection matrix $\boldsymbol{P}_i$, $i = 1, \ldots, m$. Under the perspective projection $\boldsymbol{X}_j$ are mapped to images $\boldsymbol{x}_{ij} = [u_{ij}\ v_{ij}\ 1]^T$, and the relation is written by matrix equation

$$\boldsymbol{W} = \begin{bmatrix} \lambda_{11}\boldsymbol{x}_{11} & \ldots & \lambda_{1n}\boldsymbol{x}_{1n} \\ \vdots & \ddots & \vdots \\ \lambda_{m1}\boldsymbol{x}_{m1} & \ldots & \lambda_{mn}\boldsymbol{x}_{mn} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_1 \\ \vdots \\ \boldsymbol{P}_m \end{bmatrix} \begin{bmatrix} \boldsymbol{X}_1 & \ldots & \boldsymbol{X}_n \end{bmatrix} = \boldsymbol{P}\boldsymbol{X}, \quad (1)$$

where λ_{ij} is non-zero scale factor and called projective depth. In the following we refer the matrix $\boldsymbol{P}$ and $\boldsymbol{X}$ as the JPM and the JSM respectively. Since the right hand side of the above equation has at most rank 4 once we estimate a consistent set of the depth λ_{ij} the rescaled measurement matrix $\boldsymbol{W}$ is factorized into a product of $3m \times 4$ JPM and $4 \times n$ JSM. One practical factorization method for rank deficient matrix is the SVD because it is not only numerically stable but also, in the presence of noise, can estimate best low rank approximation so that following *algebraic error* is minimized:

$$\|\boldsymbol{W} - \boldsymbol{P}\boldsymbol{X}\|_F^2 = \sum_i^m \sum_j^n \left\|\lambda_{ij}\boldsymbol{x}_{ij} - \boldsymbol{P}_i\boldsymbol{X}_j\right\|^2. \tag{2}$$

In general, there is no prior knowledge about the depth, thus projective factorization is consisted two main tasks: the depth estimation and the subspace computation.

3. SUBSPACE METHOD

This section describes the proposed column and row space approach for estimating the depth parameter using the subspace constraint, which are derived from the fact that the column and the row vector of the measurement matrix scaled by the projective depth must lie in four dimensional subspace spanned by the column vector of the JPM and the row vector of the JSM respectively. The derivation of the depth estimation method is roughly as follows. It starts from the assumption that JPM and JSM is approximately estimated. Then the error function is rearranged so that it depends on the projective depth and either the JPM or the JSM. Finally the depth parameters can be obtained from the solution of minimization of the rearranged error function. Our formulation is equivalent to [7] in the complete data case where the SVD is employed for the subspace computation.

3.1 Column space approach

We want to estimate a consistent set of the depth parameters for minimizing algebraic error (2). To begin with, we express the j-th column vector of the scaled measurement matrix as

$$\boldsymbol{P}\boldsymbol{X}_j = \boldsymbol{U}_j\boldsymbol{\mu}_j, \qquad (j = 1, \ldots, n) \tag{3}$$

where $\boldsymbol{U}_j = \mathrm{diag}\left[\boldsymbol{x}_{ij}\right]$, $\boldsymbol{\mu}_j = [\lambda_{1j},\ \lambda_{2j},\ \ldots, \lambda_{mj}]^T$. The above equation denotes the fact called the subspace constraint the vector $\boldsymbol{U}_j\boldsymbol{\mu}_j$ must lie in the column space of the JPM. The minimization of the algebraic error (2) is equivalent to the minimization of the error between both sides of (3):

$$\min_{\boldsymbol{P}} \min_{\mu_j} \sum_j \left\|\boldsymbol{U}_j\boldsymbol{\mu}_j - \boldsymbol{P}\boldsymbol{X}_j\right\|^2. \tag{4}$$

If we know $\boldsymbol{P}$ the least squares solution $\boldsymbol{X}_j$ for the above minimization problem is given by

$$\boldsymbol{X}_j = \left(\boldsymbol{P}^T\boldsymbol{P}\right)^{-1}\boldsymbol{P}^T\boldsymbol{U}_j\boldsymbol{\mu}_j. \tag{5}$$

To enforcing the subspace constraint, instead of minimizing the algebraic error (4), by using this $\boldsymbol{X}_j$ we try to minimize the residue of the projection of $\boldsymbol{U}_j\boldsymbol{\mu}_j$ onto $\boldsymbol{P}$. Substituting (5) into (4) we have

$$\min_{\mu_j} \sum_j \left\|\left(\boldsymbol{I} - \boldsymbol{P}\left(\boldsymbol{P}^T\boldsymbol{P}\right)^{-1}\boldsymbol{P}^T\right)\boldsymbol{U}_j\boldsymbol{\mu}_j\right\|^2, \quad \text{subject to } \left\|\boldsymbol{U}_j\boldsymbol{\mu}_j\right\| = 1, \tag{6}$$

where the additional constraint is imposed to avoid trivial solution so that all $\boldsymbol{\mu}_j$ equal to zero. Since the objective function of (6) is projection of $\boldsymbol{U}_j\boldsymbol{\mu}_j$ onto the subspace orthogonal to the column space of $\boldsymbol{P}$, minimizing it is equivalent to maximizing the projection onto the column space of $\boldsymbol{P}$, we have

$$\max_{\mu_j} \sum_j \left\|\left(\boldsymbol{P}\left(\boldsymbol{P}^T\boldsymbol{P}\right)^{-1}\boldsymbol{P}^T\right)\boldsymbol{U}_j\boldsymbol{\mu}_j\right\|^2, \quad \text{subject to } \left\|\boldsymbol{U}_j\boldsymbol{\mu}_j\right\| = 1. \tag{7}$$

The main reason to adopt the formulation (7) is that, by means of SVD, it can be solved very efficiently (See [7]).

For the case of missing data, we consider the reduced objective function which contains significant observations only. Consider the situation in which the point $\boldsymbol{X}_j$ is observed some subset (say $m_j \leq m$) of the cameras. Denoting the $3m_j \times 4$ JPM consisting of the subset $\widetilde{\boldsymbol{P}}_j$ and corresponding observation $\widetilde{\boldsymbol{U}}_j$, maximization for reduced objective function is expressed as

$$\max_{\widetilde{\mu}_j} \sum_j \left\|\left(\widetilde{\boldsymbol{P}}_j\left(\widetilde{\boldsymbol{P}}_j^T\widetilde{\boldsymbol{P}}_j\right)^{-1}\widetilde{\boldsymbol{P}}_j^T\right)\widetilde{\boldsymbol{U}}_j\widetilde{\boldsymbol{\mu}}_j\right\|^2, \quad \text{subject to } \left\|\widetilde{\boldsymbol{U}}_j\widetilde{\boldsymbol{\mu}}_j\right\| = \sqrt{\frac{m_j}{m}}, \tag{8}$$

where $\widetilde{\boldsymbol{\mu}}_j$ is the depth for the available observation. Notice that the constraint imposed on the scale of the depth parameters is replaced so that the number of views where the point is visible is reflected to the weighting for the observations.

3.2 Row space approach

In contrast to the column space approach described in the preceding section the row space approach proposed in this section operates in the subspace spanned by JSM. The minimization of the algebraic error which is correspond to (4) can be expressed, by introducing the matrix

$$\boldsymbol{V}_{(1,i)} = \mathrm{diag}\left[u_{ij}\right], \quad (j = 1, \ldots, n), \tag{9}$$

$$\boldsymbol{V}_{(2,i)} = \mathrm{diag}\left[v_{ij}\right], \quad (j = 1, \ldots, n), \tag{10}$$

$$\boldsymbol{V}_{(3,i)} = \boldsymbol{I}_{n\times n}, \tag{11}$$

as follows:

$$\min_{\boldsymbol{X}} \min_{\nu_i} \sum_i \sum_k \left\|\boldsymbol{V}_{(k,i)}\boldsymbol{\nu}_i - \boldsymbol{P}_{(k,i)}\boldsymbol{X}\right\|^2, \tag{12}$$

where $\boldsymbol{\nu}_i = [\lambda_{i1},\ \lambda_{i2},\ \ldots, \lambda_{in}]^T$ and $\boldsymbol{P}_{(k,i)}$, $k = 1, 2, 3$ are the depth parameter and the k-th row of $\boldsymbol{P}_i$ respectively. By means of $\boldsymbol{X}^T\left(\boldsymbol{X}\boldsymbol{X}^T\right)^{-1}$, the pseudo inverse of $\boldsymbol{X}$, we can eliminate $\boldsymbol{P}_i$ from the above equation in the same manner as (6) :

$$\min_{\boldsymbol{\nu}_i} \sum_i \sum_k \left\| \left(\boldsymbol{I} - \boldsymbol{X}^T \left(\boldsymbol{X}\boldsymbol{X}^T \right)^{-1} \boldsymbol{X} \right) \boldsymbol{V}_{(k,i)} \boldsymbol{\nu}_i \right\|^2, \quad \text{subject to} \sum_k \left\| \boldsymbol{V}_{(k,i)} \boldsymbol{\nu}_i \right\| = 1. \tag{13}$$

Since (13) cannot deal with missing data directly we derive depth estimation formula based on the subspace constraint for overcoming it in a similar way as the preceding section. For constructing reduced objective function we assume $n_j, (\leq n)$ points are visible in the i-th image and define the $4 \times n_j$ partial JSM $\widetilde{\boldsymbol{X}}_i$ and its observation $\widetilde{\boldsymbol{V}}_{k,i}$. Substituting them to (13) we may have reduced objective function about significant depth $\widetilde{\boldsymbol{\nu}}_i$ desired here, however, there also exists maximization problem equivalent to it. Consequently, after some algebraic manipulation, the depth $\widetilde{\boldsymbol{\nu}}_i$ is estimated as follows:

$$\max_{\widetilde{\boldsymbol{\nu}}_i} \sum_i \sum_k \left\| \left(\widetilde{\boldsymbol{X}}_i^T \left(\widetilde{\boldsymbol{X}}_i \widetilde{\boldsymbol{X}}_i^T \right)^{-1} \widetilde{\boldsymbol{X}}_i \right) \widetilde{\boldsymbol{V}}_{(k,i)} \widetilde{\boldsymbol{\nu}}_i \right\|^2, \quad \text{subject to} \sum_k \left\| \widetilde{\boldsymbol{V}}_{(k,i)} \widetilde{\boldsymbol{\nu}}_i \right\| = \sqrt{\frac{n_i}{n}}. \tag{14}$$

Roughly speaking, the depth estimation formula (13) is regarded as dual to (6), and hence, ideally at least, both of them should have almost same behavior except for the computational cost depending on the size of the design matrix. Nevertheless it should be pointed out that the estimation using (6) depends on the choice of the coordinate systems in the images while (13) is independent from it (proof is found in [6]). We will experimentally show the coordinate dependency of the column space approach (6) (see below).

4. FACTORIZATION

The subspace constraints derived in the above section enable us, by using bilinear approach, to estimate the consistent depth parameters from either of JPM or JSM. The bilinear approach alternates the depth estimation and subspace (i.e. JPM or JSM) computation until convergence is reached, therefore we need to compute it from the incomplete observations for dealing with image sequence with occlusion. To do this we impose the CBC (camera basis constraints) proposed by Tardif et al. [8], and solve it by using the generalized eigen approach proposed by the authors [9].

The subspace computation using the CBC method runs as follows. A sufficient set of the basis constrains translated from the partial reconstructions from the complete sub-blocks of the measurement matrix is integrated to a single design matrix. Then JPM is estimated to solve the generalized eigen problem of the design matrix. If we adopt the row space approach JSM is linearly estimated by means of intersection.

4.1 CBC method

Suppose l complete sub-blocks without missing data are selected for computing partial reconstructions. The k-th sub-block $\boldsymbol{W}_k$ is factorized, by using the SVD, to give k-th partial reconstruction:

$$\boldsymbol{W}_k = \boldsymbol{A}_k \boldsymbol{\Sigma}_k \boldsymbol{B}_k^T, \tag{15}$$

where $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$ are candidate of a partial JPM and a partial JSM respectively. By introducing $\boldsymbol{\Pi}_k$ for amputating the projection matrix whose index is not within the sub-block $\boldsymbol{W}_k$, the partial reconstruction $\boldsymbol{A}_k$ can be related to the JPM:

$$\boldsymbol{A}_k \boldsymbol{Z}_k = \boldsymbol{\Pi}_k \boldsymbol{P}, \tag{16}$$

where $\boldsymbol{Z}_k$ is a projective transformation for mapping $\boldsymbol{A}_k$ to the global coordinate system. The whole set of the equation (16) called *camera basis constrains* imposes constraints on the globally consistent $\boldsymbol{P}$ and $\boldsymbol{Z}_k$, which are estimated by solving

$$\min_{\boldsymbol{P},\boldsymbol{Z}_k} \sum_{k=1}^{l} \|\boldsymbol{\Pi}_k \boldsymbol{P} - \boldsymbol{A}_k \boldsymbol{Z}_k\|_F^2 = \sum_{k=1}^{l} \left\| \begin{bmatrix} \boldsymbol{\Pi}_k & -\boldsymbol{A}_k \end{bmatrix} \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{Z}_k \end{bmatrix} \right\|_F^2. \tag{17}$$

This method estimates $\boldsymbol{P}$ and $\boldsymbol{Z}_k$ simultaneously [8], however, the size of the design matrix grows proportionally to the number of the constraints. Furthermore the estimation of the aligning matrix $\boldsymbol{Z}_k$ is meaningless because, for the subspace computation, we only need the JPM. Thus we eliminate $\boldsymbol{Z}_k$ to reduce the number of the unknowns and the size of the design matrix by means of pseudo inverse of $\boldsymbol{A}_k$. Notice that since $\boldsymbol{A}_k$ is a part of an orthogonal matrix, the pseudo inverse of it is equal to $\boldsymbol{A}_k^T$. Substituting $\boldsymbol{Z}_k = \boldsymbol{A}_k^T \boldsymbol{\Pi}_k \boldsymbol{P}$ the minimization problem (17) is written by

$$\min_{\boldsymbol{P}} \sum_{k=1}^{l} \left\| \left(\boldsymbol{I} - \boldsymbol{A}_k \boldsymbol{A}_k^T \right) \boldsymbol{\Pi}_k \boldsymbol{P} \right\|_F^2, \quad \text{subject to} \sum_{k=1}^{l} \|\boldsymbol{\Pi}_k \boldsymbol{P}\|_F = 1, \tag{18}$$

where constraint is added for avoiding meaningless solution (say $\boldsymbol{P}$ equal to zero). Again we can obtain the equivalent maximization problem:

$$\max_{\boldsymbol{P}} \sum_{k=1}^{l} \left\| \left(\boldsymbol{A}_k \boldsymbol{A}_k^T \right) \boldsymbol{\Pi}_k \boldsymbol{P} \right\|_F^2, \quad \text{subject to} \sum_{k=1}^{l} \|\boldsymbol{\Pi}_k \boldsymbol{P}\|_F = 1. \tag{19}$$

The JPM $\boldsymbol{P}$ is obtained by solving the generalized eigen problem of following matrices:

$$\boldsymbol{M} = \sum_{k=1}^{l} \begin{bmatrix} \boldsymbol{A}_{k,1}\boldsymbol{A}_{k,1}^T & \boldsymbol{A}_{k,1}\boldsymbol{A}_{k,2}^T & \cdots & \boldsymbol{A}_{k,1}\boldsymbol{A}_{k,m}^T \\ \boldsymbol{A}_{k,2}\boldsymbol{A}_{k,1}^T & \boldsymbol{A}_{k,2}\boldsymbol{A}_{k,2}^T & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{A}_{k,m}\boldsymbol{A}_{k,1}^T & \boldsymbol{A}_{k,m}\boldsymbol{A}_{k,2}^T & \cdots & \boldsymbol{A}_{k,m}\boldsymbol{A}_{k,m}^T \end{bmatrix}, \tag{20}$$

$$\boldsymbol{\Pi}^T \boldsymbol{\Pi} = \sum_{k=1}^{l} \boldsymbol{\Pi}_k^T \boldsymbol{\Pi}_k, \tag{21}$$

where the matrix $\boldsymbol{A}_{k,i}, (i = 1, \ldots, m)$ is the projection matrix corresponding to the i-th image if the k-th sub-block contains it; otherwise $\boldsymbol{A}_{k,i} = \boldsymbol{0}_{3\times4}$. Notice that $\boldsymbol{M}$ is the $m \times m$ normal matrix of the design matrix in (19), hence the size of it is independent from the number of the basis constraints.

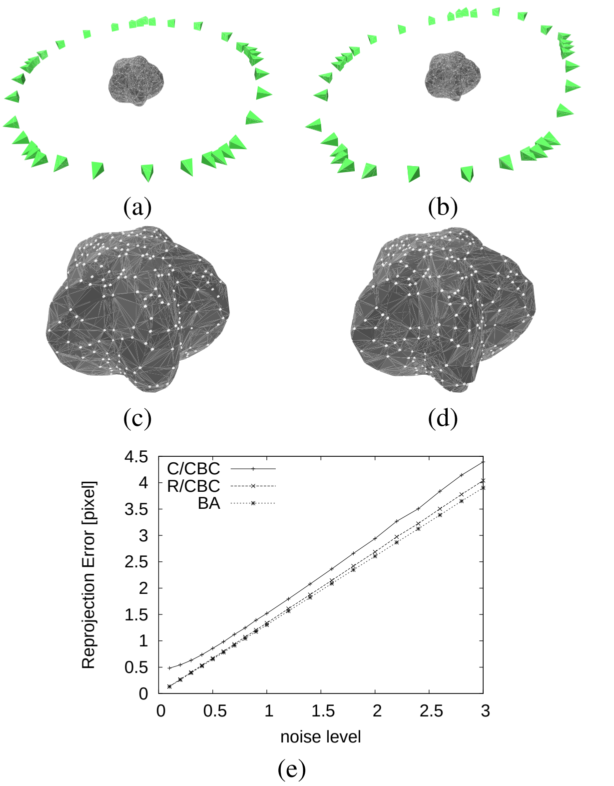

Fig. 1: **Simulation :** (a) Ground-truth Camera Layout. The cameras are represented by pyramids with apex at the Ground-truth camera location. (b) A reconstruction result of the column space approach followed by self calibration [10] ($\sigma = 1.0$). (c, d) The close-up of the figure (a) and (b) respectively. (e) The RMS reprojection error plotted against different noise levels. The abbreviations in the graph C/CBC, R/CBC, and BA stand for the column space approach, the row space approach, and nonlinear bundle adjustment.

5. EXPERIMENT

We evaluate the performance on projective reconstruction in RMS *geometric* reprojection error. Denoting the k-th row of the i-th projection matrix $\boldsymbol{P}_{(k,i)} (k = 1, 2, 3)$, it is measured as follows:

$$\mathrm{Err}_{2\mathrm{D}} = \sqrt{\frac{1}{mn}\sum_i \sum_j \left[\left(u_{ij} - \frac{\boldsymbol{P}_{(1,i)}\boldsymbol{X}_j}{\boldsymbol{P}_{(3,i)}\boldsymbol{X}_j}\right)^2 + \left(v_{ij} - \frac{\boldsymbol{P}_{(2,i)}\boldsymbol{X}_j}{\boldsymbol{P}_{(3,i)}\boldsymbol{X}_j}\right)^2\right]}, \tag{22}$$

where, in the presence of missing data, the summation and the averaging are performed over the significant observations and their number respectively.

5.1 experiment with synthetic data

In order to evaluate the performance of the proposed method, we performed several simulations using synthetic data. For the experiment we use a Desktop PC with intel Core2Duo E6550 2.33GHz and 1GB memory. The scene consisted of 512 feature points distributed on a object, and the camera moved around the object and captured 36 frames (see figure 1). The known entries of measurement matrix was about 31%. The synthetic image coordinates were corrupted by Gaussian noise with mean zero.

We carried out 50 trials for each σ. Figure 1 indicates reconstruction result respect to noise levels. The reprojection error varies roughly linearly with noise, and the row space approach (R/CBC) yields similar results as nonlinear bundle adjustment [4] (BA) while the column space approach (C/CBC) is less accurate than R/CBC.

The major reason for the less accuracy of C/CBC is that we ceased the iteration when the geometric error (22) is increased even if algebraic error decreased. The most likely cause for this behavior is that, at each iteration, the enforcement of the subspace constraint does not guarantee improvement in the sense of reduction of the geometric error. The constraint imposed on the depth parameters in (8) assures to avoid the extremely meaningless solution $\boldsymbol{\mu}_j = \boldsymbol{0}$, however, it is still possible to select the solution in which one element of $\boldsymbol{\mu}_j$ (say λ_{kj}) is zero if it decreases the overall algebraic error by setting $\boldsymbol{P}_k = \boldsymbol{0}$ and $\lambda_{kj} = 0, (j = 1, \ldots, n)$. Although, in realistic noise level, even C/CBC yields practical quality for several application (e.g. visual modeling), appropriate regularization [11, 12] is needed to avoid such meaningless solutions. One advantage of the formulation (8) to these more "theoretical" approaches may be the computational simplicity due to the cost that is independent from the length of image sequence.

5.2 experiment with real data

In this experiment we use the Wadham College sequence (courtesy of the Univ. Oxford Visual Geometry Group [13])

Table 1: **Wadham College:**

	C/CBC	R/CBC	BA
$\mathrm{Err}_{2\mathrm{D}}$ [pixel]	2.24E-01	2.26E-01	2.23E-01
CPU time [sec]	18.15	7.59	-

As compared with C/CBC, the method R/CBC required less computational time while it giving almost equivalent result.

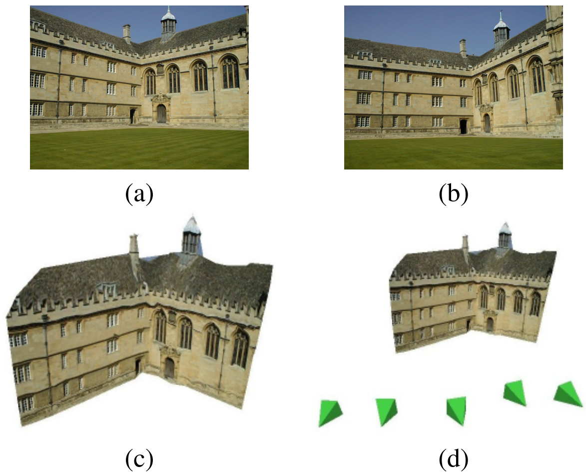

Fig. 2: **Wadham College :** (a) and (b) two images from the Wadham College sequence [13]. (c) and (d) two view of Euclidean reconstruction with texture mapping, where the projective reconstruction is upgraded by self calibration. The cameras are represented by pyramids with apex at the estimated camera location.

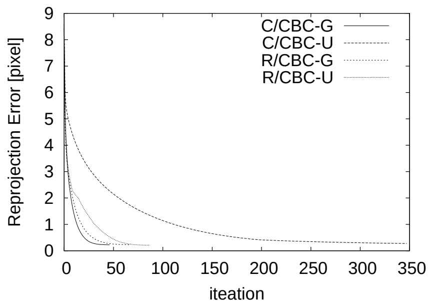

Fig. 3: **Error descent curve for the Wadham College sequence:** All methods were started with same initial solution $\lambda_{ij} = 1.0$. As compared with the other method, the method C/CBC-U required large number of iteration. In contrast to the row space approach (R/CBC-U, R/CBC-G), the number of iteration of the column space approach (C/CBC-U, C/CBC-G) required to convergence varied considerably according to the standardization. Notice that the error is plotted not against the computational time but the iteration count. The method R/CBC-G is faster than method C/CBC-G due to the difference between the computational cost per iteration.

for evaluating effectiveness the proposed method. The image sequence consisted of 5 images, with 1331 3D points and 3019 feature points and the percentage of missing data is about 55%. Our proposed method was successfully applied to experiment with the above data. The result of the experiment shown in Figure 2 and Table 1. The method C/CBC takes 47 iterations to converge in 18.15 s while R/CBC takes 68 iterations to converge is 7.59 s.

5.3 choice of the image coordinate system

As mentioned in the above section, the subspace spanned by JPM depends on the choice of the coordinate systems on the image. In this section we, therefore, present an experiment that examines the effect of it on the the behavior of the depth estimation methods. To do this we compared two choice: method [1, 7] scales image coordinate uniformly while the other [4, 14] shifts the origin to the centroid of the feature points before the scaling. We refer the choice of the image coordinate as *standardization* [1], and express the method [1, 7] and the method [4, 14] by adding suffixes "-U" and "-G" respectively, for example, the abbreviation "C/CBC-G" stands for the column space approach working on the coordinate system selected by the latter method.

The result of the experiment shown in Figure 3 was that we observed significant dependence of the convergence rate of the column space approach on the standardization while, as expected, the row space approach is almost independent from it. To summarize, the author strongly recommend to apply the standardization method [4, 14], especially when the column space approach is adopted for depth estimation.

6. CONCLUSION

In this paper, we have proposed a subspace based iterative projective factorization method that can deal with image sequences with occlusion. The experimental results on both real and synthetic data have confirmed that our proposed method yields practical quality for 3D reconstruction.

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