Impovement of Image Reconstruction from Kinoform using Error-Diffusion Method

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ABCTRACT

A computer-generated hologram(CGH) is made for three-dimensional image reconstruction of a virtual object which is a difficult to irradiate the laser light directly. One of the adverse effect factors is quantization of wave front computed by program when a computer-generated hologram is made. Amplitude element is not considered in Kinoform, it needs processing to reduce noise or false image. So several investigation was reported that the improvement of reconstructed image of Kinoform. Means to calculate the most suitable complex amplitude distribution are iterative algorithm, simulated annealing algorithm and genetic Algorithm. Error diffusion method reconstructed to separate the object as for the noise that originated in the quantization error. So it is efficient method to obtain high quality image with not many processing.

Keywords: Pattern recognition, Computer-generated hologram, Correlation function

1. INTRODUCTION

A computer-generated hologram(CGH)[1] is made for indicating complex value which is computed input object by Fourier transform. The reconstructed image can't choose a position near the origin because reconstructed object is reproduced in two places and origin symmetry. So several investigation was reported that the improvement of reconstructed image of Kinoform. Generally means to calculate the most suitable complex amplitude distribution are iterative algorithm.[3][4] But the computing time is enormous. So it is a problem to reduce computing time.

In this paper, error diffusion method which can greatly shorten the computing time without loss of image quality is applies to Kinoform.[2] Namely, after input image are transformed with Fourier transform, the error diffusion method is applied when Kinoform is generated. This method removes quantization error noise of reconstructed image from area where input object doesn't exist. We propose method to compute diffusion coefficient as a basic stage to compute the best parameter. The Limited condition of position of input object are eased by making to complex number the diffusion coefficient of error diffusion method.

2. Synthesis Method of Kinoform

In the field of watermarking, In this paragraph, computing Kinoform is expressed. Input object is "A", and it is reconstructed as shown Fig .1.

Input object of Kinoform is added random phase. Amplitude of input Object is $f(k,l)$, and random phase is $\phi_R(k,l)$. Complex amplitude $f_R(k,l)$ is given by

$$f_R(k,l) = f(k,l)\exp(-j\phi_R(k,l)) \qquad (1)$$

However (k,l) is coordinate on plane object, and $\phi_R(k,l)$ is uniform random number between $-\pi$ and π . $F(u,v)$ is transformed $f_R(k,l)$ by discrete Fourier transform(DFT)

$$F(u,v) = DFT[f_R(k.l)] \qquad (2)$$

However, DFT[・] means DFT of ・.

After phase information is extracted according to next expression $F'(u,v)$ is quantized with necessary number of quantization.

$$F'(u,v) = Q[\arg\{F(u,v)\}] \qquad (3)$$

However Q[・] means DFT of ・. For example, in case of that number of quantization is 2,it is CGH like Document 5). But in this paper, number of quantization is three or more.

Kinoform is computed by display of grayscale image.

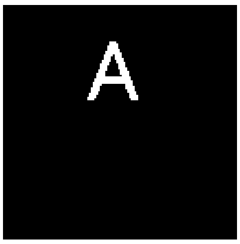


Fig.1 Input Object and its reconstructed position

3.THE ERROR DIFFUSION METHOD FOR Kinoform

3.1 Procedure of the error diffusion method

Generally, error diffusion method used quantization to real number value. But in this paper, the error diffusion method of complex number is enhanced to quantize complex number. Real part and imaginary part are quantized.

(1)Scaling of complex number

Before quantizing complex number, we scale to complex number to make order at the same level quantized complex number and the former complex number.

$$F_1(u,v) = \frac{F(u,v)}{\max|F(u,v)|} \quad (4)$$

(2)Quantization of complex number

Complex number are quantized following expressions. Generally Kinoform records only phase information of complex number. So amplitude information assumed to 1 and record only phase information.

$$F_2(u,v) = \frac{F_1(u,v)}{|F_1(u,v)|} \quad (5)$$

However in case of that phase number are limited, expression is given by

$$F_2(u,v) = Q[\arg\{\frac{F_1(u,v)}{F_1(u,v)}\}] \quad (6)$$

(3)Calculation and diffusion of quantization error

Quantization error are generated following expression,

$$s(u,v) = F_2(u,v) - F_1(u,v) \quad (7)$$

The s(u,v) are diffused to not quantized pixels with adding weight. Diffusion is toward four directions like Fig.2. The weight diffusion coefficient(a,b,c,d) are complex number and fill the next expression.

$$|a|+|b|+|c|+|d|=1 \quad (8)$$

Quantization made by raster scanning.

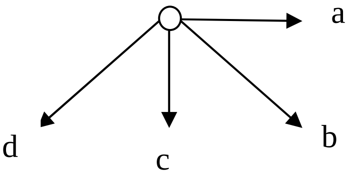
To diffuse quantization error following expression, with $F_1(u,v)$ quantization error as new. $F_1'(u,v)$

$$F_1^{'}(u+1,v) = F_1(u+1,v) + s(u,v)a \quad (9)$$

$$F_1^{'}(u+1,v+1) = F_1(u+1,v+1) + s(u,v)b \quad (10)$$

$$F_1^{'}(u,v+1) = F_1(u,v+1) + s(u,v)c \quad (11)$$

$$F_1^{'}(u-1,v+1) = F_1(u-1,v+1) + s(u,v)d \quad (12)$$



|a|+|b|+|c|+|d| =1

Fig2. Diffusion coefficient

Fig.2 Diffusion coefficient

$$a = \cos(\frac{}{N}) + j\sin(\frac{}{N}) \quad (13)$$

$$b^{'} = \cos(\frac{2\pi(k+l)}{N}) + j\sin(\frac{2\pi(k+l)}{N}) \quad (14)$$

$$c^{'} = \cos(\frac{2\pi l}{N}) + j\sin(\frac{2\pi l}{N}) \quad (15)$$

$$d^{'} = \cos(\frac{2\pi(l-k)}{N}) + j\sin(\frac{2\pi(l-k)}{N}) \quad (16)$$

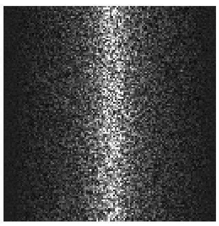
N is pixel of original image with input object, u and v are coordinate on hologram phase(The top of the left of the image is assumed to be (0,0)). a, b, c and d are obtained $a^{'}$, $b^{'}$, $c^{'}$ and $d^{'}$ following expression.

$$a = a^{'}/(|a^{'}|+|b^{'}|+|c^{'}|+|d^{'}|) \quad (17)$$

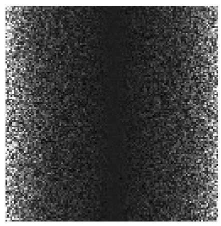
$$b = b^{'}/(|a^{'}|+|b^{'}|+|c^{'}|+|d^{'}|) \quad (18)$$

$$c = c^{'}/(|a^{'}|+|b^{'}|+|c^{'}|+|d^{'}|) \quad (19)$$

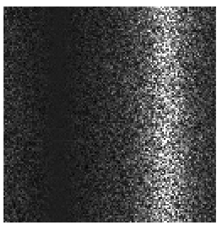
$$d = d^{'}/(|a^{'}|+|b^{'}|+|c^{'}|+|d^{'}|) \quad (20)$$



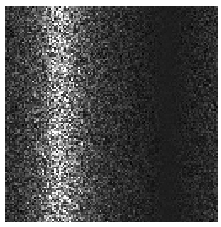
(a) x=0, a=-1+j0



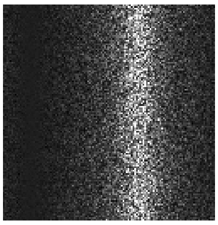
(b) x=N/2, a=-1+j0

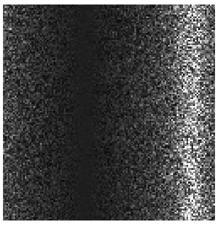


(c) k=N/4, a=-0+j



(d)k=N3/4, a=-0+j





a=--1/$\sqrt{2}$ − j1/$\sqrt{2}$ a=-1/$\sqrt{2}$ − j1/$\sqrt{2}$

Fig.3 Reconstructed image without input

4. EFFECT OF THE ERROR DIFFUSION METHOD BY DIFFUSION COEFFICIENT OF COMPLEX NUMBER

In this paragraph, we examine the relation to the generation of the noise and diffusion coefficient.

4.1 The relation of the generation of the noise and diffusion coefficient.

(1)About a(is a diffusion coefficient)

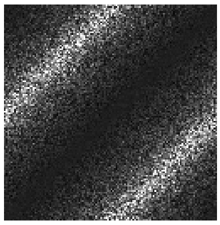
Reconstructed images which is change several a are Fig.3. However input object doesn't exist, and b=c=d=0. In case of a=-1+j0, it is good that object sets position like center as k=N/2 following Fig.3(a). k=N/2 in case of a=+1+j0, k=N/4 in case of a=0-j1, k=3N/4 in case of a=0+j1, are good following Fig.3(b)(c)(d). And it is good that object sets position like center k=N/8 in case of $a = -1/\sqrt{2} - j1/\sqrt{2}$ and l=3N/8 in case of $a = 1/\sqrt{2} - j1/\sqrt{2}$

(2)About b(is a diffusion coefficient)

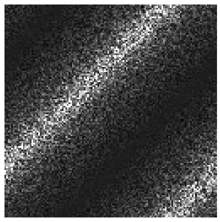
Reconstructed images which are change several b are Fig.4. However input object doesn't exist, and a=c=d=0. In case of c=-1+j0, it is good that object sets position like center as k+l=N/2 following Fig.4(a). k+l=N/2 in case of c=+1+j0, j=N/4 in case of c=0-j1, k+l=3N/4 in case of c=0+j1, are good following Fig.3(b)(c)(d).

(3)About c(is diffusion coefficient)

Reconstructed images which are change several c are Fig.5. However input object doesn't exist, and a=b=d=0. In case of c=-1+j0, it is good that object sets position like center as l=N/2 following Fig.5(a). l=N/2 in case of c=+1+j0, j=N/4 in case of c=0-j1, l=3N/4 in case of c=0+j1, are good following Fig.5(b)(c)(d). And it is good that object sets position like center l=N/8 in case of $c = -1/\sqrt{2} - j1/\sqrt{2}$ and l=3N/8 in case of $c = 1/\sqrt{2} - j1/\sqrt{2}$

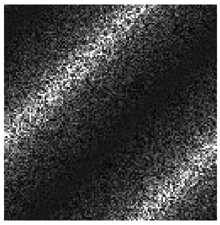


(a) k+ l = 0, a = -1+j0



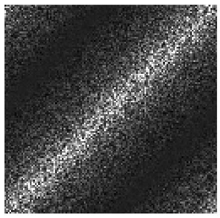
(b) k+ l = N/4,3N/4

a= 1+j0



(c) k+ l = N/4,5N/4

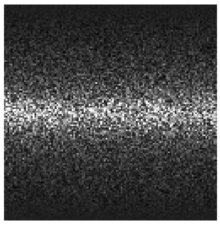
a= 0-j



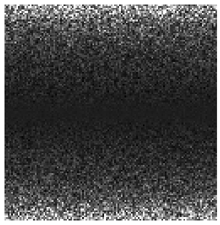
(d) k+ l = 3N/4, 5N/5

a= 0+j

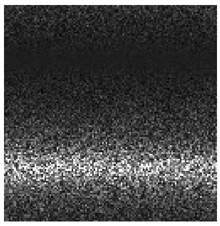
Fig.4 Reconstructed image without input image



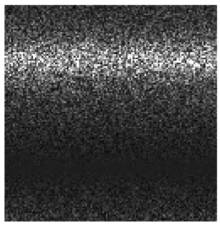
(a) l = 0, a= -1+j0



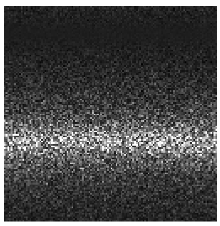
(b) l = N/2, a=1+j0



(c) l = N/4, a= 0 - j

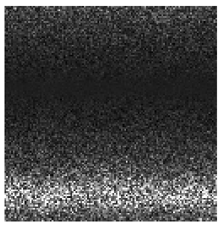


(d) l =3 N/4, a= 0 + j



(e) l = N/8,

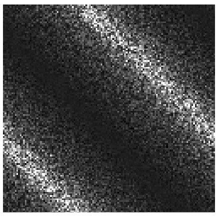
a= -1/$\sqrt{2}$ − j1$\sqrt{2}$



(f) l =3 N/8,

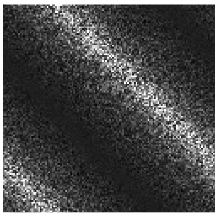
a= 1/$\sqrt{2}$ − j1$\sqrt{2}$

Fig.5 Reconstructed image without input image

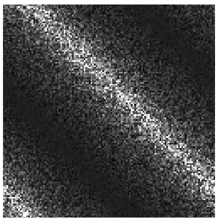


(a) –k+l =0, - N,

a= -1+j0

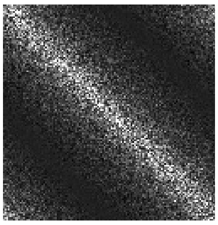


(b)- k+ l = -N/4, 3N/5



(c)- k+ l = N/4,5N/4

a= 0-j



(d)- k+ l =3 N/4,5N/5

a= 0+j

Fig.6 Reconstructed image without input image

(4)About d(is diffusion coefficient)

Reconstructed images which are change several d are Fig.6. However input object doesn't exist, and a=c=d=0. In case of d=-1+j0, it is good that object sets position like center as –k+l=N/2 following Fig.6(a). –k+l=N/2 in case of d=+1+j0, -k+l=N/4 in case of d=0-j1, -k+l=3N/4 in case of d=0+j1, are good following Fig.6(b)(c)(d).

We know that it is possible to move place which isn't exist noise an arbitrary place in case of quantizing diffusion coefficient. It is considered that diffusion coefficient can be calculated for deciding place of input object.

4.2 About combination of diffusion coefficient more than 2

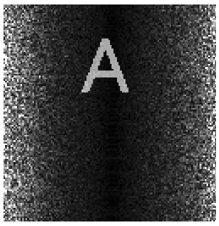
Input image is Fig1 whose size is 128(H)×128(V) and center coordinate is (k.l)=(64,32).

Fig.7 is image which has only one the error diffusion coefficient. Fig.7(a) is image in case of only diffusion coefficient a, Fig.7(b) is only b, Fig7(c) is only c, Fig.7(d) is only d. For these case, it is cleared that object and noise separate to computed coefficient using expression(13)~(20).

Fig.8 is image using two diffusion coefficient. Fig8(a) use two diffusion coefficient(a, c) and b=d=0. Fig8(b) use two diffusion coefficient(b, d) and a=c=0. For these case, it is cleared too that object and noise separate to computed coefficient using expression(13)~(20).

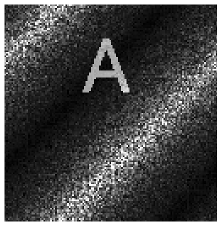
Fig.9 is image using all diffusion coefficient, using a, b, c and d. And it is cleared too that object and noise separate to computed coefficient using expression(13)~(20).

Table.1 shows presence of diffusion coefficient and image quality of reconstructed image. Image quality is given by following expression.



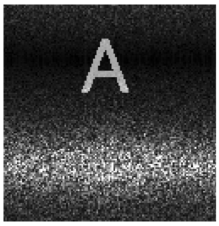
(a) only using a

b=c=d=0



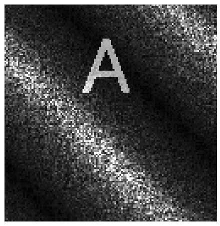
(b) only using b

a=c=d=0



(c) only using c

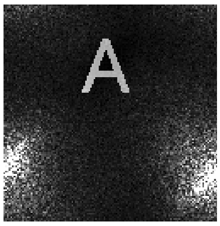
a=b=d=0



(d) only using d

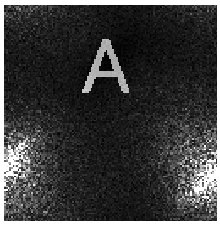
a=b=c=0

Fig.7 Reconstructed image with only one diffusion coefficient



(a) only using a and c

a=b=d=0



(b) only using b and d

a=b=c=0

Fig.8 Reconstructed image with two diffusion coefficients

$$E = \alpha_1 E_1 + \alpha_2 E_2 \tag{21}$$

$$E_1 = \left\langle \left| |g|^2 - \frac{\langle |g|^2 \rangle}{\langle |r|^2 \rangle} |r|^2 \right|^2 \right\rangle \tag{22}$$

$$E_2 = \left\langle \left| \frac{|g| - \langle |g| \rangle}{\sigma_{|g|}} - \frac{|r| - \langle |r| \rangle}{\sigma_{|r|}} \right|^2 \right\rangle \tag{23}$$

|g| is amplitude of original image. |r| is amplitude of reconstructed image. <・> is average of |・|. σ is standard deviation of affixing character.

E_1 contribute to image contrast and E_2 contribute to image difference. And computing E is done in area which is 16(H)×16(V) and input image "A" exists.

The method using only a and c (b=d=0), Fig.8(a), obtain best reconstructed image. So this combination is best diffusion coefficient.

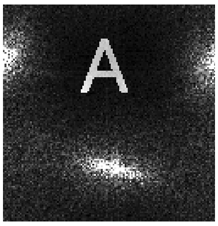


Fig.9 Reconstructed image with four diffusion coefficients

Table.1 Comparison of Reconstructed images depends on existance of diffusion coefficient

a	b	b	d	E
1+j0	0	0	0	0.024359
0	0-j	0	0	0.031651
0	0	0-j	0	0.021597
0	0	0	0-j	0.032315
1/2+j0	0	0-j1/2	0	0.013825
0	0-j1/2	0	0-j1/2	0.031829
1/4+j0	0-j1/4	0-j1/4	0-j1/4	0.022944

Fig.10 Input Object and its reconstructed position

Table.2 Comparison of Reconstructed images depends on existance of diffusion coefficients

a	b	b	d	E
1+j0	0	0	0	0.046472
0	1+j0	0	0	0.091132
0	0	1+j0	0	0.049011
0	0	0	1+j0	0.092302
1/2+j0	0	1/2+j0	0	0.029953
0	0-j1/2	0	0-j1/2	0.061777
1/4+j0	1/4+j0	1/4+j0	1/4+j0	0.036397

Fig.11 Reconstructed image depends on existence of diffusion coefficient

For this result shows similar opinion if input image or position change, Fig10 shows input image including object, Tabel.2 shows result of reconstructed image with combination of diffusion coefficient. But computing E is done in area which is 32(H) × 32(V) and input image , ”Lena”, exists. Center coordinate(k, l) of “Lena” is (64, 64). The method using only a and c (b=d=0), Fig.8(a), also obtain E which is smallest. So it is cleared that this combination which use a and c not using b, d is best diffusion coefficient in Kinoform as shown in Fig.11.

So diffusion coefficient a, b, c and d is calculated following expression.

$$a = \frac{\cos(2\pi k / N) + j\sin(2\pi k / N)}{2} \quad (24)$$

$$b = 0 \quad (25)$$

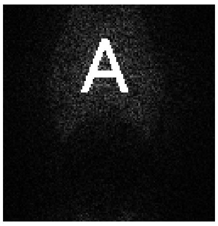
$$c = \frac{\cos(2\pi l / N) + j\sin(2\pi l / N)}{2} \quad (26)$$

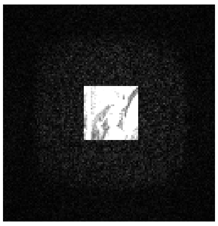
$$d = 0 \quad (27)$$

4.3 Comparison with iterative algorithm

In this paper, CGH are computed with iterative algorithm to compare effectiveness the error diffusion method in Kinoform. Repetition counts are 1000 using method is iterative algorithm in Document5). And the error diffusion method with only diffusion coefficient a and c(b=d=0). Fig.12 shows result using iterative algorithm. Fig.8(b) and Fig.11 shows reconstructed image using Kinoform. It is not possible to separate object and noise following Fig.12, but object and noise are separated with the error diffusion method. So it is clear that reconstructed image is improved using the error diffusion method. And Table.3 shows comparison of E which is image quality. So it is cleared the error diffusion method superior to iterative algorithm in computing CGH.

But it is said that the error diffusion method is effective in area which is separated object and noise.





(a) Input object “A” (b) Input Object “Lenna”

Fig.12 Reconstructed image using iterative algorithm

Table.3 Comparison of image quality

(iterative algorithm and error diffusion method)

Input Object	iterative algorithm	Error diffusion method
"A"	**0.058526**	**0.013825**
"Lenna"	**0.037208**	**0.029935**

5. CONCLUTION

In this paper, reconstructed image is improved in Kinoform using the error diffusion method. Effectiveness of the error diffusion method in Kinoform which is separation of object and noise is ascertained that is also the error diffusion method in CGH. The result of using complex number as diffusion coefficient in the error diffusion method, it was cleared that the position of the object can be chosen comparatively freely. And it was cleared that the error diffusion method is able to obtain excellent reconstructed image compared with the iterative algorithm because of little processing.

It is cleared that error diffusion method in Kinoform is effective in improvement of reconstructed image.

2. REFERENCES

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