Improvement of reconstructed image from computer generated psuedo holograms using iterative method

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ABSTRACT

Computer-Generated Hologram (CGH) is generally made by Fourier Transform.

CGH is made by an optical reconstruction. Computer-Generated Pseudo Hologram (CGPH) is made up Complex Hadamard Transform instead of CGH which is made by the Fourier Transform. CGPH differs from CGH in point of view the possibility of optical reconstruction. There is an advantage that it cannot be optical reconstruction, in other word, physical leakage of the confidential information is impossible.

In this paper, a binary image was converted in Complex Hadamard Transform, and CGPH was made. Improvement of the reconstructed image from CGPH is done by error diffusion method and iterative method. The result that the reconstructed image is improved is shown.

Keywords: Computer-Generated Hologram, Hadamard Transform, Error diffusion method, Iterative method

1. INTRODUCTION

[1] Hologram is technique of the record and reconstruction of image information by Laser which applies the interference and diffract. CGH is mostly made by FFT because FFT can record and reconstruct of image information by its calculation which is obtained the diffraction and interference. As above, CGH is similar processing with hologram by Laser, and it indicates that a physical analysis of the confidential information is possible. Hadamard Transform cannot be optical reconstruction, in other word, physical leakage of the confidential information is impossible. In fact, it is an effective technique to Data hiding. Then, we used Hadamard Transform. [2] Hadamard Transform is compose of Hadamard matrix which is consist of ±1 ,and use the base function which requested by that matrix. Conversion formula is needs only addition and subtraction method because it is set up by the sum total of matrix product of ±1 and the image information.

In this paper, Complex Hadamard Transform, which is extending of Hadamard Transform within the range of complex number, is used. CGPH is made up Complex Hadamard Transform. Phase information was quantized to binary for the purpose of the simplicity of the display of CGPH. The noise reduction method is done to the reconstructed image for the reduction of the noise according to the quantization of the phase.

2. HADAMARD TRANSFORM

2.1 Hadamard Transform

[3] Hadamard Transform is composed of Hadamard matrix, and use the base function which is set up with that matrix. Hadamard matrix is described as follows.

$$H(0) = [1] \quad (1)$$

$$H(n) = \begin{bmatrix} H(n-1) & H(n-1) \\ H(n-1) & -H(n-1) \end{bmatrix} \quad (2)$$

Where n=1,2,3,…. Hadamard matrix of Size N×N($N = 2^n$) is composed by Eq.(1) and Eq.(2). At this time, the size of the input image should be the same as that of Hadamard matrix. 2D Hadamard Transform is described as Eq.(3) of follows.

$$F = \frac{1}{n} HfH^T \quad (3)$$

where F is output, H is Hadamard matrix, f is input and H^t is transposed matrix of Hadamard matrix. In 2D Hadamard Transform, Hadamard Transform is done in the x direction and the y direction respectively.

$$HH^T = I \qquad I \text{ is unit matrix.} \quad (4)$$

Inverse-Hadamard Transform can take out the input image by the character of Eq.(4),and is as almost similar processing as Hadamard Transform. Inverse Hadamard Transform is described as follows.

$$f = nHFH^T \quad (5)$$

2.2 Complex Hadamard Transform

Complex Hadamard Transform is conversion that adds ±i to the element of Hadamard matrix, and is expanded within the range of complex number. Complex Hadamard matrix is described as follows.

$$H(0) = [1] \quad (6)$$

$$H(n) = \begin{bmatrix} H(n-1) & iH(n-1) \\ iH(n-1) & H(n-1) \end{bmatrix} \quad (7)$$

Complex Hadamard Transform is as similar processing as Eq.(3),and use complex conjugate matrix instead of transposed matrix. The character similar to Eq.(4) is approved to Complex Hadamard Transform because complex conjugate matrix is similar to inverse matrix. Then, that conversion and inversion are done.
Transform formula is described as follows.

$$F = \frac{1}{n} HfH^* \quad (8)$$

$$f = nHFH^* \quad (9)$$

H^* is complex conjugate matrix.

3. NOISE REDUCTION METHOD

In this chapter, after the input image is converted, processing where it goes to the noise of the reconstruted image is described.

3.1 Error diffusion method

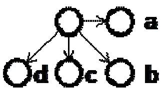
[4] Error diffusion method by raster scan is applied to CGH that makes phase information binary because of the purpose of the facilitation of the display of the hologram object.

In this section, basic of Error diffusion method that use raster scan is described.
It is scaled by Eq.(10) so that the real part in the complex value may become 1.

$$F_s(u,v) = \frac{F(u,v)}{\max\left|\mathrm{Re}[F(u,v)]\right|} \quad (10)$$

where (u,v) is a discrete ordinate of the Hdamard Transform and $F(u,v)$ is that image.The scaled reason is that the effect of the Error diffusion becomes insufficient when max|Re[$F(u,v)$]| is smaller or bigger than 1.

As for this scaled value $F_s(u,v)$, phase information is quantized to binary, and then a quantization error occurs. Error diffusion is done to a quantization error. The algorithm of Error diffusion is described as follows.



$|\boldsymbol{a}|+|\boldsymbol{b}|+|\boldsymbol{c}|+|\boldsymbol{d}|=1$

Fig.1 diffusion coefficient

$$F'(u,v) = \begin{cases} +1 & (\pi/2 \le |\arg[F_s(u,v)]|) \\ -1 & (otherwise) \end{cases} \quad (11)$$

$$s(u,v) = F'(u,v) - F_s(u,v) \quad (12)$$

$$F_s(u+1,v) = F_s(u,v) + a \cdot s(u,v) \quad (13)$$

$$F_s(u+1,v+1) = F_s(u+1,v+1) + b \cdot s(u,v) \quad (14)$$

$$F_s(u,v+1) = F_s(u,v+1) + c \cdot s(u,v) \quad (15)$$

$$F_s(u-1,v+1) = F_s(u-1,v+1) + a \cdot s(u,v) \quad (16)$$

where $s(u,v)$ is a quantization error. Error diffusion is done by multiplying diffusion coefficient (a,b,c,d) to four direction as shown in Fig.1.Error diffusion to the undefined area is not done. In a word, Error diffusion is not done wherever Eq.(16) is in $u=0$, Eq.(13) and Eq.(14) are in $u=N-1$, or Eq.(14)~(16) is in $v=N-1$.Eq(13)~(16) is a type of procedural language, which means a right member is substituted for a left member.

3.2 Iterative method

Inverse Complex Hadamard transform is done, and then CGPH which resultant reconstructed image is corrected is combines one after another. Iterative method is a technique of repeating the above process.

The reconstructed image is improved by repeating Complex Hadamard transform and inverse for the input image. Repeat count is 10^2.

4. SIMULASION RESULT

4.1 CGPH and Reconstructed image

In this section, binary image which is shown as Fig.2 (the original image/ size 128×128) is encoded by Complex Hadamard transform. ”F” in the original image is a input object. The original image, CGPH and the reconstructed image by Complex Hadamard transform are shown as follows.

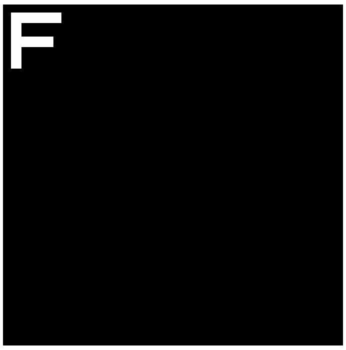


Fig.2 original image

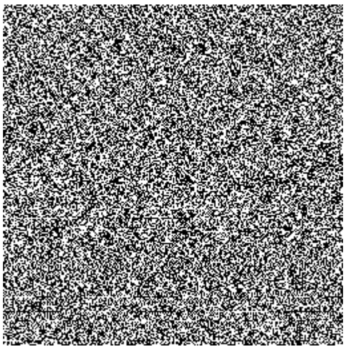


Fig.3 CGPH

Fig.4 reconstructed image

It can be confirmed that the reconstructed image is the original image to which the noise occurred by comparing Fig.2 with Fig.4.

4.2 Application of Noise Reduction Method

In this section, the noise reduction method in Chapter.3 is done to Fig.4 (reconstructed image). Those images to which the noise reduction method is done are shown as follows.

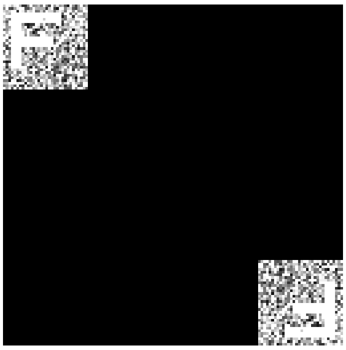


Fig.4 reconstructed image

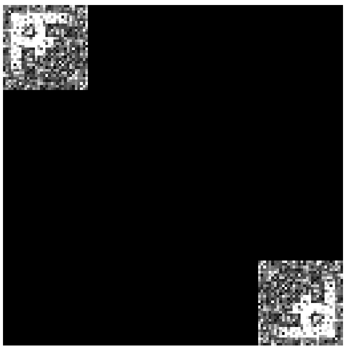


Fig.5 error diffusion method

Fig.6 iterative method

It can be confirmed that nearby noise of "F" in Fig.5 has diffused by comparing Fig.4 with Fig.5. In Fig.6, it can be confirmed that the noise under "F" has been improved by comparing.

4.3 Image Evaluation

The images of section4.2 are evaluated by Mean Squared Error (MSE). The evaluation is good when E is small. The formula of E is described as follows.

$$E = \alpha_1 E_1 + \alpha_2 E_2 \tag{17}$$

$$E_1 = \left\langle \left| |g|^2 - \frac{\langle |g|^2 \rangle}{\langle |r|^2 \rangle} |r|^2 \right|^2 \right\rangle \tag{18}$$

$$E_2 = \left\langle \left| \frac{|g| - \langle |g| \rangle}{\sigma_{|g|}} - \frac{|r| - \langle |r| \rangle}{\sigma_{|r|}} \right|^2 \right\rangle \tag{19}$$

where amplitude of the original image is |g|, amplitude of the reconstructed image is |r|, average value of $|\cdot|$ is $\langle \cdot \rangle$ and E_2 is contributed to diffusion of amplitude of an image.

The evaluation of the images of section4.2 is shown at Table.1.

It is indicated that Iterative method is the most efficient method when the evaluated value is confirmed with Table.1. The evaluated value of error diffusion method is not improved, but it is shown that the noise has been improved when Fig.5 is confirmed.

Table.1 evaluation of the image improvement

noise rejection method	E(evaluation)
no processing	0.231561151
error diffusion method	0.245669073
iterative method	0.150741395

5. CONCLUSION

In this paper, CGPH by Complex Hadamard transform was made, and the reconstructed image was obtained. Noise reduction by Error diffusion method and Iterative method is done to the reconstructed image.

Evaluation value is not good on the noise reduction when noise reduction by Error diffusion method is made, but it can be slightly confirmed that nearby noise of “F” in Fig.5 has diffused by comparing Fig.4 with Fig.5. Image improvement by Iterative method can be confirmed from evaluation value and visual evaluation. So Noise reduction by Iterative method turned out to effective method to CGPH.

There are a number of diffusion methods of Error diffusion method, and image improvement by those methods is an issue in the future. A image improvement by iterative method has been already confirmed, and then an image improvement by it combined with other noise reduction method is an issue in the future.

6. REFERENCES

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