

# INVERSE HALFTONING USING KALMANN FILTERING

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## ABSTRACT

The inverse halftoning is processing to restore the image made binary to former step image. There are smoothing and gaussian filtering in the technique so far. However, there are still a lot of insufficient points in past inverse halftoning. The removal of the noise and the edge enhancement are closely related in inverse halftoning. It is difficult to do both the noise reduction and the edge enhancement in high accuracy at the same time in the technique so far. The technique that can achieve both the removal of the noise and the emphasis of Edge at the same time is expected as future tasks.

Then, it was tried to apply the Kalmann filtering to inverse halftoning. In the actual experiment, the effectiveness of the application of the Kalmann filtering to inverse halftoning comparing it with the technique so far was shown.

## 1.Introduction

Inverse halftoning is processing to restore the image made binary to former step image. There are Gaussian filter etc. as a past technique. However, It is difficult for inverse halftoning to do the emphasis of Edge and the removal of the noise. There are a lot of insufficient points in the technique so far.

Then, it was tried to apply the Kalmann filter to the inverse half tone processing in the actual experiment.

Effectiveness is shown, and, up to now, the karmann filter has been used also in the field of the civil engineering and economics in the aerospace engineering and the control engineering now. Additionally, the effectiveness of the use of the Kalmann filter is shown in various fields of the multimedia such as voice recognition and GPS under the noise environment. In the actual experiment, the effectiveness of the application of the Kalmann filter to the inverse-halftoning comparing it with the technique so far is shown.

Moreover, processing a inverse-halftoning only by the Kalmann filter will take shape of giving the Kalmann filter after it smoothes it as a preprocessing because it is difficult.

## 2.Halftoning

The image used to experiment is a standard image "Lenna" of 256×256 pixels. A systematic dither method of the Bayer type was given to the image of Figure 1, Figure 2 was made binary, and the random dither of the Floyd&Steinberg type was given to the image of Figure 1

and Figure 3 was made binary.

## 3.Inverse-halftoning by technique in the past

### 3.1 Smoothing

Process smoothing to pointed pixel and 8pixels around the pixel. At this point, define original image's gray-scale as  $f(i,j)$ , grayscale of image after processing smoothing as  $g(i,j)$ , filtering matrix as  $m(i,j)$  in coordinate  $(i,j)$ . And shown following equation.

$$g(i, j) = f(i, j) * M(i, j) \quad (1)$$

Now, filtering matrix is shown by following equation.

$$M(i, j) = \frac{1}{10} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad (2)$$

The reason for processing smoothing about 8 pixels around pointed pixel is becoming obscure in edge of image in case area of processing smoothing is wide. And for same reason, weight of pointed pixel is defined as 2 in filtering matrix. Moreover, the image when Smoothing is done to the image to which Figure 2 chart 3 is restored is shown in Figure 4.

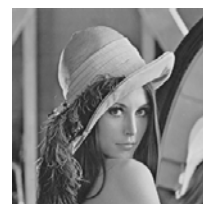


Fig1 Original picture image



Fig2 Restoration image1



Fig3 Restoration image2



(a) Processing image1 (b) Processing image2

Fig4 Result of smoothing

### 3.2 Median filtering

A central value of the density in the area is arranged in small the order, and the density value with new noteworthy point of a median filter is the 51st density value while the mean value of the neighborhood area was adjusted to the density value of the center in Smoothing.

Figure 5 shows the image that gives a median filter to the image restored by Figure 2 and Figure 3. After extent in which Smoothing was done had been restored because a inverse-half toning was not able to be processed with a median filter alone, a median filter was given.

### 3.3 Gaussian filtering

Gaussian filtering is a filter that smoothes it by folding in expression (3).

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \quad (3)$$

The original picture image is assumed the  $f(i,j)$ . Smoothing image  $s(i,j)$  is requested from expression (4).

$$s(i, j) = g(i, j) * f(i, j) \\ = \sum_m \sum_n g(i+m, j+n) f(i, j) \quad (4)$$

\* shows the folding in integration. It is assumed  $\sigma=0.55$  in the actual experiment. Figure 6 is an image when Gaussian filter is actually given to the image of Figure 2 and Figure 3.

## 4. Kalmann filtering

### 4.1 Modeling of image

Expression (5) is used for the model with the original picture image.

$$X(t) = FX(t-1) + W(t-1) \quad (5)$$

F shows the state transition procession of  $9 \times 9$ . X is a state variable, and W is an internal noise. Moreover, t shows time.

Next, state variable X is assumed  $X_i (i = 0, 1, \dots, 8)$  and assumed to be arrangement shown in Figure 7.

$X_0$	$X_1$	$X_5$
$X_1$	$X_8$	$X_6$
$X_2$	$X_4$	$X_7$

Fig7 Arrangement of state variable

$X_i(t) (i = 0, 1, \dots, 4)$  is assumed to be an untouched transition of pixel value at time of the previous state.  $X_i(t) i = 5, 6, 7$  used and presumed the pixel shown in Table 1.

$X_8(t)$  is the presumption value of a final original picture image is presumed

By the surrounding pixel  $x_i(t-1), i = 0, 1, \dots, 8$ .

To give the surrounding pixel needed by the presumption of  $x_{23}(t)$ ,  $x_i(t), i = 5, 6, 7$  is presumed. These pixels are presumed by using a past pixel.

Table 1: Pixel used to presume .

Filtered pixel	Support
$X_5(t)$	$X_i(t-1) (i = 1, 5, 6)$
$X_6(t)$	$X_i(t-1) (i = 5, 6, 7)$
$X_7(t)$	$X_i(t-1) (i = 4, 6, 7)$

The following expression is used for the model of the observation image.

$$Y(t) = HX(t) + V(t) \quad (6)$$

Y shows the observation image. H shows the observation procession of  $4 \times 9$ . V is an observation noise.



(a)Processing image1 (b)Processing image2

Fig5 Result of using median filter



(a)Processing image1 (b)Processing image2

Fig5 Result of using Gaussian filter

## 4.2 Algorithm

When the Kalmann filter is applied to the image model of 4.1, the algorithm shown as follows is used.

(1) Filter equation

$$\begin{aligned} \hat{X}(t+1/t) &= F\hat{X}(t/t) \quad (7) \\ \hat{X}(t/t) &= \hat{X}(t/t-1) \\ &+ K(t)[Y(t) - H(t)\hat{X}(t/t-1)] \quad (8) \end{aligned}$$

(2) Kalmann gain

$$K(t) = P(t/t-1)H^T[HP(t/t-1)H^T + Q_v]^{-1} \quad (9)$$

(3) Presumption error margin covariance procession

$$\begin{aligned} P(t+1/t) &= FP(t/t)F^T + Q_w \quad (10) \\ P(t/t) &= P(t/t-1) - K(t)HP(t-1) \quad (11) \end{aligned}$$

(4) Initial condition

$$\begin{aligned} \hat{X}(0/-1) &= \bar{X}(0) \quad (12) \\ P(0/-1) &= \\ E\{[X(0) - \hat{X}(0/-1)][X(0) - \hat{X}(0/-1)]^T\} & \quad (13) \end{aligned}$$

$\hat{X}$  shows the presumption value.  $Q_w$  shows the covariance procession of observation noise  $V(t)$ .  $Q_v$  shows the covariance procession of observation noise  $V(t)$

## 4.3 Computational method of parameter

The Kalmann filter is a linear establishment system that assumes input and presumption value B to be an output as for A. The computational method of the Kalmann filter is shown below.

Initial condition

$$\hat{x}_{0/-1} = \bar{x}_0, \quad P_{0/-1} = 0 \quad (14)$$

Kalmann gain

$$K_t = P_{t/t-1}H_t^T[H_tP_{t/t-1}H_t^T + R_t]^{-1} \quad (15)$$

(15)It twines and  $K_0$  is calculated.

The value of  $y_0$  at  $t=0$  is observed.

$$\hat{x}_{t/t} = \hat{x}_{t/t-1} + K_t[y_t - H_t\hat{x}_{t/t-1}] \quad t = 0,1\dots(16)$$

$$P_{t/t} = P_{t/t-1} - K_tH_tP_{t/t-1} \quad t = 0,1\dots(17)$$

(16),(17)It twines and  $\hat{x}_{0/0}, P_{0/0}$  is requested.

$$\hat{x}_{t+1/t} = F_t\hat{x}_{t/t} \quad (18)$$

$$P_{t+1/t} = F_tP_{t/t}F_t^T + G_tQ_tG_t^T \quad (19)$$

(18),(19)It twines and  $\hat{x}_{1/0}, P_{1/0}$  is requested.

$$K_t = P_{t/t-1}H_t^T[H_tP_{t/t-1}H_t^T + R_t]^{-1} \quad (20)$$

(20)It twines and  $K_1$  is requested.

The presumption value and the presumption error margin covariance procession can be calculated according to a same procedure.

Because the Kalmann filter corrects an old presumption value whenever a new observation value is obtained, and calculates a new resumption value like this, it is an algorithm that is appropriate for the online state presumption by the computer.

The method of presuming state transition procession  $F$ , covariance procession  $Q_w$ , and  $Q_v$  is as follows.

$$X_5(t) = [F_{51} \quad F_{55} \quad F_{56}] \begin{bmatrix} X_1(t-1) \\ X_5(t-1) \\ X_6(t-1) \end{bmatrix} + W_5(t) \quad (21)$$

The expected value is taken.

$$\begin{aligned} \rho_{ji}(l) &= E[X_j(t)X_i(t-l)] \\ Q_{wji}\delta(l) &= E[W_j(t)X_i(t-l)] \quad (22) \end{aligned}$$

$$\rho_{55}(l) = [F_{51} \quad F_{55} \quad F_{56}] \begin{bmatrix} \rho_{15}(l-1) \\ \rho_{55}(l-1) \\ \rho_{65}(l-1) \end{bmatrix} + Q_{w55}\delta(l) \quad (23)$$

$$\delta(l) = \begin{cases} 1, & l = 0 \\ 0, & l \neq 0 \end{cases} \quad (24)$$

When you do a similar operation.

$$\rho_{51}(l) = \begin{bmatrix} F_{51} & F_{55} & F_{56} \end{bmatrix} \begin{bmatrix} \rho_{11}(l-1) \\ \rho_{51}(l-1) \\ \rho_{61}(l-1) \end{bmatrix} \quad (25)$$

$$\rho_{56}(l) = \begin{bmatrix} F_{51} & F_{55} & F_{56} \end{bmatrix} \begin{bmatrix} \rho_{16}(l-1) \\ \rho_{56}(l-1) \\ \rho_{66}(l-1) \end{bmatrix} \quad (26)$$

$l = 1$  is substituted for (25),(26).

$$\begin{bmatrix} \rho_{55}(0) & \rho_{15}(1) & \rho_{55}(1) & \rho_{65}(1) \\ \rho_{51}(1) & \rho_{11}(0) & \rho_{51}(0) & \rho_{61}(0) \\ \rho_{55}(1) & \rho_{15}(0) & \rho_{55}(0) & \rho_{65}(0) \\ \rho_{56}(1) & \rho_{16}(0) & \rho_{56}(0) & \rho_{66}(0) \end{bmatrix} \begin{bmatrix} 1 \\ -F_{51} \\ -F_{55} \\ -F_{56} \end{bmatrix} = \begin{bmatrix} Q_{w55} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The expected value is taken.

$$Q_{w5i} = \rho_{5i}(0) - \begin{bmatrix} F_{51} & F_{55} & F_{56} \end{bmatrix} \begin{bmatrix} \rho_{1i}(l) \\ \rho_{5i}(l) \\ \rho_{6i}(l) \end{bmatrix} \quad (27)$$

When neither  $X_j(t)$  nor  $X_i(t-l)$  show the same pixel.

$$\rho_{ji}(l) = E[X_j(t)X_i(t-l)] \cong E[Y_j(t)Y_i(t-l)] \quad (28)$$

When  $X_j(t)$  and  $X_i(t-l)$  show the same pixel.

The presumption value of the decentralization of the original picture image is set for the value of  $Q_{w88}$  to become small.

Next, the decentralization of the observation noise is requested.

$$\sigma_v^2 = \sigma_y^2 - \rho_{ii}(0) \quad (29)$$

Covariance procession of  $Q_v$  observation noise.

$$Q_v = \begin{bmatrix} \sigma_v^2 & 0 & 0 & 0 \\ 0 & \sigma_v^2 & 0 & 0 \\ 0 & 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 & \sigma_v^2 \end{bmatrix} \quad (30)$$

## 4.4 Experiment result

Figure 8 shows the image when the Kalmann filter is given to the image of figure 2 and figure3 respectively.

Table 2 shows Signal-Noise ratio.

Signal-Noise ratio was used as a method of evaluating an objective image in this experiment.

Signal-Noise ratio is the one that the level of the noise was shown according to the ratio of the signal and the noise.

Signal-Noise ratio can be calculated by dividing the standard deviation of the signal by the standard deviation of the error margin as shown in expression (14).

$$PSNR = 10 \log_{10} \frac{\sqrt{\sum f(x,y)^2}}{\sum_{x=0}^N \sum_{y=0}^M \{f(x,y) - f'(x,y)\}^2} (db)$$

$f(x,y)$  A indicates the step value of the original picture image of noteworthy pixel  $(x,y)$ .  $f'(x,y)$  shows the step value of the image to which noteworthy pixel  $(x,y)$  is restored.

The noise has been removed from the technique so far

Figure 8. Moreover, it is understood that the outline part is emphasized compared with the technique so far.

The effectiveness of the understanding of the improvement of Signal-Noise ratio in the past from the data of Table 2 compared with the technique, and the application of the Kalmann filter to the reverse-half tone processing was able to be shown.

Table 3 shows smoothing done before the Kalmann filter is given and the result of using various filters is shown.

The restoration image with the best time when a median filter had been combined with the Kalmann filter as understood in the table was able to be obtained.

Table1

	Floyd	Jarvis
smooth	22.13	21.84
median	24.48	23.58
gaussian	24.40	23.76
Kalmann	25.75	24.58

Table2

	Bayer	Err dfsf
smooth + Kalmann	24.5	27.4
median + Kalmann	26.8	29.6
gaussian + Kalmann	20.4	20.2

## 5. Summary

In this report, it proposed the technique for applying the Kalmann filter to the reverse-half tone processing.

It was able to be confirmed to the proposal technique that there was effectiveness for the restoration of the image.

Moreover, the restoration image with the best time when a median filter had been combined with

the Kalmann filter was able to be obtained.



Fig8 Processing result with Kalmann filter

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