HI-SPEED COMPUTER-GENERATED HOLOGRAM ALGORITHM

Hyun-Jun Choi, Young-Ho Seo, Ji-Sang Yoo, Dong-Wook Kim

3DMRL. Kwangwoon University
Seoul, Korea
E-mail: hjchoi@kw.ac.kr

ABSTRACT

This paper proposes an algorithm that increases the speed of generating a Fresnel hologram using a recursive addition operation covering the whole coordinate array of a digital hologram. The 3D object designed to calculate the digital hologram used the depth-map image produced by computer graphics (CG). The proposed algorithm is a technique that performs CGH (computer generated hologram) operation with only the recursive addition from the hologram's whole coordinates by analyzing the regularity between the 3D object and the digital hologram coordinates. The experimental results showed that the proposed algorithm increased operation speed by 30% over the technique using the conventional CGH equation.

Keywords: digital hologram, 3D, hi-speed, fresnel transform

1. INTRODUCTION

CGH was proposed by Brown and Lohmann in 1966[1]. This method can obtain an interference pattern through an arithmetic operation on a PC after approximating optical signals. When the CGH method is used, a digital hologram can easily be obtained from an object in real space or a virtual object. However, to calculate the digital hologram using the CGH method, approximately 900 seconds are required if a general PC is used to display a significant computation amount (3D Object measuring approximately 1 cm x 1 cm x 1cm) in space. To improve the problem, [2] proposed a method that increases the operation speed by recursively adding only the difference of the distance between the 3D object and the digital hologram to be generated. However, a disadvantage to [2]'s method is where errors are accumulated and many multiplication operations are required. In [3], a technique was proposed where the general CGH operation, which used to be realized from the whole coordinate array of the digital hologram to be generated, is carried out only on the first coordinate of the x-axis ((0,0), (1,0),(2,0),(3,0)…(n,0)) and the remaining operation of coordinate is carried out by adding the precalculated values and the previous operation results to the results of the first operation.

This paper has analyzed the recursive addition proposed in [3] to find the regularity between the 3D image and the digital hologram coordinate array, and reduced the number of whole operations carried out on the first coordinate of the x-axis of the digital hologram, and the partial multiplication and the partial addition performed on overall coordinates.

2. COMPUTER-GENERATED HOLOGRAM

The digital hologram system uses electronic equipment instead of optical equipment to record the interference pattern of holography in the CCD camera and transmit it via video signal. The image is reconstructed by illuminating laser beam to the interference pattern displayed on an SLM (Spatial Light Modulator) from the receiver. A hologram may be acquired by using the optical system, however, it can also be acquired by an operation where the optical system is arithmetically modeled. The hologram obtained through such arithmetic operation is called CGH.

The conventional CGH creation equation is defined as follows,

$$I_\alpha = \sum_j^N A_j \cos(k\sqrt{(px_\alpha - px_j)^2 + (py_\alpha - py_j)^2 + z_j{}^2} + \phi_\alpha + \phi_j) \quad (1)$$

Here, α and j are defined as hologram and 3D object, k is defined into $2\pi/\lambda$ as the wave number of the reference wave; and p represents the pixel pitch of the hologram; x_α and y_α as the coordinates of the hologram, and x_j, y_j, and z_j represent the coordinates of the 3D object.

Fig. 1 shows the coordinate array of the 3D object and the digital hologram to apply the CGH method. The coordinate array shown in Fig. 1 is the 3D object measuring 2×2, an example where the digital hologram measuring 4×4 in size is generated. To generate a digital hologram at this time, the operation shown in Eq. (1) must be carried out for 2×2×4×4=64 times.

When Eq. (1) is approximated to the first term after the Taylor expansion, we can arrange it as follows.

$$I_\alpha = \sum_j^N A_j \cos(\frac{2\pi}{\lambda}(z_j + \frac{p^2}{2z_j}((x_\alpha - x_j)^2 + (y_\alpha - y_j)^2)) + \phi_\alpha + \phi_j) \quad (2)$$

Terms of Eq. (2) can be simplified as follows.

$$I_\alpha = \sum_j^N A_j \cos(2\pi(\theta_z + \theta_H) + \phi_\alpha + \phi_j)$$
$$(\theta_z = \frac{z_j}{\lambda}, \theta_H = \frac{p^2}{2\lambda z_j}(x_{\alpha j}{}^2 + y_{\alpha j}{}^2)) \quad (3)$$

Here, $x_{\alpha j}$ and $y_{\alpha j}$ mean $(x_\alpha - x_j)$ and $(y_\alpha - y_j)$, respectively.

The phase $\theta_H(x_{\alpha j}+n, y_{\alpha j}, z_j)$ at one point $(x_\alpha+n, y_\alpha)$ of the digital hologram may be expressed as follows.

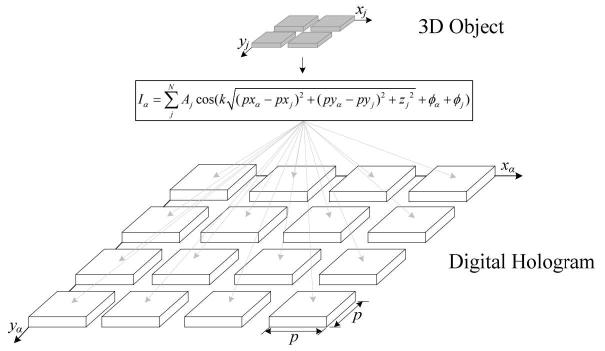


Fig. 1. CGH operation of the conventional method

$$\begin{aligned}&\theta_H(x_{\alpha j}+n, y_{\alpha j}, z_j)\\&=\frac{p^2}{2\lambda z_j}((x_{\alpha j}+n)^2+{y_{\alpha j}}^2))=\frac{p^2}{2\lambda z_j}({x_{\alpha j}}^2+{y_{\alpha j}}^2)+\frac{p^2}{2\lambda z_j}(2nx_{\alpha j}+n^2)\\&=\theta_H(x_{\alpha j}, y_{\alpha j}, z_j)+\Gamma_{xn}\end{aligned} \tag{4}$$

Here, Γ_{xn} is defined as follows.

$$\Gamma_{xn}=\frac{p^2}{2\lambda z_j}(2nx_{\alpha j}+n^2) \tag{5}$$

If the n-values are substituted for Eq. (5), in the case of n=1, Γ_{x1} is

$$\Gamma_{x1}=\frac{p^2}{2\lambda z_j}(2x_{\alpha j}+1) \tag{6}$$

And in the case of n=2, Γ_{x2} is

$$\begin{aligned}\Gamma_{x2}&=\frac{p^2}{2\lambda z_j}(4x_{\alpha j}+4)\\&=\frac{p^2}{2\lambda z_j}(2x_{\alpha j}+1)+\frac{p^2}{2\lambda z_j}(2x_{\alpha j}+1)+\frac{p^2}{2\lambda z_j}\times 2\\&=\Gamma_{x1}+\Gamma_{x1}+\Delta_x\end{aligned} \tag{7}$$

Here, Δ_x is defined as follows:

$$\Delta_x=\frac{p^2}{2\lambda z_j}\times 2 \tag{8}$$

Again, when Γ_{x3} is calculated in the case of n=3,

$$\begin{aligned}\Gamma_{x3}&=\frac{p^2}{2\lambda z_j}(6x_{\alpha j}+9)\\&=\frac{p^2}{2\lambda z_j}(4x_{\alpha j}+4)+\frac{p^2}{2\lambda z_j}(2x_{\alpha j}+1)+\frac{p^2}{2\lambda z_j}\times 4\\&=\Gamma_{x2}+\Gamma_{x1}+2\Delta_x\end{aligned} \tag{9}$$

When n=N, Γ_{xN} may be generalized as follows

$$\Gamma_{xN}=\Gamma_{x(N-1)}+\Gamma_{x1}+(N-1)\Delta_x \tag{10}$$

If the value of $\Gamma_{x1}$and Δ_x are precalculated in Eq. (10), only the first coordinate values of the values of the *x*-axis of the digital hologram are calculated using Eq. (2), and starting from (*x* + 1)th, CGH operation can be carried out by adding the values of Γ_{x1}, Δ_x, and the previously calculated $\Gamma_{x(N-1)}$ values. At this time, CGH operation shown in Eq. (2) is defined as the whole operation, and the partial operation shown in Eq. (10) used to compute the hologram values on the same *x*-axis, as partial multiplication and partial addition. Fig. 2 shows the method of [3] described above. The shaded block in the figure means the results of the whole operation, and the ordinary block, the results of a partial operation.

For example, in the case of a general CGH operation, the whole operation is carried out 1024×1024×200×200=41,943,040,000 times when a 1024×1024 digital hologram is created from a 200×200 3D object. But as for the method proposed in [3], the whole operation is repeated 1024×200×200=40,960,000times, and partial multiplication and partial addition are performed 1023×1024×200×200= 41,902,080,000times. In other words, this method reduces the number of whole operations by more than 99.9% by replacing the complicated whole operation with recursive partial multiplication and partial addition.

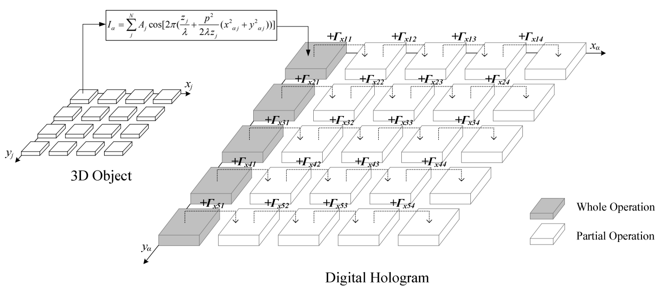


Fig. 2. CGH operation using recursive addition

3. PROPOSED ALGORITHM

If the original equation is substituted for Γ_{x1} and Δ_x of the Eq. (10), it becomes the same as follows.

$$\Gamma_{xN}=\frac{p^2}{2\lambda z_j}(2x_{\alpha j}+1)+\Gamma_{x(N-1)}+\frac{2p^2}{2\lambda z_j}(N-1) \tag{11}$$

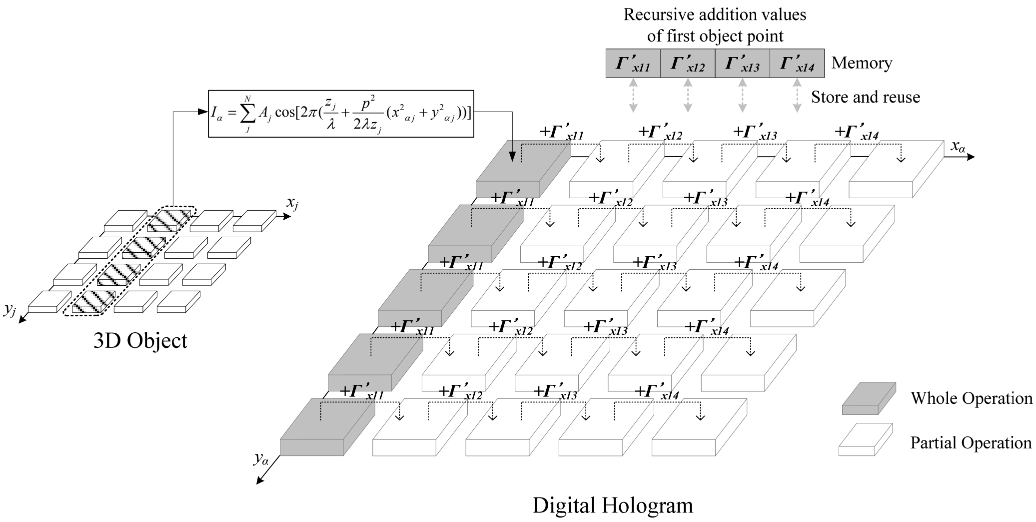


Fig. 3. The recursive addition CGH operation of x-axis is carried out on the point existing in the same column of the 3D object

Eq. (11) shows that when one point of the 3D object is calculated from the first *x*-axis (*x*-axis that begins as (0,0) coordinate) of the hologram, the variable that changes the value after Γ_{x2} is only N. That is, if one point of the 3D object is computed, the Γ_{xN} value that was computed from the first x-axis of the hologram is the same as the Γ_{xN} values of the other *x*-axis. Accordingly, the Γ_{xN} that were calculated when the first *x*-axis holograms were computed can also be used as it is when another *x*-axis hologram is calculated.

The application of the aforementioned method can be expanded to a case where a CGH operation is carried out on the points that exist on the same column of the 3D object as shown in Fig 3. However, the precalculated Γ_{xN} values cannot be used again because, even though the x_j, x_a, *N* values are the same when the points existing on the same column are recursively added using Eq. (11), z_j is changed. To solve this, we can process the Eq. (11) like the below.

In the case of N=1, Γ'_{x1} is

$$\Gamma'_{x1} = \frac{1}{z_j}(\frac{p^2}{2\lambda}(2x_{\alpha j}+1)) = \frac{1}{z_j}(\Gamma_1) \qquad (12)$$

In the case of N=2, Γ'_{x2} is

$$\Gamma'_{x2} = \frac{1}{z_j}(\frac{p^2}{2\lambda}(2x_{\alpha j}+1)) + \frac{1}{z_j}(\frac{p^2}{2\lambda}(2x_{\alpha j}+1)) + \frac{1}{z_j}(\frac{2p^2}{2\lambda})$$
$$= \frac{1}{z_j}(\Gamma_{x1} + \Gamma_{x1} + \Delta_x) \qquad (13)$$

In the case of N=3, Γ'_{x3} is

$$\Gamma'_{3x} = \frac{1}{z_j}(\frac{p^2}{2\lambda}(2x_{\alpha j}+1)) + \frac{1}{z_j}(\frac{p^2}{2\lambda}(2x_{\alpha j}+1) + \frac{p^2}{2\lambda}(2x_{\alpha j}+1) + \frac{2p^2}{2\lambda})$$
$$+ \frac{1}{z_j}(\frac{2p^2}{2\lambda}\times 2)$$
$$= \frac{1}{z_j}(\Gamma_{x1} + \Gamma_{x2} + 2\Delta_x) \qquad (14)$$

And Γ'_{xN} in the case of n=N can be generalized as follows.

$$\Gamma'_{xN} = \frac{1}{z_j}(\Gamma_{x(N-1)} + \Gamma_{x1} + (N-1)\Delta_x) \qquad (15)$$

As shown in Eq. (10) and (15), the Γ'_{xN} values used when carrying out CGH operations on the 3D objects of the same column, except *z*, are identical. It means that when the recursive addition CGH operation of *x*-axis is carried out on the points existing in the same column of the 3D object, the remaining terms, excluding z_j can reduce operation time by using the Γ'_{xN} values of the first point as it is.

4. SIMULATION RESULT

In order to verify the proposed algorithm, conventional CGH operation as shown in Eq. (1) in an environment displayed on Table 1, the technique of [3] described in Section 2, and the recursive addition algorithm covering the *x*-axis coordinate array of the hologram proposed in the paper were implemented.

Table 1. Experimental setup

Items		Specification
PC set-up	OS	MS Windows XP
	CPU	Intel Core2 Quad 2.4GHz
	RAM	2.0GB
	Compiler	MS Visual C++ 6.0
Optical parameters	3D object size (x_j, y_j , z_j)	200×200×8-bit
	Hologram resolution (x_a, y_a)	1024×1024
	Reference wave length (λ)	633*nm* (red)
	Reconstruction distance	1000*mm*
	Pixel pitch (p)	10.4*um*×10.4*um*

The 3D object used 200×200 depth-map image created by CG, and the light intensity of each point was replaced by the pixel value of the depth-map image. The digital hologram to be generated was set in the red-channel (633*nm*) and the gray-scale (8-bit) measuring 1024×1024, considering the reconstruction environment.

Fig. 4 shows the results of measuring the CPU time of the CGH algorithm proposed in this paper and the previous methods. For the purpose of comparison, the CPU time of the algorithm proposed and previous methods were measured while increasing the number of points of the 3D object. As shown in Fig 8, the recursive addition operation covering the whole areas of the proposed hologram has increased the operation speed by more than 30% compared with the previous method.

Fig. 5 shows examples of the images used for the test. In Fig. 5, (a) is a 3D object image, (b) is a digital hologram generated by the proposed method, (c) is a holographic image reconstructed by PC simulation[4], and (d) is a holographic image reconstructed by an optical system. The optical system used SLM measuring 1280×1024 with a pixel pitch of 13.62*um*×13.62*um*, and reconstructed the system with a 532*nm*-wavelength light source.

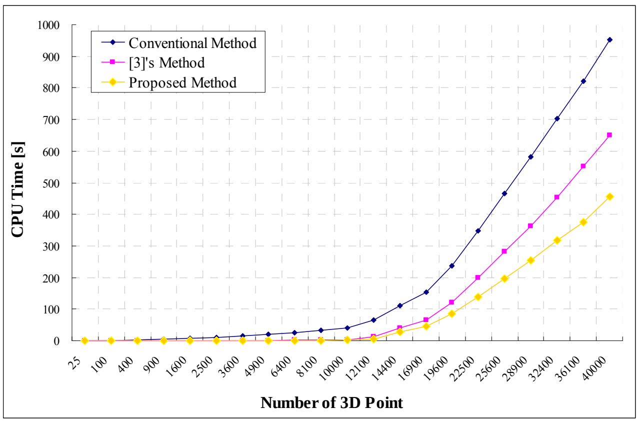
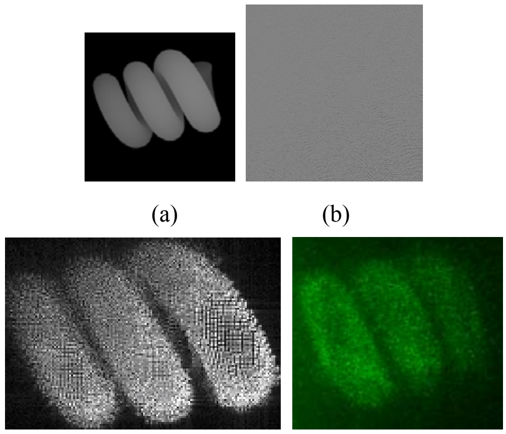


Fig. 4. Simulation result

5. CONCLUSION

This paper has proposed an algorithm that can increase the operation speed of the CGH algorithm used when the real object or virtual object is transformed into a digital hologram.

The proposed algorithm is a technique that can reduce the overall operation time by carrying out the CGH operation that was previously carried out on the whole coordinate array of the digital hologram covering one point of the 3D object only on the first coordinate, by recursively adding the previously calculated values to resultant values calculated from the first coordinate for the remaining coordinates. The proposed algorithm was compared with the previous method, and the results showed that the operation speed has increased by about 30%.



(c) (d)

Fig. 5. Example images; (a) 3D object, (b) generated digital hologram by proposed method, (c) reconstructed image using PC, (d) reconstructed image using optical system

ACKNOWLEGMENT

This work was supported by the Seoul Development Institute funded by the Seoul R&BD(NT080528) Program.

REFERENCES

[1] B. R. Brown and A. W Lohmann, Complex spatial filtering with binary masks, Applied Optics, vol. 5, pp. 967-969, June 1966.

[2] H. Yoshikawa, S. Iwase, and T. Oneda, Fast computation of Fresnel holograms employing difference, Proceedings of SPIE, vol. 3956, 2000.

[3] T. Ito, N. Masuda, K. Yoshimura, A. Shiraki, T. Shimobaba, and T. Sugie, Special-purpose computer HORN-5 for a real-time electroholography, Optics Express, vol. 13, no. 6, pp. 1923-1932, March 2005.

[4] S. Stein and L. T. Vegard, HoloVision, http://www2.edge.no/projects/index.php?expn