

## DC-DC 컨버터의 LMI기반 슬라이딩 모드 제어기 설계

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### LMI fuzzy based sliding mode control for DC-DC converter

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**Abstract** - Nowadays DC-DC converter has been used widely in electronic production. It has a high requirement in wide input voltage, load variations, stability, providing a fast transient response and lower overshoot. However, it is not easy to be controlled because of its nonlinearity. In this paper, the nonlinear model of DC-DC converter is approximated by four linear models and sub-controllers are designed by using the LMI guaranteeing the stability of the sub-systems at the same time. For the robust of the control system, an integral sliding mode control (ISMC) is applied together with LMI fuzzy controller. The proposed controller supports a fast and almost no overshooting transient response for the DC-DC converter control.

#### 1. 서 론

Generally, there are two kinds of DC-DC converter control ways. One is model based control and the other is non model based control. The model based control must use nonlinear control methods [1][2]. The non model based controllers of dc-dc converters are mostly fuzzy controllers [3][4]. Most of them have used a Mamdani type fuzzy controller which requires determining of the membership functions based on the experiences of an expert. In [3][4], simple fuzzy controller was designed but has no flexibility in its application. It is very conservative in its design and performance. The mathematical model of aDC-DC converter can be obtained easily. So, it is highly desirable to use the mathematical model of a DC-DC converter. The T-S fuzzy type controller is based on mathematical model and used with linear control theory under a stability condition. The T-S fuzzy combines the linear controllers under the stability condition using LMI. Although fuzzy LMI controller has the robust, the system's transient responses are still not good enough. LMI fuzzy control system, likes other fuzzy systems, is so dependent on rules which have already predetermined peak values of fuzzy sets. So that, if values of elements in fuzzy sets are out of these predetermined peak values, the system will work on unknown states. To solve this problem, in the traditional method, a filter which limited input signal in the upper and lower boundary is added. However, the added filter will introduce an additional disturbance in the real system and makes the transient response bad. In this paper, an ISMC is applied to overcome this problem. During the reaching phase, there is no guarantee of robustness in conventional sliding mode control. To overcome this disadvantage, ISMC is arisen and works well.

#### 2. 본 론

##### 2.1 LMI Fuzzy control

Consider a nonlinear system as follows.

$$\dot{x}(t) = f(x) + g(x)u(t) \tag{1}$$

Depending on the operating points, a nonlinear system can be expressed as follows.

The i-th model is the case that  $z_1(t)$  is  $M_{i1}$  and ... and  $z_p(t)$  is  $M_{ip}$

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \tag{2}$$

where  $i=1,2, \dots, r$  and  $M_{ij}$  is the fuzzy set and  $r$  is the number of model rules

Given a pair of  $(x(t), u(t))$ , the fuzzy systems are inferred as follows:

$$\dot{x}(t) = \frac{\sum_{i=1}^r w_i(z(t)) \{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^r w_i(z(t))} = \sum_{i=1}^r h_i(z(t)) \{A_i x(t) + B_i u(t)\} \tag{3}$$

$$y(t) = \frac{\sum_{i=1}^r w_i(z(t)) C_i x(t)}{\sum_{i=1}^r w_i(z(t))} = \sum_{i=1}^r h_i(z(t)) C_i x(t) \tag{4}$$

**THEOREM 1** The equilibrium of the continuous fuzzy control system described by (3) and (4) is globally asymptotically stable if there exists a common positive definite matrix  $P$  such that

$$\begin{aligned} G_{ii}^T P + P G_{ii} &< 0 \\ \left(\frac{G_{ij} + G_{ji}}{2}\right)^T P + P \left(\frac{G_{ij} + G_{ji}}{2}\right) &\leq 0 \end{aligned} \tag{5}$$

if and only if the  $i$ th rule and  $j$ th rule have no overlap where  $i < j$ .

Assume there is a stable state feedback controller  $u_i = -F_i X(t)$

$$u(t) = -\frac{\sum_{i=1}^r w_i(z(t)) F_i x(t)}{\sum_{i=1}^r w_i(z(t))} = -\sum_{i=1}^r h_i(z(t)) F_i x(t) \tag{6}$$

for the  $i$ th fuzzy rule.

Multiplying the inequality (5) on the left and right by  $P^{-1}$  and defining a new variable  $X = P^{-1}$  and  $M_i = F_i X$ , we rewrite the conditions as

$$-X A_i^T - A_i X + M_i^T B_i^T + B_i M_i > 0 \tag{7}$$

Now the problem changes to find the positive definite matrix  $X$  and  $M$  to satisfy the condition (7). This can be solved by LMIs by computer easily.

##### 2.2 LMI Fuzzy control for ideal DC-DC converter

The ideal boost converter circuit [4] is as following Fig.1.

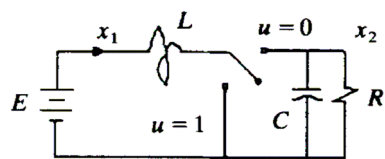


Fig. 1. The ideal boost converter circuit.

$$\begin{aligned}\dot{x}_1 &= -(1-u)\frac{1}{L}x_2 + \frac{E}{L} \\ \dot{x}_2 &= (1-u)\frac{1}{C}x_1 - \frac{1}{RC}x_2\end{aligned}\quad (8)$$

where  $x_1$  is the current,  $x_2$  is the output voltage.

For the equilibrium point  $x_{1r} = I_d$ ,  $x_{2r} = V_d$  and equilibrium point reference input  $r$ , substitute the totally control input  $u = u_0 + r$ ,  $x_1 = e_1 + x_{1r}$  and  $x_2 = e_2 + x_{2r}$  into (8), and then the error reference system as following:

$$\begin{aligned}\dot{e}_1 &= -(1-r)\frac{1}{L}e_2 + \frac{x_2}{L}u_0 \\ \dot{e}_2 &= (1-r)\frac{1}{C}e_1 - \frac{1}{RC}e_2 - \frac{x_1}{C}u_0\end{aligned}\quad (9)$$

$$z_1 = \frac{x_2}{L}, \quad z_2 = -\frac{x_1}{C}\quad (10)$$

If set (10), equation (9) can be rewritten as:

$$\begin{aligned}\dot{e}_1 &= -(1-r)\frac{1}{L}e_2 + z_1u_0 \\ \dot{e}_2 &= (1-r)\frac{1}{C}e_1 - \frac{1}{RC}e_2 - z_2u_0\end{aligned}\quad (11)$$

As the monotonic of  $z_1$  and  $z_2$  are same, so that, the membership function of the system can be simply written as:

$$\begin{aligned}M_1 &= \frac{z_1 - z_{1\min}}{z_{1\max} - z_{1\min}}, M_2 = \frac{z_{1\max} - z_1}{z_{1\max} - z_{1\min}} \\ M_3 &= \frac{z_{2\max} - z_2}{z_{2\max} - z_{2\min}}, M_4 = \frac{z_2 - z_{2\min}}{z_{2\max} - z_{2\min}}\end{aligned}\quad (12)$$

where  $z_{1\min}$ ,  $z_{1\max}$ ,  $z_{2\min}$  and  $z_{2\max}$  are the boundary of  $z_1$  and  $z_2$ . Equation (11) can be of the following form:

$$\dot{e} = \sum_{i=1}^r M_i(A_i \times e + B_i \times \sum_{j=1}^r M_j u_{0i})\quad (13)$$

$$A_1 = A_2 = A_3 = A_4 = \begin{bmatrix} 0 & -(1-r)\frac{1}{L} \\ (1-r)\frac{1}{C} & -\frac{1}{RC} \end{bmatrix}, \quad B_1 = [z_{1\max} \quad z_{2\max}],$$

$$B_2 = [z_{1\max} \quad z_{2\min}], \quad B_3 = [z_{1\min} \quad z_{2\max}] \quad \text{and} \quad B_4 = [z_{1\min} \quad z_{2\min}]$$

Using LMI toolbox calculates out controller parameters  $K$ . By LMI, the control input is defined by (6) as

$$u_{0i} = k_{1i}e_1 + k_{2i}e_2\quad (14)$$

Use inequality (7) and matlab LMI toolbox to calculate out the parameters  $k_{1i}$  and  $k_{2i}$  where  $i=1,2,3,4$ . In the following parts, the totally controller for nominal system is defined as  $u_n(t)$  and has the format like (6).

### 2.3 Combination of LMI fuzzy and ISMC

Based on the SMC matching condition the system with disturbance is as follows:

$$\begin{aligned}\dot{x}_1(t) &= -(1-u(t)-d(t))\frac{1}{L}x_2(t) + \frac{E}{L} \\ \dot{x}_2(t) &= (1-u(t)-d(t))\frac{1}{C}x_1(t) - \frac{1}{RC}x_2(t)\end{aligned}\quad (15)$$

where  $d$  is the disturbance.

The sliding surface is defined as:

$$S = x_2(t) - x_{2n}(t) + x_1(t) - x_{1n}(t)\quad (16)$$

where  $x_{1n}$  and  $x_{2n}$  are states of nominal system.

Assume  $u(t) = u_n(t) + u_s(t)$  and derivate of  $S$  is:

$$\begin{aligned}\dot{S} &= \frac{1-u_n(t)}{C}e_{1n}(t) + \left(\frac{u_n(t)-1}{L} - \frac{1}{RC}\right)e_{2n}(t) \\ &\quad + \left(\frac{x_2(t)}{L} - \frac{x_1(t)}{C}\right)(u_s(t) + d(t))\end{aligned}\quad (17)$$

where  $e_{1n}(t) = x_1(t) - x_{1n}(t)$ ,  $e_{2n}(t) = x_2(t) - x_{2n}(t)$ ,  $u_n(t)$  is the nominal control input and  $u_s$  is the sliding control input.

The sliding controller finally is given out as:

$$\begin{aligned}u_s(t) &= \left(\frac{CL}{Lx_1 - Cx_2}\right)\left(\frac{1-u_n(t)}{C}e_{1n}(t) + \left(\frac{u_n(t)-1}{L} - \frac{1}{RC}\right)e_{2n}(t)\right) \\ &\quad + d_{\max} \text{sign}(s)\end{aligned}\quad (18)$$

where  $d_{\max}$  is the maximal absolute value of the disturbance.

### 2.4 Simulation

Use the model of (9) and controller design process in the above section with the typical parameters:  $R=100\Omega$ ,  $L=330mH$ ,  $E=12V$ ,  $C=2200\mu F$ ,  $x_{1r} = I_d = 0.48A$  and  $x_{2r} = V_d = 24V$ . Based on SMC matching condition, a very big disturbance which is a sine waveform and the peak value is 727.27. Fig.2 shows the voltage output only using LMI fuzzy controller. Fig.3 shows the voltage output using LMI fuzzy based ISMC controller.

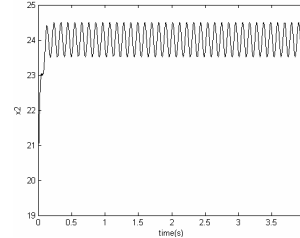


Fig. 2. The voltage output only using LMI fuzzy controller.

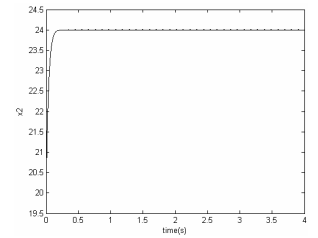


Fig. 3. The voltage output using LMI fuzzy based on ISMC

## 3. 결 론

In this paper, the Fuzzy LMI based ISMC controller is used for DC-DC power converters. LMI fuzzy solved the initial big input for DC-DC converter from ISMC, while ISMC solved the problem of LMI fuzzy which is so dependent on fuzzy rules. The final results show that the combination control is efficient and perfect.

### [참 고 문 헌]

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