Application of Kirchhoff Transformation Algorithm to Simulating the Groundwater Flow System at Yucca Mountain

S. H. Ji, Y. J. Park*, E.A. Sudicky*, Y. K. Koh
Korea Atomic Energy Research Institute, 1045 Daedeokdaero, Yuseong-gu, Daejeon, 305-353, Korea
*University of Waterloo, 200 University Avenue West, Waterloo, ON N2L 3G1, Canada
shji@kaeri.re.kr

1. Introduction

Describing variably saturated groundwater flow that is generally represented by Richards' equation still remains a challenge because it is difficult to solve it numerically for many realistic physical conditions. This numerical difficulty in solving Richards' equation originates from its unique mathematical characteristic and strong nonlinearity through the constitutive relations among the fluid pressure, saturation and relative permeability. When a steady-state variably saturated groundwater system is simulated, solving Richards' equation becomes even more troublesome because the difficulty of obtaining good initial guess is increased in steady-state problems [1]. For this reason, a steady-state solution of Richards' equation, which is critical as the starting point for subsequent transient simulations for a real-world application, is usually approximated from marching out through transient simulations, and thus most of the simulation time is employed in the calculation of a steady-state fluid flow field.

We developed a methodology to accurately and efficiently compute a direct steady-state solution for Richards' equation. The suggested numerical approach solved a nonlinear Richards' equation through a linearization of a steady-state Richards' equation by using the Kirchhoff transformation and a complimentary transformation, and a discretization of it using a control volume finite element/finite difference approach. Then, it was verified by a comparison with analytical solutions and applied to simulating the groundwater flow system at Yucca Mountain area.

2. Kirchhoff Transformation

The equation governing a variably saturated flow in steady state is given by Richards' equation,

$$-\nabla\cdot\left(\overline{\overline{K_s}}k_r\cdot\nabla(\psi+z)\right)+Q=0, \qquad (1)$$

and the boundary conditions for the solution of equation (1) are given by:

$$\psi=\psi_0 \qquad on\ \Gamma_1$$

$$\overline{q}=-\overline{\overline{K_s}}k_r\cdot\nabla(\psi+z)=\overline{q_0} \qquad on\ \Gamma_2.$$

To solve equation (1), constitutive relations are required that relate the primary unknown ψ to the secondary variables S_w and k_r. The van Genuchten-Mualem (VM) and Brooks-Corey (BC) constitutive relations have been widely accepted but Gardener-Russo (GR) relations have been also adapted in many analytical and numerical approaches for its simplistic nature. If we assume x, y, z in the Cartesian coordinates are in the principal direction, applying the Kirchhoff transformation, $\varphi(\psi)=\int_{-\infty}^{\psi}k_r(s)\,ds$ to equation (1) yields:

$$-\nabla\cdot\left(\overline{\overline{K_s}}\cdot\nabla\varphi\right)-\frac{\partial}{\partial z}(v_\varphi\varphi)+Q=0 \qquad (2)$$

subject to the Kirchhoff-transformed boundary conditions:

$$\varphi=\varphi(\psi_0) \qquad on\ \Gamma_1,$$

$$\overline{q^*}=-\overline{\overline{K_s}}\cdot\nabla\varphi-(v_\varphi\varphi)_z=\overline{q_0} \qquad on\ \Gamma_2,$$

where, $v_{\varphi} = K_s k_r / \varphi$ that is introduced by a complementary transform to reduce the nonlinearity of the discrete matrix equation $A(\Phi)\Phi = b(\Phi)$ from equation (2). Then, the transformed equation (2) is discretized by using a control volume finite element method that can be evaluated by using different numerical approaches such as a simple finite difference approximation [2].

3. Results

The accuracy and the performance of the proposed approach were evaluated by comparing our solutions to transient numerical solutions and analytical solutions in 1- and 3-D domains. which clearly shows that the numerical solution by the transformation approach is more accurate than those by the transient simulation approaches and that it is also consistently more efficient. Moreover, the results indicate that the proposed approach is even more efficient in the heterogeneous soils than the homogeneous soils, because the Newton-Raphson iterative linearization requires a smaller time step size to converge in the heterogeneous systems.

Then, the proposed approach was applied to solving a steady-state regional groundwater flow system at Yucca Mountain area in Fig. 1. Fig. 2 shows the results.

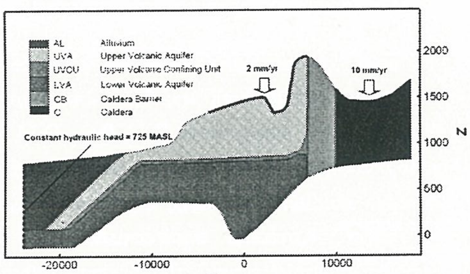

Fig. 1. Simulated domain

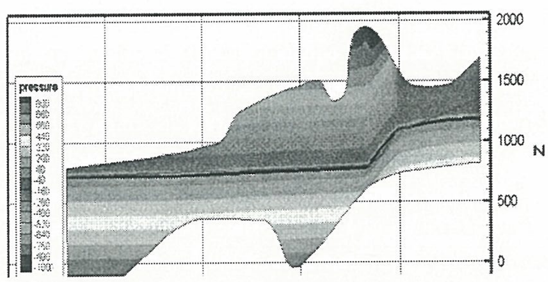

Fig. 2. Calculated hydraulic head distribution

Its accuracy and performance were compared to a transient simulation approach. The simulated domain consists of six hydrogeologic units and constitutive relations are described by the VM model. The results are that the simulation time is 218 CPU seconds for the transformation approach and 1.40×10^{6} CPU seconds for the transient simulation approach even though the simulated pressure head and saturation distributions for both approaches are similar.

4. Conclusion

Our numerical experiments show that the Kirchhoff transformation approach to simulate the steady-state variably saturated groundwater flow produces more rapid convergence and smaller errors than the classical transient simulation approach. Especially, when the heterogeneity of the domain and/or the nonlinearity of the constitutive relations are strong the Kirchhoff transformation approach shows more efficient performace. Considering that the suggested Kirchhoff transformation approach is irrelevant to the quality of initial conditions, it has the potential to lead to more robust solutions of Richards' equation for real-world problems with large scale.

REFERENCES

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